PRINCIPLES OF BEHAVIORAL ECONOMICS

Prof. Sujata Kar

Department of Management Studies
IIT Roorkee

Week 48

Lecture 48

Hello everyone, so I was discussing different time-inconsistent behaviors in the previous module. This is the 48th lecture of the course on Principles of Behavioral Economics, and I am going to continue with such discussions while introducing the concept of naive hyperbolic discounting. So we begin with an example again to show time-inconsistent behavior among individuals. And then present a model which may try to explain those behaviors that are not actually explained by DUM or where DUM is incapable of explaining them. The food stamp program was originally introduced in 1939 as a way to alleviate hunger in the United States.

Known as the Supplemental Nutrition Assistance Program, or in short, SNAP. The program provides each participant with a card that can be used to purchase food, much like a debit card. Recipients receive a monthly transfer of money to their account, generally on the first of the month. Thereafter, they can use that money to buy food. Close to 50 million Americans collect SNAP benefits to pay for at least part of their food budget, with a cost of over $75 billion per year.

The average food stamp recipient does not spend any of her other income, in addition to the SNAP benefit, each month on food. This finding was originally considered a sign of an effective program when discovered by economists. Suppose that recipients receive some amount of money, say S, in SNAP benefits that must be spent on food. If this then resulted in recipients spending exactly S on food, the restriction on spending of SNAP benefits is clearly binding, and the recipients would have been better off if they had just been given cash to spend on anything.

Alternatively, if they spend K, which is greater than S, on food, the rational model implies that recipients could not be made better off by giving them cash instead. Because whatever is being transferred, they are already spending more than that. So if they had been provided cash, they would also spend it on food only and not on anything else. Suppose a rational person, considering her consumption over the month, would solve equation 3.

$$max_{\{x,c_0,....c_{30}\}} v(x) + \sum_{t=0}^{30} \delta^t u(c_t)$$

This is actually equation 3, where basically this is consumption over a period of 30 days; x is

the consumption or purchases of all other commodities—all commodities other than food. So, VX is the value generated by consumption of those goods and services, and UCT is the utility generated by consumption of food. They are summed over a period of 0 to 30 days, and each utility is discounted by the discount factor raised to the power of the time period. So, here, X represents all non-food consumption. VX represents the utility of non-food consumption irrespective of time within the month.

Ct represents consumption of food at time t, and Uc is the instantaneous utility of consumption of food per day. The consumer is given income of W, so an income of W is there and SNAP benefit of S must solve equation 3, the equation that we just discussed in the previous slide, subject to two constraints, constraint 1 and constraint 2.

$$\sum_{t=0}^{30} c_t \geq s$$

$$x + \sum_{t=0}^{30} c_t \leq s + w$$

The constraints are, first of all, the consumption of food over a 30-day period should be greater than or equal to the SNAP benefit represented by S. And the second constraint—so this is my constraint 1, and this is constraint 2.

The second constraint is that an individual's overall expenditure—so expenditure on X, and this is the expenditure on consumption— must be less than or equal to the SNAP benefit plus the income that he or she receives, W. This is to satisfy the constraints imposed by SNAP. So, basically, an individual must solve equation 3 subject to C1 and C2 to satisfy the constraints imposed by SNAP. The constraints C1 are reproduced here.

So, C1 tells us that the recipient must consume at least her SNAP benefit in food. If she is spending more money than just the SNAP allotment on food, then the constraint is not binding. In the sense that whether we have this constraint or not, it actually does not matter. The constraint is not binding.

That is the case when Ct is greater than S. The constraint in equation 2, that is C2, tells us that she cannot spend more money than she has on hand for food. If instead the government simply gave the recipient S in cash that could be used for anything, the recipient must solve equation 3 subject to equation C2, eliminating the food restriction contained in equation C1 because, as we are trying to say, this is not binding. But if C1 wasn't binding under the SNAP program, then the cash transfer results in exactly the same optimization problem as the SNAP program.

Because the individuals are spending as much on food as they receive from the SNAP program, and sometimes even more. So if it is not binding, then giving them cash or providing the amount in terms of cards that must be spent only on food would yield the same results. Thus, the recipient would consume the same amount of food and be just as well off as if she had received a cash transfer. Alternatively, if the recipient spends no more than S on food under the SNAP program, then C1 is binding, and the recipient would behave differently if given only a cash transfer. This added restriction on the solution must result in a lower level of utility than if the recipient were simply given cash.

If this were the case, the recipient could be made better off with a smaller cash transfer, making the policy inefficient. The US government is concerned with the rising number of SNAP participants who are overweight or obese. Encouraging them to eat more than they would with a simple cash transfer could be viewed as counterproductive. Thus, economists viewed the news that recipients spent some of their own money on food in addition to the SNAP benefit as good news. Recipients must be as well off as they would have been with a simple cash transfer, and we are not encouraging excess consumption, which might lead to obesity or overweight problems.

The problem began when someone decided to explore how a recipient would react to a cash transfer equal in value to the SNAP benefit. So basically, for some regions, an experiment was conducted where, instead of giving the SNAP benefit, people were given an equivalent amount of cash transfer. In 1989, the US Department of Agriculture conducted an experiment in which they randomly assigned households in Alabama and San Diego, California, to receive either the traditional SNAP benefit or an equivalent amount in cash. Although the differences were small, on average, those receiving the SNAP benefits consumed about 100 to 200 more calories than those receiving cash-despite both groups spending. Although the differences were small, on average, those receiving SNAP benefits consumed about 100 to 200 more calories than those receiving cash, despite both groups spending in excess of the benefit on food. So both groups spent more than the benefit they received. But the SNAP group actually consumed more calories.

These two facts together contradict the standard economic model of choice. Later work by Parke E. Wilde and Christine K. Ranney shed more light on the puzzle. They found that for the period they studied, spending on food in the first two days after receiving the food stamp benefit spikes to about $5 per person per day. Spending for the rest of the month hovers around $2 per person per day. So in the initial days, they would spend more, and later on, they would spend less.

Moreover, they find that for a large portion of those receiving food stamps, the calories consumed per day drop more than 10% in the last week of the month relative to all other weeks, which means that food consumption or calorie consumption declined in the last week of the month. If we consider modeling equation 3, this is equation 3, which is reproduced here, and the two constraints we have talked about, C1 and C2, we should find a solution that satisfies delta raised to the power j, u prime c j equals delta raised to the power k u prime c k. We introduced the Euler equation yesterday.

$$\delta^j u'(c_j) = \delta^k u'(c_k)$$

If you remember, for two consecutive periods, it was like u prime c t,

where u' refers to the first derivative of the utility function with respect to consumption in period t and u' c t plus 1. We considered the ratio of them, and that was equal to 1 upon 1 plus r. Alternatively, if we consider a greater number of periods, like if we have this is ct and this is ct plus 3, then we might have here delta squared because 1 delta would be here and delta cubed would be here. So, something cancels out. Ideally, this is the marginal rate

of this is the marginal utility of consumption in period j, and this is the marginal utility of consumption in period k. So, the neoclassical analysis tells us that the marginal utility of consumption should be the same across all periods. Accordingly, we have this equal sign. Here, as I already told, u' c is the instantaneous marginal utility or slope of the instantaneous utility function evaluated at c. People tend not to discount too much over short periods of time, like a single day.

So, delta should be close to 1. In this case, equation 4—this was our equation 4—says that for any two periods j and k, if they are relatively close together, cj should be relatively close to ck. Suppose j is less than k; the farther they are from each other in time, the greater will be the consumption in period J relative to period K. So, more consumption in the recent period compared to a further period.

This is commonly referred to as consumption smoothing, in the sense that when they are close by, there is not much difference. Differences are observed only when the time periods are far apart. So, here we introduce this line. This is the usual utility function, and what it shows is that if the discount factor by day is almost equal to 1. The marginal utility function for one day should be almost identical to that of the previous day.

So here we have this is d raised to the power minus jk, and this is d raised to the power minus 1k. So the slopes are pretty close, and the consumption levels are also pretty close on those two days. So this leads to an optimal level of consumption that is almost identical. So these two consumption levels are very close to each other. Here, the curve you see represents the instantaneous utility function with a familiar increasing but concave form.

This kind of utility function was introduced while discussing prospect theory. This is only in the domain of gain, so we do not have the lower portion. The solid line sloping upward has a slope delta raised to the power minus jk, where k is some constant representing the slope at the optimal time, optimal level of consumption in period 0. So, this is the line slope delta raised to the power minus jk. This line is tangent to the utility curve at one point cj, representing

the optimal consumption at time period j. Optimal consumption in the next period occurs at the line when the slope delta raised to the power minus j minus 1 k is represented on the graph by c plus c j plus 1. Given the slight change in slope between these two lines, the amounts of consumption should be rather close together. Alternatively, n periods after period j, so this is like delta raised to the power minus j minus nk, which may be very different from consumption in period j. j. So, you can see that the tangency is here.

So, this is quite different from the consumption in period either j minus 1, j plus 1, or period j. In any case, consumption from one period to the next should not change much. That's the basic point here. It should decline somewhat, but the rate of decline should create a smoothly declining function. Alternatively, in the SNAP data, we observe a sharp decline in food consumption in the last week.

This signals yet again that the behavior contradicts the standard economic model of choice. The rational model suggests that recipients will have the foresight to conserve their money to consume similar amounts every day. Rather than consume a lot up front and then run out of food and money in the last week of the month. So this is basically not for very rational individuals. Rational individuals would actually conserve their money to consume similar amounts every day.

A time-consistent model of decision-making would not predict this behavior, nor can it be reconciled. So that is why we bring in the concept of naive hyperbolic discounting. So we would be introducing or always having this term 'naive' in the sense that This hyperbolic discounter—hyperbolic discounting—has already been introduced in the last module. So these hyperbolic discounters are actually naive.

So they do not know that they are making irrational decisions or maybe making decisions that are not considered to be perfect or rational by some understanding. Robert H. Strotz first proposed a model of time-inconsistent preferences due to discounting that varies by the time horizon. Generally, he examined cases in which discount factors delta in the near term are relatively small, indicating that consumption now is much more valuable than in the near future. However, discount factors in the distant future are much closer to 1.

George W. Ainsley proposed a model that with some later modifications is commonly called hyperbolic discounting which replaces delta t, the discount factor or delta raised to the power t, with a hyperbolic discounting factor where the discount factor is written as 1 plus alpha t raised to the power minus beta upon alpha. where alpha and beta are both greater than 0.

$$h(t) = (1 + \alpha t)^{-\left(\frac{\beta}{\alpha}\right)}$$

The consumer maximizes here consumption stream C1 to C2 to any number of days. The total utility generated from the consumption that is equal to this is the discount

factor multiplied by the utility associated with each time consumption. Here, i equals 1 to t. So, as you can see, for i equals 1, I will have 1 plus alpha raised to the power minus beta upon alpha. If i equals 2, then my discount factor will become 1 plus 2 alpha raised to the power minus beta upon alpha, always multiplied by the utility. Similarly, we will have for the third period, fourth period, and so on.

$$max_{\{c_1,c_2,\dots\}} U(c_1, c_2, \dots) = \sum_{i=1}^{T} (1+\alpha t)^{-\left(\frac{\beta}{\alpha}\right)} u(c_i)$$

So, time appears here in multiplication form with alpha. Now, consider two questions. Would you rather have one apple today or two apples tomorrow? Would you rather have one apple one year from now or two apples one year and one day from now? So here, this is like a choice between today and tomorrow.

And here, it is a choice between one year from now or one year and one day from now. If the majority prefers one apple today in question one, then utility from one apple is greater than utility from two apples discounted to the present time.

$$u(1) > \delta u(2)$$

If the majority prefers two apples in question 2, then this is a clear violation of the stationarity property. So, in question 2, if they say that they prefer delta U366 greater than delta

this is delta raised to the power 366, 365 U 365, then this is a clear violation of the stationarity property. Because stationarity properties say that your choices would depend completely on the difference between the two time periods, which is like t prime minus t, if you remember. So, in both cases, t prime minus t is 1 here as well as here. So, if you like it here. then your preference should be the same when it comes to one year and one day from now.

$$\gamma u(1) < \gamma \psi u(2)$$

Let psi be the daily discount factor for decisions regarding consumption one year from now and gamma the discount applied for waiting one year. Now you can see that if people prefer two apples after one year and one day over two apples after one year, then we can say that gamma U1 is less than gamma psi u2 which is like gamma is the discount factor for one year after one year you are getting one apple but that is less preferred to a situation where after one year and one day so you first apply the one-year discount, then you apply the one-year and one-day discount multiplied by the utility from two apples, so this is now actually greater than gamma U1. Gamma cancels out. And this is my equation 1 from the previous slide.

So, 1 and 2 can be reconciled if delta is less than psi. That is, the daily percentage discount decreases over time. Hyperbolic discount can accommodate this pattern.

$$(1)^{-\left(\frac{\beta}{\alpha}\right)} = 1 \quad \& \quad \delta = (1+\alpha)^{-\left(\frac{\beta}{\alpha}\right)} < 1$$

The hyperbolic discount rate for period 0, that is today, and period 1, tomorrow, would be 1 raised to the power minus beta raised to the power alpha so basically if you remember we had 1 plus alpha t raised to the power minus beta upon alpha

So if t equals 0, we are left with only 1. So, 1 raised to the power minus beta upon alpha equals 1, period 1. That is tomorrow. It is like 1 plus alpha raised to the power minus beta upon alpha. That should be less than 1 for any positive value of alpha. Alpha and beta are positive numbers. So, this is less than 1.

The larger the alpha, the smaller is delta. Now, the discount that applies to 1 year and 1 year 1 day are, first of all, you can see this is 1 plus alpha 365. This is 1 year raised to the power minus beta upon alpha, and 1 year 1 day is delta. 1 plus alpha 366 raised to the power minus beta upon alpha, respectively.

$$(1+\alpha 365)^{-\left(\frac{\beta}{\alpha}\right)} \quad \& \quad (1+\alpha 366)^{-\left(\frac{\beta}{\alpha}\right)}, \quad \text{respectively.}$$

Therefore, psi which is the daily discount that applies after 1 year would be a ratio of these two raised to the power minus beta upon alpha

and since alpha 366 and alpha 365 are very close to each other. So, we can expect that this is approximately equal to 1.

$$\psi = \left(\frac{1+\alpha 366}{1+\alpha 365}\right)^{-\left(\frac{\beta}{\alpha}\right)} \approx 1 > \delta$$

And thus, greater than delta. In general, the discount factor applied from one period to the next takes the form psi equals 1 plus alpha t plus 1.

$$\psi = \left(\frac{1+\alpha(t+1)}{1+\alpha t}\right)^{-\left(\frac{\beta}{\alpha}\right)}$$

As you can see, it is t plus 1 divided by 1 plus alpha t raised to the power minus beta upon alpha, the usual one.

Now, I come back to or go back to the SNAP example. Under hyperbolic discounting, the maximization problem is, again, we are maximizing x and the consumption stream for a period of 30 days, vx, that is the value or utility from all the goods and services other than food, then this is my discount factor, 1 plus alpha t raised to the power minus beta upon alpha multiplied by period-specific utility summed up over 0 to 30 periods.

$$max_{\{x,c_0,c_1,..,c_{30}\}} v(x) + \sum_{t=0}^{30} (1+\alpha t)^{-\left(\frac{\beta}{\alpha}\right)} u(c_t)$$

And the first order condition gives us 1 plus alpha j minus raised to the power minus beta upon alpha multiplied by u prime C j, again the marginal utility of consumption in period j, is equal to 1 plus alpha k raised to the power minus beta upon alpha u prime C k. So, this is exactly the same as what we obtained from, say, the discounted utility model. There we had delta j u prime C j equals delta raised to the power k u prime C k. Here, instead of delta j and delta k, I have these as the discount factors. The rest remains the same.

$$max_{\{x,c_0,c_1,..,c_{30}\}} v(x) + \sum_{t=1}^{30} (1 + \alpha(t-1))^{-\left(\frac{\beta}{\alpha}\right)} u(c_t)$$

We have previously proved that gamma 0, 1 is 1 plus alpha raised to the power 1. So, basically, when we are considering two periods, say gamma 0 and 1, the 1 for 0 will be in the denominator. If you just remember, when we are considering simple gamma for the 1 year, 1 day, then it was like alpha 1 plus alpha 366 divided by 1 plus alpha 365 raised to the power minus beta upon alpha. So, in a similar fashion, when it is 0 and 1, then I have 1 in the denominator, the lower period or the initial period and 1 plus alpha,

which is basically alpha multiplied by 1 raised to the power minus beta upon alpha, which would be less than psi jk where we have 1 plus alpha k divided by 1 plus alpha j raised to the power minus beta upon alpha. This would be, so we claim that psi jk is less than or greater than psi 0, 1. For j and k greater than 1, greater than equal to 1 and psi j, j plus 1 converges to 1 as j gets large. Thus a SNAP recipient who behaves according to hyperbolic discounting when receiving her benefits will plan to consume a lot today

but then hope to smooth out her consumption over the rest of the month. So when the days are close by then he or she considers spending a much larger amount. Unfortunately a time-inconsistent recipient does not live by her plan. Instead when she arrives in the second period she solves like again we have this 30 days or here now actually 29 days of consumption.

$$(1 + \alpha(j-1))^{-\left(\frac{\beta}{\alpha}\right)} u'(c_j) = (1 + \alpha(k-1))^{-\left(\frac{\beta}{\alpha}\right)} u'(c_k)$$

This is consumption of other goods and services we are now beginning from t equals to 1 instead of beginning from t equals to 0 and if I have t equals to 1 here then putting t equals to 1 here I will be leaving left with only I will be left with 1 so though this is time period 1 but in order to have a similar flow of discount factors and consumption utility generated by consumption multiplied by the discount factor. I am replacing t with t minus 1 here.

The rest of the things remain the same.

$$(1+\alpha j)^{-\left(\frac{\beta}{\alpha}\right)}u'(c_j) = (1+\alpha k)^{-\left(\frac{\beta}{\alpha}\right)}u'(c_k)$$

And the first order condition gives us 1 plus alpha j minus 1 raised to the power minus beta upon alpha. So, initially or previously this was simply alpha j. Now, this is replaced by alpha j minus 1. And similarly, previously it was alpha k and now it is replaced by alpha k minus 1. Consequently, the new discount factor for period 1 and 2 are period 1 and 2, we have k minus 1 upon j minus 1, which is like equal to 1 plus alpha divided by 1 equals to psi 0, 1.

$$\hat{\psi}(1,2) = \left(\frac{1+\alpha(k-1)}{1+\alpha(j-1)}\right)^{-\left(\frac{\beta}{\alpha}\right)} = \left(\frac{1+\alpha}{1}\right)^{-\left(\frac{\beta}{\alpha}\right)} = \psi(0,1)$$

$$< \psi(1,2) = \left(\frac{1+2\alpha}{1+\alpha}\right)^{-\left(\frac{\beta}{\alpha}\right)}$$

But this would be smaller than psi 1 2, where I have 1 plus 2 alpha divided by 1 plus alpha raised to the power minus beta upon alpha. Therefore, previously, recipients had anticipated applying a discount of psi 1, 2. They thought of applying a discount of psi 1, 2 between the first and second period. Now, instead, they apply psi hat 1, 2, which is which is equivalent to psi 0, 1.

So all I want to say is that if you remember psi 0,1 in the case of Ted and Matthew's example was 30%, and later on, it was 10%. So the recipients of SNAP also first thinks that in the first period they would definitely apply 30% but from next period onwards they would start applying 10%. But when the next period arrives then they treat it as the fresh beginning and again applies 30%. Again, when they arrive in the second period, then they consider it to be a fresh start and again apply 30%.

So, each period they basically revise their discount rate, daily discount rate, and even though in the initial time they thought of applying a discount rate of 10% for consecutive periods, they end up discounting at a much higher rate of 30% that is bringing forward consumption to the present time. Higher discount rate implies that individuals are more impatient. They would consume more today as a result of which in the initial period there are higher consumption. Then in the later periods, there are lower consumption and in the last week, the consumption is much lower.

$$\psi(0,1) = \left(\frac{1+\alpha}{1}\right)^{-\left(\frac{\beta}{\alpha}\right)} < \psi(j,k) = \left(\frac{1+\alpha k}{1+\alpha j}\right)^{-\left(\frac{\beta}{\alpha}\right)}$$

So that is how the hyperbolic discounting or naive hyperbolic discounting model is able to explain the SNAP example. This is how the SNAP recipients consume more in period 1 than they had planned leaving less for the future when they anticipated smoothing their consumption. In the following period, they again decide to consume more than they had planned in either of the previous periods, again putting off consumption smoothing until a later date. Recipients continue to put off smoothing until they reach a point where they no longer have enough food to last the rest of the month, and their consumption is forced to drop significantly.

This is what we can show diagrammatically. This is the planned consumption, which is somewhat smoother compared to the actual consumption, which is steeper. Ideally, it tries to show that the drop is much steeper. We will later compare naive hyperbolic discounting graphically against the discounted utility model. And that would actually show us very clearly how steep or drastic the behavior of the naive hyperbolic discounter is when specifically faced with this kind of time-inconsistent preference.

That is all for the current module. These are the references used. Thank you.