PRINCIPLES OF BEHAVIORAL ECONOMICS

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Week 44

Lecture 44

Hello everyone, this is the 44th lecture of the course on Principles of Behavioral Economics. We are discussing intertemporal choice models, and in this module, we are going to discuss the discounted utility model. We have already talked about Bohm Bawerk's trade-offs and Irving Fisher's indifference curve analysis. They were early pre-DUM, that is, discounted utility model developments in the theory of intertemporal choice that took different psychological factors into account, which were swept away by the DUM, condensing them into a single construct: the discount rate.

Samuelson introduced the DUM in 1937 in a short article modestly titled A Note on Measurement of Utility. Apart from commenting that the comparison of intertemporal trade-offs required a cardinal as opposed to ordinal measures of utility, it extended Fisher's indifference curve analysis, essentially limited to the comparison of two periods to multi-period situations. So, giving basically a more formal structure to Irving Fisher's indifference curve analysis, extending it to multi-period situations. The nature of the model can be best described in mathematical terms.

It specifies an intertemporal utility function where utility is a function of consumption spread over t periods which describes the utility at time t of the consumption profile starting in period t and continuing up to period capital T. So, you can see that ct, ct plus 1, ct plus 2, and so on. T can take any value. The model incorporates the general axioms of the neoclassical model described in previous modules regarding the completeness, transitivity, and independence principles. These principles we discussed in the context of prospect theory.

The DUM then goes on to assume that a person's intertemporal utility function can be described by the following specific functional form.

$$U_{t(c_t, \dots, c_T)} = \sum_{k=0}^{T-t} D(k)u(c_{t+k})$$

where $D(k) = \left(\frac{1}{1+\rho}\right)^k$

$\frac{1}{(1+\rho)^2}$

So, this is the functional form where D k is equal to 1 upon 1 plus rho raised to the power k. Now, if you remember in the previous modules, we have talked about a discount factor of 1 plus r. This was for one period. For two periods, that would be 1 upon 1 plus r raised to the power 2. So, in a similar fashion, we have here 1 upon 1 plus rho raised to the power k. r is replaced by rho.

We call it the discount factor. What are they discounting? They are discounting the future flow of utilities arising out of consumption. This implies that d0 equals 1. So, if dk is 1 upon 1 plus rho raised to the power k, then when k is equal to 0,

then d0 equals 1, then d1 would be 1 upon 1 plus rho, and so on. Therefore, we have this expression.

$$U_{t(c_t, \dots, c_T)} = \sum_{k=0}^{T-t} D(k)u(c_{t+k})$$

$$= c_t + \frac{c_{t+1}}{1+\rho} + \frac{c_{t+2}}{(1+\rho)^2} + \dots + \frac{c_T}{(1+\rho)^{T-t}}$$

This is the original expression which can be expanded and when dk is replaced by this expression, then we have ct. First of all, see this can be expanded as d0u and ct plus 0. T plus 0 is basically T. Then D1 UCT plus 1, and so on.

So, D naught is 1. That is why we have only CT, or we can also write it as UCT. We are assuming here that utility depends on Utility, say it is a linear function of consumption, so uct equals to ct. In a similar fashion, we have d1 equals to 1 upon 1 plus rho, so ct plus 1 upon 1 plus rho and so on.

The term u ct plus k can be interpreted as the person's instantaneous utility function, that is their perceived well-being in period t plus k. The term dk refers to the person's discount function that is the relative weight that the person attaches in time period t to their well-being in period t plus k. Finally, the term rho refers to the person's discount rate, that is, the rate at which they discount expected future utilities. This term therefore combines all the various psychological factors involved in time preference that were discussed earlier. It will aid an understanding of the workings of the model, as well as the implications and anomalies discussed later, if a simplified example is examined at this stage.

Let us take the following consumption profile, measured in thousands of rupees per year over a period of the next three years: 20, 20, 20, which implies that every year you are going to spend 20,000. We will also assume, for simplicity, that these consecutive equal amounts of consumption yield equal amounts of utility. So, again, this is a simplistic assumption. If uct equals to ct, Alternatively, U(20) equals 20.

Since the measurement of utility is in arbitrary units of utils or utiles, we can consider the utility profile as consisting of the terms 20, 20, 20. It is assumed that the model is discrete and that the utilities are all received at points in time at the end of each period, rather than being continuous flows throughout each period. So, basically, this is Three years, then you are receiving the first 20 at the end of the first year, the second 20 at the end of the second year, and the third 20 at the end of the third year. This may appear unrealistic, but the DUM can be modified to transform it into a continuous-time model as well.

The utility function becomes an integral of a negative exponential function. We will use the discrete form of the model for the time being. If the consumer discounts future utility at the rate of 10% per year, the current utility of the consumption and utility profiles can now be calculated as: see the first period, it is 20 divided by 1 plus 0.1. So, we are beginning with period 1, not considering period 0 specifically, because the 20 is coming at the end of the first period. So, that is why this is raised to the power 1. Then, at the end of the second year, we have 20 divided by 1 plus 0.1 raised to the power 2, and then this is raised to the power 3. By summing all of them, this is 49.74.

$$U_t(20, 20, 20) = \frac{20}{1+0.1} + \frac{20}{(1+0.1)^2} + \frac{20}{(1+0.1)^3} = 49.74$$

This calculation illustrates an important feature of the DUM. It closely resembles the compound interest formula used in calculating net present value.

This analogue with the common technique for evaluating financial investments, has been largely responsible for the rapid assimilation of the model by economists. Koopmans, in 1960, showed that the model could be derived from a set of axioms or basic principles that are superficially plausible. The model also gained normative status, as it was shown that an exponential discounting function was consistent with rationality in the sense that it resulted in dynamically consistent choices. So, since this model was leading to rational decision-making. So, it also gained the normative status.

Other economists including Lancaster, Fishburn and Meyer have since provided alternative axiom systems for the DUM, further increasing its perceived legitimacy and popularity. Now, what Samuelson proposed was the mathematics of the exponential utility model. We have U C0 C1. equals to. So, basically, the total utility from consumption across two periods is the utility from consumption in period 0 plus consumption in period 1, multiplied by the discount factor.

$$\frac{\partial U(C_0, C_1)}{\partial C_0} = u'(C_0),$$

So, here u ct is a period utility function for period t and the future utility is discounted by a positive number delta which is less than 1. Here, delta is the discount factor. let u prime c denote the derivative of u c, then I will be having delta u c naught c 1 upon delta c naught is equal to u prime c naught.

$$\frac{\partial U(C_0, C_1)}{\partial C_1} = \delta u'(C_1)$$

So, this is again, if you remember from the previous, probably the second module, that this is the partial derivative with respect to consumption in period 1, where we are trying to find out that if only the consumption in period 0 changes, holding the consumption in period 1 constant, then how total utility is going to change.

And this is denoted by u prime c naught. And, in a similar fashion, this is the partial derivative with respect to consumption in period 1. So, only consumption in period 1 is changing, and how this is impacting the total utility. We denote it by u prime c1, and since this has a delta in front of it, the delta remains here as well.

$$\frac{u'(C_0)}{\delta u'(C_1)} = 1 + r$$

Therefore, u prime c naught divided by delta u prime c1 is equal to 1 plus r, which is the Euler equation for the time discounting model. This relationship was derived in Module 1 or 2, or Module 41 or 42—that is, the initial modules on the intertemporal choice models. Here, the left-hand side is the ratio of the utility in two periods, while the right-hand side is the ratio of their prices. We also discussed that 1 plus R is basically the relative price of consumption in periods 1 and 2. The two-period model is easy to understand and to analyze, and for some application problems,

the insight we obtain from a more general multi-period model with three periods and more may be essentially the same. For other application problems, however, results from a two-period model may not be appropriate. For example, consider a businessperson who will retire in three years. He is thinking about how much to consume and save this year. One option is to enjoy consuming more this year and start saving for retirement next year.

This option looks attractive. Namely, there is a temptation to procrastinate saving for retirement. To think about this type of problem, it is necessary to use models that deal with three periods or more. Let us look at a multi-period model. The budget constraint equation would be something like initially we had only these two periods.

Now that is extended to T periods. So we simply keep on writing them.

$$C_0 + \frac{1}{1+r} C_1 + \cdots + \frac{1}{(1+r)^T} C_T$$
$$= Y_0 + \frac{1}{1+r} Y_1 + \cdots + \frac{1}{(1+r)^T} Y_T$$

With 1 upon 1 plus r raised to the power capital T multiplied by Ct. And this consumption over a large number of periods, multiple periods, must be equal to the present value of all

the income streams across all the periods. This intertemporal budget constraint is similar to that of the two-period model in that it states that

The discounted present value of current and future consumption must not exceed the present discounted value of current and future labor income. The consumer in a multi-period model maximizes his utility function. Again, the utility from all the consumption spread across all possible years is subject to the intertemporal budget constraint. The Euler equation for this optimization is, now we are basically equating like delta u delta Ct plus k upon delta u delta Ct, which is essentially, I mean this is what is written. They are the same two things.

$$\frac{\partial U/\partial C_{t+k}}{\partial U/\partial C_t} = \left(\frac{u'(C_{t+k})}{u'(C_t)}\right) = \frac{1/(1+r)^{t+k}}{1/(1+r)^t} = \frac{1}{(1+r)^k}$$

Delta u delta Ct plus k is u prime Ct plus k. And then we equate them with their relative prices. So this is 1 upon 1 plus r raised to the power t plus k. And the relative price of this is 1 upon 1 plus r raised to the power t. Then cancelling things out, we have 1 upon 1 plus r raised to the power k. This Euler equation shows that the marginal rate of substitution of consumption in period t and consumption in period t plus k is equal to their relative prices again.

And here the relative price is 1 upon 1 plus r raised to the power k. As the time-discounting utility function for a multi-period, we consider this stream of utilities.

$$U(C_0, \ldots, C_t) = u(C_0) + \delta u(C_1) + \delta^2 u(C_2) + \cdots + \delta^T u(C_T)$$

So, u c naught plus delta u c 1 plus delta squared u c 2 and so on. So, now what is the difference? You can see that 1 upon 1 plus r is replaced by delta, or which we are actually calling 1 upon 1 plus rho here in this module. We call this exponential discounting model because the utility from period t is discounted by the exponential function delta t of delta raised to the power t of delta.

In this model, the Euler equation that equates the marginal rate of substitution of consumption in period t and consumption in period t plus k to their relative price is simply 1 plus r raised to the power k. So, actually, if you remember the Euler equation, then initially we had it with just two periods. So, we had u prime ct in the numerator and delta u prime ct plus 1 in the denominator. That was set equal to 1 plus r.

$$\frac{u'(C_t)}{\delta^k u'(C_{t+k})} = (1+r)^k$$

Now we have delta raised to the power k because we are comparing basically the marginal rate of substitution between period t and period t plus k. So, in the previous case, k was equal to 1. Now we are generalizing it to any value of k. As a result of which we have delta raise to the power k u prime Ct plus k and this is equals to 1 plus r raise to the power k. If we set k equals to 1 in this equation then we obtain the Euler equation for two neighboring periods with which we are already pretty much familiar. In traditional economics, especially in macroeconomics, the exponential discounting model has been used as the standard model.

Both the permanent income hypothesis and life cycle hypothesis, which are the key consumption theories in macroeconomics, depend on the above equation. As a simple case, we assume that delta into 1 plus r equals 1. Here, delta into 1 plus r is derived from the Euler equation. We had u prime ct. Divided by delta raised to the power k u prime ct plus k equals 1 plus r raised to the power k. This implies that u prime ct divided by u prime ct plus k

is equal to delta raised to the power k 1 plus r raised to the power k, right? And if k is equal to 1, then this would become delta into 1 plus r raised to the power k. So, if we assume that delta into 1 plus r equals 1, then the marginal utility does not change over two neighboring periods. If this equals 1, if we set this equal to 1, then what we have is that u prime ct equals u prime ct plus 1, right? Because here k is equal to 1, here k is equal to 1.

So, this implies that the marginal utility does not change over two neighboring periods, right? The marginal utility is thought to satisfy the law of diminishing marginal utility that the marginal utility decreases as consumption increases and that is how we have u double

prime c less than or equal to 0, meaning the second derivative of the utility function is less than or equal to 0. Here we assume the strict inequality In this context, if it is not changing, then this means u double prime c equals 0 holds.

Then, for the marginal utility to stay the same over two neighboring periods, consumption should stay the same. Because this holds for all two neighboring periods, consumption does not change in any period. In the budget constraint equation, this is the budget constraint equation that we have. We can let Ct be constant at C and solve for C to obtain the optimal level of consumption. So all these Cs will be replaced by a single C, and accordingly, we can solve for the optimal level of consumption, which is going to remain fixed for all the periods.

The optimal level of consumption depends on the right hand side of the budget constraint equation, which is the present discounted value of current and future labor income, which is basically the permanent income and not on income in each period beyond its contribution to permanent income. The permanent income hypothesis applies this model to the response of consumption to temporary changes in income due to business cycle fluctuations and other such factors. Even when income decreases significantly because of a recession, as long as the change is temporary, consumption should not change. So this is the basic precept of the permanent income hypothesis or the life-cycle theory that we had discussed while discussing mental accounting.

And as a result, there would be some drop at some point in time when the change is temporary. We can expect that there would be some other point in time when this drop would be made up for. And as a result, overall, consumption is or is expected to remain the same. The consumer is predicted to borrow or draw funds from assets. that he has accumulated in order to keep consumption at the same level.

The life-cycle hypothesis applies this model to the whole life of a consumer. Even when income is low during school years and retirement years, consumption is predicted to be the same as in working years. When the interest rate is high and delta multiplied by 1 plus r is greater than 1, then u prime Ct is greater than u prime Ct plus 1. Because the marginal utility decreases over time, The optimal level of consumption is predicted to increase over time.

Alternatively, this implies that when borrowing is expensive, people borrow less. So, with this, I conclude this topic on the discounted utility model. Thank you.