

PRINCIPLES OF BEHAVIORAL ECONOMICS

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Week 29

Lecture 29

Hello everyone, this is the 29th lecture of the course on Principles of Behavioral Economics. We have been discussing prospect theory. So, once we discussed the original prospect theory, then we started discussing the improvements that were offered, and they are called later, the cumulative prospect theory, in short, CPT. I also described or started describing how the later version, or advanced version, cumulative prospect theory offers improvements

compared to the original prospect theory, alternatively called OPT. We can also call it just prospect theory. So, I mentioned that there are particularly two aspects where it has contributed. One is bringing in non-linear preferences. The second one is source dependence.

Also, this new prospect theory, or cumulative prospect theory, allows incorporation of multiple outcomes in a prospect. So, we continue with our discussion on cumulative prospect theory. Talking about nonlinear preferences, the principle of diminishing sensitivity applies to the weighting functions where it entails that the impact of a given change in probability diminishes with its distance from the boundary.

In the evaluation of outcomes, the reference point serves as a boundary that distinguishes gains from losses. In the evaluation of uncertainty, there are two natural boundaries, certainty and impossibility that correspond to the endpoints of this certainty scale. So, as you understand that any probabilities would be actually, you know, lying between 0 and 1 with the two endpoints inclusive. So, the boundaries are certainty when the probability is equal to 1 and impossibility when the probability is equal to 0. Similarly, you know these are the boundaries of uncertainty and similarly the weighting decision or weighting functions also would be having this kind of boundaries.

For example, an increase of 0.1 in the probability of winning a given prize has more impact when it changes the probability of winning from 0.9 to 1. or from 0 to 0.1 than when it changes the probability of winning from 0.3 to 0.4 or from 0.6 to 0.7. This thing we discussed also yesterday with an example of probability changing by 0.01, right, but the impact remains the same. So, what it implies is that diminishing sensitivity therefore gives rise to a waiting function that is concave near 0. and convex near 1.

So, if I try to draw this curve with 0 here and 1 here, so I have sort of a box where this is 1 and this is also 1, then what it says is that it is concave near 0 and then convex near 1. For uncertain prospects, the principle yields subadditivity for very unlikely events and superadditivity near certainty. The function is not well behaved near the endpoints and very small probabilities can be either greatly over weighted or neglected altogether. KT suggested the following as a two-part power function of the value scale. So, this is how the value scale can be expressed.

$$v(x) = \begin{cases} x^\alpha & \text{if } x \geq 0 \\ -\lambda(-x)^\beta & \text{if } x < 0 \end{cases}$$

Where we have x is the outcome, x raised to the power alpha for all positive outcomes, that is when x takes a value greater than equal to 0 or we can say all non-negative outcomes. And when there are negative outcomes, the value function takes the form minus lambda multiplied by minus x raised to the power beta for all x less than 0. Alpha is the coefficient of diminishing marginal sensitivity to gains, and beta is the coefficient of diminishing marginal sensitivity to losses. Lambda refers to the coefficient of loss aversion. We will also check examples where we can apply this value function.

The functional form of the weighting functions are w plus p equals to this form and w minus p takes a different, I mean takes a very similar form, but only difference is that the parameters are different here. Here it is gamma, and for negative v, it is referred to as delta. where gamma and delta determine the curvature of the function and they are different for losses and gains. A non-linear regression procedure was used to estimate the parameters of equations.

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1-p)^\gamma)^{1/\gamma}}$$

$$w^-(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}$$

The median exponent of the value function was 0.88 for both gains and losses—that is, alpha and beta—in accordance with diminishing sensitivity. So, what they observed is that alpha equals beta equals 0.88. What it also shows is that the extent of diminishing sensitivity in the domains of gains and losses is the same if the values of beta and alpha are nearly the same. The median lambda was 2.25, indicating pronounced loss aversion, and the median values of the parameters gamma and delta were 0.61 and 0.69, respectively. So, the median value of

delta being 2.25 implies that loss aversion was actually observed, and loss pinched 2.25 times more than the pressure that you would get from a gain. That is what at least PT—or Kahneman and Tversky—observed through their experiments. Similarly, for the weighting functions, gamma and delta values were obtained as 0.61 and 0.69 respectively. Now, as I previously drew, this is basically the shape of the weighting function. You can see this line is actually refers to W plus and this is W minus.

The smooth curves can be interpreted as weighting functions. The inverted S shaped curve is steepest near the endpoints and shallow in the middle of the range. So it is concave here and it is steep. Again this is steep here and convex and this is shallow near the middle range. What does it imply is that there is diminishing sensitivity in the middle range or when probability changes in the middle range then the decision weights actually do not change in equal proportions

while there are over weighting for very small probabilities and similarly there is the impact of certainty for very high probabilities. For both positive and negative prospects, people overweight low probabilities and underweight moderate and high probabilities. Thus, people are relatively insensitive to probability difference in the middle range. Tversky and Kahneman performed a study with graduate students to reveal their preferences in terms of certainty equivalence for a number of prospects. The results are shown in the following slide.

But I just need to mention here that in the following slides, we will be having values of certainty equivalence denoted by CEs and EV referred to expected value. So, we need to remember that certainty equivalence, if they are greater than EV, this indicates risk seeking

for both gains and losses. While if CE is less than EV, this indicates risk aversion for both gains and losses. If CE equals EV, then the individual is basically risk-neutral. This table contains the empirical evidence.

Prospect	Description	EV	Median CE	Attitude to risk
(0, 0.05; 100, 0.95)	Gain, high p	95	78	Averse
(0, 0.05; -100, 0.95)	Loss, high p	-95	-84	Seeking
(0, 0.50; 100, 0.50)	Gain, medium p	50	36	Averse
(0, 0.50; -100, 0.50)	Loss, medium p	-50	-42	Seeking
(0, 0.95; 100, 0.05)	Gain, low p	5	14	Seeking
(0, 0.95; -100, 0.05)	Loss, low p	-5	-8	Averse

We have prospects here, followed by descriptions, then expected values, then certainty equivalents, the median values of certainty equivalents, and attitude toward risk. I just need to mention, or maybe you are supposed to recall from the previous slide, that when we have CE greater than EV then that indicates risk seeking for both gains and losses. When CE is less than EV this indicates risk aversion for both gains and losses. CE less than EV means we have risk-averse, and CE greater than EV means we have risk-seeking. These are the prospects. In the beginning, we have a prospect where One can win 100 with a probability of 0.95.

This is, of course, a strictly positive prospect with high probability. The expected value is 95, which we can obtain by multiplying 100 by 0.95 plus 0 by 0.05, which of course is going to give us 95. The median certainty equivalent was observed to be 78. Now, following this, the individuals are, on average, risk-averse. Now, in a similar fashion, if we just consider the negative of that prospect, where one loses 100 with a probability of 0.95, then again the loss probability is high.

The expected value is minus 95. The certainty equivalent is minus 84 which means there is risk seeking since certainty equivalent is greater than expected value. Next, we consider two prospects where the probabilities are medium. The first one is for gain, and the second

one is for loss. Here you gain 100 rupees, here you lose 100 rupees, and the expected values would, of course, be 50 and minus 50.

Again, it is observed that the certainty equivalent is lower. So, you are basically willing to accept a lower amount in order to avoid this prospect, or this prospect is equivalent to this amount of certain gain. So, since the certain gain that you are asking in lieu of this prospect, this risky prospect, is lower than the expected value of the risky prospect. We call them risk-averse.

So, again, these individuals are risk-averse, and when it comes to losses, then their attitudes are again risk-seeking because the certainty equivalent is greater than the expected value. of loss and finally these are the two prospects where the probabilities are low you can see that they are just 0.05. Again, in one, you gain 100; in another, you lose 100. The expected values are again obtained by simply multiplying 100 by its probability, so it is 5 here and minus 5 here. Now you can see that The certainty equivalent is actually greater than the expected value, which means here the individuals are risk-seeking. So, this implies that when the probability of winning a larger amount is actually very small, then people would rather prefer that risky prospect as compared to a sure gain of a smaller amount.

In a similar fashion, when we consider this, then there is this loss of minus 100 with probability 0.05, while you are going to lose minus 5 for sure. Then people go for a certainty equivalent. The median value is minus 8. Since the median value of the certainty equivalent is less than the expected value here, the individuals are risk-averse. So, one example is just mentioned here: the subjects are prepared to pay an average of 78 dollars to obtain an expected value of 95 dollars, showing risk aversion. So, through this empirical evidence, actually Kahneman and Tversky tried to establish their argument that they put forward in the cumulative prospect theory about individuals' attitudes toward risk as reflected in the preferences empirically or through their experiments.

Now, again, we get back to the improvements offered by CPT. So, it is just summarized here. We compared it with the expected utility theory or EUT. The comparison can be under three dimensions, or we can consider three different aspects. First of all, the objects of choice in expected utility theory, probability distributions over wealth are basically the objects of choice,

while in cumulative prospect theory, prospect frames in terms of gains as well as losses. Then, the second aspect, specification, or dimension is the evaluation rule. In the case of expected utility theory, it is simply the expected utility as we calculate it. And in the case

of cumulative prospect theory, it is a two-part cumulative aspect. functional. The first part has a value function or value scale, and the second is a decision weighting scale.

And the third specification is the characteristics of the functions that map uncertain events and possible outcomes into their subjective counterparts. In the case of expected utility theory, utility is a concave function of wealth and nothing more than that, it says. So, it just says that utility is a concave function for both gains and losses. But cumulative prospect theory says that the value function is S-shaped and the weighting functions are inverse S-shaped.

So, the value function would look like this for gains and losses. While the weighting functions we have already drawn are inverted S-shaped. This is the 45-degree line. The curvature of weighting function explains the characteristic reflection pattern of attitudes to risky prospects. Overweighting of small probabilities explains the popularity of lotteries and insurance.

Underweighting of high probabilities explain the prevalence of risk aversion in choices between probable gains and sure things and the prevalence of risk-seeking in choices between probable and sure losses. Risk aversion for gains and risk-seeking for losses are further enhanced by the curvature of the value function in two domains. The pronounced asymmetry of the value function, which we have labeled loss aversion, explains the extreme reluctance to accept mixed prospects.

The shape of the weighting function explains the certainty effect. The value scale Vx previously quoted from KT's original work had r equals to 0. For r not equals to 0, the value scale then can be written as Vx equal to x minus r raised to the power α for x greater than equal to r and Vx equals minus λ into r minus x raised to the power β if x is less than 0. Now, you see that if r is equal to 0,

then this form becomes x raised to the power. If r is actually equals to 0 then for x greater than equals to 0 Vx becomes x raised to the power α . α similarly for r equal to 0 when x is less than 0 then Vx becomes minus λ minus x raised to the power β , right. And these are the value functions or value scales that KT mentioned in their original paper. Now, here r is, of course, the reference point. We are now going to talk about an application.

$$v(x) = (x - r)^\alpha \quad \text{if } x \geq r$$

$$v(x) = -\lambda(r - x)^\beta \quad \text{if } x < r$$

This was also Tversky and Kahneman's original work, published in 1981, where they are describing one problem and they described it in two different ways. So, the problem is of the Asian disease. This is the first statement. We call it Asian Disease Problem. Imagine that the US is preparing for the outbreak of an unusual Asian disease, which is expected to kill 600 people.

Now, two alternative programs to combat the disease have been proposed. Assume that the exact scientific estimate of the consequences of the programs is as follows. If program A is adopted, 200 people will be saved. If program B is adopted, there is a one-third probability that 600 people will be saved and a two-thirds probability that no one will be saved.

Which of the two programs would you favor? When people were asked, 72% chose A and 28% chose B. Now, let us just write down the prospect. The prospects are prospect A is simply 200 because it states that if A is adopted, then 200 people will be saved. So, 200 will be saved with a probability of 1. So, that is why we do not need to mention anything.

So, it is like 201 and 400, 0. So, 400 people will die for sure and 200 people will be saved for sure. So, this is simply written as 200, right? And similarly, for prospect B or for program B, when program B is adopted, then there is one-third probability that 600 people will be saved and we can always write that there is two-third probability that no one will be saved.

So as you know that we can write it explicitly and we can also drop it. So this is how we would be writing prospect B. Now the same problem when stated a little differently, we call it Asian Disease Problem 2. The initial portions would be similar that imagine that the US is preparing for the outbreak of an unusual Asian disease which is expected to kill 600 people. Two alternative programs to combat the disease have been proposed.

Assume that the exact scientific estimate of the consequences of the programs are as follows. Now, from here, the statements actually change. If program C is adopted, 400 people will die. Again, 400 people are going to die for sure. If program D is adopted, there

is a one-third probability that nobody will die and a two-thirds probability that 600 people will die.

Which of the two programs would you favor? So here, it was observed that 78% preferred D while 22% preferred C. Now, if we compare statement 1 and statement 2, you would see that if I compare A with C, Then A said that 200 people will be saved for sure, and C is saying that 400 people will die for sure. So these are actually the same thing. Similarly, B said that 600 people will be saved with a probability of one-third, while there is a probability of two-thirds that nobody would be saved.

In a similar fashion, D says that if a one-third probability is there, nobody will die. So, nobody will die implies that 600 people are going to be saved. And then, there is a two-thirds probability that 600 people will die. So, a two-thirds probability that nobody is going to live. Again, it is the same thing.

However, we would be writing D as 600, 2 by 3 because here we are actually writing things in terms of how many are dying. So, this is actually not the prospect of D. Prospect D is actually written as 0 to—sorry, would be written as there is to the probability that 600 people will die. So, if I consider how many people are going to die, then 600 people are going to die. So, 0, one-third, nobody is going to live, and 600, 2 by 3, 600 people are going to die. So, if I drop this 0, one-third, then I will be left with only 600, 2 by 3.

Similarly, C400 actually implies that 1 and 0—sorry, 200, with 0. So, that is why we would be just writing 400. Now, name a few effects that this problem portrays. So, in this, we can see framing, certainty, framing effect primarily because the way things have framed, the answers are changing though the implications are the same. Then we see certainty effect – there is a preference for certain positive situations. A is preferred because it is stated as 200 people are going to be saved for sure. There is reflection effect because when the gains and losses are mentioned, they are just, you know, the reflection of one another. There is loss aversion because of loss aversion, people actually prefer to go for, say, A over B and then D over C. And then preferring A over B and then D over C also refers to preference reversal. Let us calculate the value functions using the functional forms and estimated parameters prescribed by the CPT and check whether it correctly predicts the expressed preferences or not. Now, problem 1 states the problem as how many lives saved.

So, this is how that is a gain frame. And that's how the reference point is 0. Problem 2 states the problem as how many die, the loss frame. So that's why the reference point is r equals

to 600. Now how many lives saved r is equals to 0 because when we begin the problem statement, then it initially says that the disease is going to kill 600 people.

So 600 people are going to be killed, 0 is going to be saved and then we start looking for programs, alternative programs, how many are going to be saved. So, since it is a gain frame, it is a positive frame. So, reference point is 0. So, when there are no chances or there are chances of 0 or saving 0 life, then how much these programs are able to save. That is why the reference point is 0 here.

Similarly in the die frame that is the loss frame again we consider r as 600 because as mentioned that in the beginning it said that the disease is going to kill 600 people, so here 600 is the reference. 600 are going to die. Now the programs, their interventions, how much basically they are going to cause losses, that's how it is being framed. now these are important points because first of all let us consider or calculate value functions for problem 1. I have mentioned problem 1 here just for ready references.

Now, given the prospects, the prospects are repeated here. The value functions are, first of all, we have value function for prospect A, that is, since π or w plus 1 is equal to 1, alternatively, π plus 1 is equal to 1, then it is already mentioned here. So I do not need to multiply anything here. I have $V200$ which is 200 raised to the power 0.88 because α takes a value of 0.88 and it gives us an approximate value 105.90. If I consider the prospect B then in prospect B there is two third chance that nobody will be saved.

So V is equals to 0 and then there is one third chance that 600 people would be saved. Now, here the reference point is also 0. So, since V is equals to 0, since reference is 0, we can always use the original KT framework and then x raised to the power α , x takes a value 0 value 0 and that is why we will not be having this component anywhere. We just focus on this component.

So, W plus 1 $\geq V$ 600, W plus 1 \geq can be written as like this, now here this is gain frame so we use the value of γ which is equal to 0.61 And this is the functional form of W plus multiplied by 600 raised to the power 0.88. And it gives us a value of approximately 67.24. Since V_A equals to 1.90 greater than V_B 67.24, A is preferred to B. And this preference is actually explained or justified by the functional forms suggested by the cumulative prospect theory.

$$\begin{aligned}
 V(A) &= v(200) = 200^{0.88} \cong 105.90 \text{ for } w^+(1) = 1 \\
 V(B) &= w^+(2/3)v(0) + w^+(1/3)v(600) \\
 &= w^+(1/3)v(600) = \frac{\left(\frac{1}{3}\right)^{0.61}}{\left(\left(\frac{1}{3}\right)^{0.61} + \left(\frac{2}{3}\right)^{0.61}\right)^{1/0.61}} \times 600^{0.88} \\
 &\cong 67.24
 \end{aligned}$$

$w^+(1) = 1$
 $\alpha = 0.88$
 $\alpha = 0$

- Since $V(A) = 105.90 > V(B) = 67.24$, A preferred to B is justified.

$$\begin{aligned}
 V(C) &= v(600 - 400) = -2.25 \times (200^{0.88}) \cong -238.28 \\
 &\text{for } w^-(1) = 1 \text{ and } \lambda = 2.25 \\
 V(D) &= w^-(2/3)v(600 - 600) + w^-(1/3)v(600 - 0) \\
 &= w^-(1/3)v(600) \\
 &= \frac{\left(\frac{1}{3}\right)^{0.69}}{\left(\left(\frac{1}{3}\right)^{0.69} + \left(\frac{2}{3}\right)^{0.69}\right)^{1/0.69}} \times [-2.25 \times 600^{0.88}] \\
 &\cong -165.43
 \end{aligned}$$

β
 $v(x) = -\lambda(x - \alpha)$
 $\beta = 0.88$
 $\lambda = 2.25$

- Since $V(D) = -165.43 > V(C) = -238.28$, D preferred to C is justified.

Now, let us consider the value functions for problem 2. Again, the problem 2 is reproduced here and these are the prospects. Now, given the prospects, and here the reference point or reference value is 600. The value functions are, first of all, for C. Again, you know that W minus 1 is also equal to 1. So, we have $V(600 \text{ minus } 400)$ and

this is now here when I have reference point and in the domain of losses KT defines Vx as $\text{minus } \lambda \text{ into } r \text{ minus } x \text{ raised to the power } \beta$. So λ takes a value 2.25 so I have $\text{minus } 2.25$ here and $r \text{ minus } x$ is 200. So, 200 raised to the power again 0.88, because β is also estimated to be 0.88. That gives me an approximate value of $\text{minus } 238.28$. Now, considering prospect D, I again have two parts.

So, a two-thirds probability is there that all 600 lives will be lost. So, $w \text{ minus } \frac{2}{3} v600 \text{ minus } 600 \text{ plus } w \text{ minus } \frac{1}{3} v600 \text{ minus } 0$. Now, as you understand, $600 \text{ minus } 600$ gives us 0. So, of course, this term drops out.

I am left with $W \text{ minus one third } V600$ because $600 \text{ minus } 0$ is 600. So, again, this is the functional form of $W \text{ minus one third}$. Here, we are using the power 0.69 because δ is estimated to be 0.69 multiplied by now 600 following this functional form is $\text{minus } 2.25$ multiplied by 600 raised to the power 0.88, and the value that we get is $\text{minus } 160.43$. Now, you see that the value of D, which is $\text{minus } 160.43$, is greater than the value of C, which is $\text{minus } 238.28$. That is why D is preferred to C, and again, the cumulative prospect theory can explain the preferences that are expressed for these two different statements by individuals. So, with this, I conclude the discussion on prospect theory. In the next module, I will briefly talk about the criticisms that are raised, discussed, or observed in the literature against prospect theory. So, these are the references for this module. Thank you.