PRINCIPLES OF BEHAVIORAL ECONOMICS

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Week 49

Lecture 49

Hello everyone, today we are going to discuss naive quasi-hyperbolic discounting. As you know, currently we are discussing intertemporal choice models, and under that, first we discussed the discounted utility model or exponential discounting, and then we discussed naive hyperbolic discounting. So, before I start discussing the alternative, which is naive quasi-hyperbolic discounting, I will begin with an example to show how the previous ones that we have already learned—the naive hyperbolic discounting and exponential discounting—

how do they differ? We will show it using an example with some numbers. So, suppose we assume that for all periods, the consumption is equal to 200, and the utility generated by the amount of consumption is also equal to 200. This is a simplistic assumption. Besides that, if you remember,

the discount factor for naive hyperbolic discount denoted by ht was 1 plus alpha t raised to the power minus beta upon alpha. So, we are assuming that alpha equals to 0.5, beta equals to 0.6 and in the exponential discounting model, we had the discount factor delta. We are assuming two alternative values of delta at 0.8 and 0.6. After that, we are calculating the stream of utilities over a period of, say, 10 periods for alternative discount factors. So, this is first the hyperbolic discounting factor.

When I have t equal to 0, then you can understand that 1 plus alpha t becomes equal to 1. When t equals 1, then we have 1 plus alpha raised to the power minus beta upon alpha. For given values of beta and alpha, we have this value of ht. Similarly, when t equals 2, then we have 1 plus alpha 2t

1 plus alpha 2t, sorry, 1 plus 2 alpha raised to the power minus beta upon alpha. And then for values of alpha and beta, which are already given, I can calculate the value of ht when t equals 2. In a similar fashion, all these numbers are calculated. And then how do we arrive

at the utility? It is simply 200, which is assumed to be constant, multiplied by the discount factor.

So, when it is 200, 1 multiplied by 200 is 200. 200 multiplied by 0.603682 is this number, and so on. In a similar fashion, we have also calculated the utilities associated with the discounting factors like when delta equals 0.8 and delta equals 0.6. We are calling them delta 1 and delta 2, just giving two names. And again, multiplying delta by the discount factor multiplied by the utility.

Is going to give me the stream of utilities and the stream of, you know, future discounts, right. So, here you can see that when t equals how we discount under the exponential discounting, that is pretty simple. We have delta, then delta, it is delta raised to the power i for i taking any number. So, when i is equal to 0, we have 1, When i, that is the time period, equals to 1, then we have delta equals to 0.8; t equals to 2, that is i equals to 2, then delta squared gives us 0.64, and so on.

In a similar fashion, I have for delta 2 equals to 0.6, then these discounting factors multiplied by the utilities which are constant at 200, gives us the stream of future utilities discounted to the present value. So, what these numbers as such do not show much, but if we plot them, then we can see how they are different. So, the red line is basically the exponential discounting with delta equals to 0.8. The green line is the exponential discounting with delta equals to 0.6, and in between lies the hyperbolic discounting.

Now, what broadly it shows is that in case of exponential discounting, you can see the lines are smooth, in the sense here it is declining at a constant exponential rate. Similarly, this is also the case. But when it comes to hyperbolic discounting, then the initial drops are larger. After that, the declines are much less as compared to this exponential discounting with delta equals to 0.6.

Now, the difference between having delta equal to 0.8 and 0.6 is mentioned here. When delta equals 0.8, given that delta equals 1 plus r, or we can also call it rho. So, then the discount rate is actually much smaller. It is 0.25 when delta equals 0.8 and 0.67 when delta equals 0.6. So, when it is smaller, we say that the discounter is actually very patient.

So, as a result, you can see there is a more equal spread of consumption over a large period of time. Or the line is much smoother compared to when delta is smaller at 0.6, which implies that the discount rate is actually higher. And hyperbolic discounting actually lies between the two when it is beginning, this is matching with a high discount rate, which implies that this exponential hyperbolic discount in the beginning is much higher. The discount rates in the beginning are much higher, but after that, they reduce the discounting factor or discounting rate.

As a result, over time, they actually converge with the exponential discounter having a lower discount rate. So, if you remember the example with Ted and Matthew, what Ted was doing—he was an exponential discounter, discounting at a rate of 10% constantly—while Matthew initially discounted at a much higher rate of 30%. And then later on, he was discounting it at 10% for the second period and for the other periods, though other periods did not matter in that example, but then he was discounting them at 0%.

So, if he starts discounting altogether after the second period, then the line becomes something like this for all other periods. It is like constant, right? So, this diagram basically explains the difference between exponential discounting and naive hyperbolic discounting. Next, we will talk about naive quasi-hyperbolic discounting. One of the primary advantages of the exponential discounting function is the simplicity with which

exponential discounting can be used in solving maximization problems. So, basically, the mathematical exposition is much simpler compared to hyperbolic discounting. By contrast, the hyperbolic discounting model can be difficult to deal with mathematically, given the functional form of the discount factor. This has led David Laibson to propose an approximation to the hyperbolic discounting function called quasi-hyperbolic discounting. Quasi-hyperbolic discounting separates the hyperbolic profile of time discounting into two different discount factors.

These two factors represent the discount applied to the utility of consumption in the second period and the discount applied to the utility of consumption for each additional period. The discount factor applied to the second period is small relative to the other discount factor, representing the notion that people discount consumption tomorrow relative to today more heavily than the day after tomorrow relative to tomorrow. So, basically, the initial time period discounting is much larger. In other words, any consumption in the future receives some penalty in the mind of the consumer.

But trading consumption between two different periods in the future does not face such a steep penalty. Thus, the model suggests this is again total utility from the current period to any number of periods: C0, C1, C2. That is consumption from the current period to any number of periods. So, the first period utility is given by the consumption in the first period, U(C0). This is the current.

$$U(c_0, c_1, c_2 \dots) = u(c_0) + \beta\delta u(c_1) + \beta\delta^2 u(c_2) + \beta\delta^3 u(c_3) + \cdots + \beta\delta^i u(c_i) + \cdots$$

So, we do not need to discount it. But from the second period onwards, two discount factors are introduced. One is beta, which remains constant for all future periods. You can see. And the second factor is, the second discount factor is delta, which keeps on increasing in power.

So, in the first period, it is delta raised to the power 1. In the second period, delta raised to the power 2. In the third period, delta raised to the power 3. And so on. Now, as you can see, for any value of delta less than 1, as we keep increasing the power of delta, this number becomes smaller and smaller.

That is additionally multiplied by one discount factor, making it even smaller. You know, this is how I would say rather naive quasi-hyperbolic discounting. This is how I would say that the naive quasi-hyperbolic discounting provides an approximation to naive hyperbolic discounting because, as we said, in the immediate future, the discount rates are higher as compared to as we proceed to later periods. So, from the first period to the second period, the discount rate is actually larger as compared to when we move further because

then these multiplications, multiplied by the constant discount factor, make them actually very small, and the initial drops are relatively larger. So, this can alternatively be written as u c naught i take it out plus the rest of them are summed up from i equals to 1 to t beta delta raise to the power i u c i.

$$0 < \beta < \delta < 1$$

In equation 1, 0 less than beta, less than delta, less than 1. So, of course, we understand that most often delta is less than 1. We will also discuss possibilities of delta greater than 1, but much later, maybe towards the end of the next module.

But more importantly, beta is less than delta. And if we want to consider the infinite time horizon problem, then t may be infinite. Here beta represents the discount applied to utility of consumption in the second period and in all future periods and delta represents the discount applied to utility of consumption for each additional period as we move further into the future. In general, if beta is less than delta, the function approximates hyperbolic discounting.

So here in this figure, this actually refers to the exponential discounting. The triangles, as mentioned here, are marked in triangles. The triangles represent quasi-hyperbolic discounting, and the squares represent hyperbolic discounting. So, you can see that, of course, quasi-hyperbolic discounting, as we have already mentioned, begins with a much larger increase in the discount rate. So, the immediate future experiences a sharper drop, but exponential discounting actually represents a much smoother consumption spread over a period of time.

Now, quasi-hyperbolic discounting is actually pretty close to hyperbolic discounting. The advantage of this form is that it closely replicates the exponential mathematical form, thus restoring the simple mathematical formulas for time-discounting problems. So, as you can see, quasi-hyperbolic discounting has in terms of mathematical exposition, a lot of similarities with exponential discounting, but as a result, we have some advantages while dealing with mathematical exposition. But at the same time, in terms of its structure, it actually resembles quasi-hyperbolic discounting.

The solution to the utility maximization problem for Equation 1, which we represented earlier, again requires that discounted marginal utility should be equal in each period. However, the differential discount implies for i greater than 1, which is like u prime c i equals beta delta raised to the power i u prime c i. Basically, for the first period or the current period, you have u c naught, so the marginal utility from the current period is u prime c naught. For the rest of the periods, for i greater than or equal to 1, you have this expression.

$$u'(c_i) = \beta\delta^i u'(c_i)$$

Thus, given a functional form for the instantaneous utility function, we can find the relationship between consumption in one period and the next. For example, suppose that people must maximize their utility of consumption over an infinite time horizon. given an initial endowment of wealth w. Suppose further that this is our utility function u c equals to c raised to the power alpha. So, that the marginal utility or slope of the utility function of consumption is given by u prime c equals to alpha c raised to the power alpha minus 1 Now, you can see that this utility function is actually not dependent on time as of now.

So u c represent any period where c raise to the power alpha is the functional form of the utility function. Then, the optimization condition implies that the optimization condition is like we equate the marginal utility from each period with each other. So, this is the very first period. Now, the current period also has a utility of c raised to the power alpha.

As you can see, to begin with, it did not attach any time subscript to the functional form of utility. So, we have alpha c naught alpha raised to the power alpha minus 1.

$$u'(c) = \alpha c^{\alpha - 1}$$

Basically, in this expression, we are just putting the times in the subscripts. But for the next period onwards, that is, from the second period onwards, I will be having two discount factors, beta and delta.

So, for the first period, I have alpha, beta; alpha is coming from here, coming from here; beta, delta remains as it is; c1 raised to the power alpha minus 1. For the second period, my discount factor is beta delta squared. So, again, alpha comes from here, then beta delta squared, and then C2 alpha raised to the power alpha minus 1, and so on. So, this can be alternatively written as we simply multiply or rather put a power of 1 upon alpha minus 1.

$$\alpha c_0^{\alpha-1} = \alpha\beta\delta c_1^{\alpha-1} = \alpha\beta\delta^2 c_2^{\alpha-1} = \cdots = \alpha\beta\delta^i c_i^{\alpha-1} = \cdots$$

$$\alpha^{\frac{1}{\alpha-1}} c_0 = (\alpha\beta\delta)^{\frac{1}{\alpha-1}} c_1 = \cdots = (\alpha\beta\delta^i)^{\frac{1}{\alpha-1}} c_i = \cdots$$

So, this cancels out, and we are left with alpha raised to the power 1 upon alpha minus 1. Similarly, for all those components which did not have a power, they will now have a power of 1 upon alpha minus 1. Those which had a power of alpha minus 1, the power would be gone. So, these are simple transformations. This implies that C1, that is the consumption in period 1, can be expressed as C naught beta delta raised to the power minus 1 upon alpha minus 1.

$$c_0 + c_1 + c_2 + \cdots = w,$$

And that, in general—so if we go for generalization of this expression—it is like the current period consumption multiplied by beta raised to the power minus 1 upon alpha minus 1, then delta raised to the power minus i upon alpha minus 1. So, here, since it was for the first period, beta delta both had the same power, but when you go for the second period, third period, then delta will have a slightly different power. Now, this is basically the first equation just represented here so that we can match The budget constraint implies that all consumptions spread over all the periods must add up to the total wealth one had begun with.

$$c_i = c_0 \beta^{-\frac{1}{\alpha-1}} \delta^{-\frac{i}{\alpha-1}} \quad \text{for } i = 1, 2, ..$$

$$\alpha^{\frac{1}{\alpha-1}} c_0 = (\alpha\beta\delta)^{\frac{1}{\alpha-1}} c_1 = \cdots = (\alpha\beta\delta^i)^{\frac{1}{\alpha-1}} c_i = \cdots$$

Substituting from above, we have C naught—basically, my consumption streams are like C naught plus C1 plus C2. So, C naught, then C1 would be plus this; C2 would be again, you know, from this expression, I can obtain the value of C2 which would be like C2 equals to C naught beta raised to the power minus 1 upon alpha minus 1 delta raised to the power minus 2 upon alpha minus 1.

So, we keep on adding for each and every period. Then we take C naught common out and then we will be left with this expression that must add up to the right-hand side, which is W. So, by applying the formula for the sum of the infinite GP series, we can derive the closed-form solution as this. So, this is the left-hand side of the equation we showed in the last slide. And this is basically the infinite GP series, which is summed up, and that formula is applied.

$$c_0 \left(1 + \beta^{-\frac{1}{\alpha-1}} \sum_{i=1}^{\infty} \left(\delta^{-\frac{1}{\alpha-1}} \right)^i \right) = w$$

So, we have this thing: 1 upon 1. 1 minus delta raised to the power minus 1 upon alpha minus 1 is the sum of the infinite GP series. We have in the numerator beta raised to the power minus 1 upon alpha minus 1. Everything multiplied by C naught equals the right-hand side, which is the total wealth. So, the idea is that all other periods' consumption could then be calculated from the above formula.

So, once I know the values of beta, alpha, delta, and the initial consumption, I can actually calculate the consumption of all other periods 2, 3, 4, 5, up to infinity. This is one major advantage of the quasi-hyperbolic discounting over hyperbolic discounting. Because in the case of hyperbolic discounting, what happens is that, suppose we model the same decision using a hyperbolic discount function, you would arrive at the following. So, in the case of hyperbolic discounting, we will have this expression.

$$c_0 \left[1 + \sum_{i=1}^{\infty} (1 + i\alpha)\right]^{-\beta/\alpha} = w$$

Now, you can see that this is not a closed-form solution because here this is dependent on the time period. So, for each and every period, we have to actually calculate separately. This cannot yield a closed-form solution. So, that is what I was trying to tell you: this is the advantage of using quasi-hyperbolic discounting over hyperbolic discounting.

That is why the quasi-hyperbolic form is much more common in practice than the hyperbolic form and In several books, you would find that quasi-hyperbolic discounting is referred to as hyperbolic discounting, and there is no further mention of hyperbolic discounting. As in many situations, economists use the quasi-hyperbolic approximation rather than the hyperbolic form. In fact, the quasi-hyperbolic form is much more common in practice than the hyperbolic form that it approximates. Uri Benzion, Amnon Rapoport, and Joseph Yagil find evidence of changing discount rates over time.

They asked 2,204 participants about their preferences between receiving bundles of money after various waiting periods. So, for example, one question asked participants to suppose they had just earned $200 for their labor. But after coming to pick up the money, they found their employer was temporarily out of funds. Instead, they were offered payment in six months. Participants were asked how much they would need to be paid at the later time to be indifferent between receiving $200 now or the higher amount later.

The amounts of money and the length of time were varied. So, this is an example of the results from that experiment. Basically, they actually mentioned alternative amounts. The amounts are 40, 200, 1000, and 5000. The same situation.

	Time Delay			
Amount	6-months	1 year	2 years	4 years
$40	0.626	0.769	0.792	0.834
$200	0.700	0.797	0.819	0.850
$1000	0.710	0.817	0.875	0.842
$5000	0.845	0.855	0.865	0.907

You want it now. Or you want it after a period—if you want it after a delay, after some period—and those periods are like six months, one year, two years, and four years. Then, how much more you would need—that was the question that was asked to them. The table displays the estimated discount factors for each of the various scenarios, similar to the scenario just described, assuming a money matrix utility function. What we observe is that for all possible timelines, the initial discount rates are actually much higher.

And after that, they do not increase by a substantial amount. After that, the discount rates, of course, increase, but then the increases are smoother. For each, we see a relatively large discount factor for the first six months of delay. After that, they become much smoother. So discounts for longer periods are much smaller, reflecting the hyperbolic nature of discounting.

If a person must delay a reward for some time, longer waits are no longer thought of as so costly. Similar experiments have been run with actual money payouts, finding additional evidence that discount factors climb over time, eventually becoming stable. Let us take an example. Suppose that you are on a diet but have to decide whether to have a slice of red velvet cake at a party some random Saturday. Eating the cake would give you a utility of 4.

If you have the cake, however, you will have to exercise for hours on Sunday, which would give you a utility of 0. We call this option C. The other option is to skip the cake, which would give you a utility of 1. And to spend Sunday relaxing in front of the television for a utility of 6, we call this option D. Thus, you are facing the choice between C and D. You discount the future hyperbolically, or rather quasi-hyperbolically, with beta equal to half and delta equal to two-thirds. So, the first question is: from the point of view of Friday, what is the utility of eating the cake and of skipping it?

Option C: $\beta\delta \times u$(eat the cake) + $\beta\delta^2 \times u$(exercise on Sunday) = 0 + $\beta\delta$ 4 + $\beta\delta^2 \times 0$ = (1/2) ×(2/3) × 4 = 4/3

Which would you prefer? So, I have two options. From Friday's perspective, option C—that I eat the cake and then exercise on Sunday—is like beta delta because Saturday is tomorrow (today is Friday), beta delta multiplied by utility from eating the cake, then beta delta squared (which is Sunday, the second day). That's why I have a delta squared utility from exercising on Sunday. On Friday, nothing is happening, so we assign a zero utility, then beta delta multiplied by 4 (the utility from eating the cake).

Beta delta squared multiplied by 0—the disutility or utility from exercising on Sunday. These numbers were given in the previous slide. And what we observe is that, given that beta equals half and delta equals two-thirds, I have 4/3 utility from option C. Now, talking about option D. I can skip—or you can skip—the cake and then relax on Sunday. Again, multiplied by tomorrow's discounting factor, multiplied by the day after tomorrow's discounting factor.

So, again for Friday, we have a 0. Then beta delta multiplied by 1. That is the utility from skipping the cake, plus beta delta squared multiplied by 6. The utility from relaxing in front of the television on Sunday, and that total amount turns out to be 5/3. So, since 5/3 is greater than 4/3, you decide to skip the cake on Friday.

So, next when Saturday arrives, the question is, from the point of view of Saturday, what is the utility of eating the cake and of skipping it? Which would you prefer? So, when

Saturday arrives, option C is eat the cake, and since it is Saturday, we do not have to discount it. This is the current period.

Sunday will be discounted with a discounting factor beta multiplied by delta, and you exercise on Sunday. So, eating the cake has a utility of 4, then beta delta multiplied by 0, the utility from exercising on Sunday; your total utility is 4. Option D: you skip the cake multiplied by beta delta, plus beta delta multiplied by relaxing on Sunday. So, when you skip the cake, you get a utility of 1, beta delta multiplied by relaxing on Sunday.

Option D: $\beta\delta \times u(\text{skip the cake}) + \beta\delta^2 \times u(\text{relax on Sunday}) = 0 + \beta\delta \times 1 + \beta\delta^2 \times 6 = (1/2) \times (2/3) + (1/2) \times (2/3)^2 \times 6 = 5/3$ - **You decide to skip the cake.**

That is 6 units of utility. And this gives me a total utility of 3. Now, 3 being less than 4, you decide to eat the cake. So, what it shows is that the behavior here is time-inconsistent. So, this probably shows that the time-inconsistent behavior is well explained by a model like quasi-hyperbolic discounting.

Hyperbolic discounting is also able to capture it, but we have already explained that. The advantage lies with quasi-hyperbolic discounting, and that is how time-inconsistent or not-so-rational behavior is viewed from the perspective of neoclassical economics. The quasi-hyperbolic discounting model is able to explain certain preference reversals or time-inconsistent behavior. With this, I conclude this module on the introduction of naive quasi-hyperbolic discounting.

Next, we will be discussing some behaviors that are neither explained by exponential discounting nor by behavioral models of intertemporal choice. Thank you.