PRINCIPLES OF BEHAVIORAL ECONOMICS

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Week 28

Lecture 28

Hello, and this is the 28th lecture of the course on Principles of Behavioral Economics. So, as I was discussing decision weights of the prospect theory in the last module, here in this module we are going to discuss cumulative prospect theory. Now, as mentioned previously, cumulative prospect theory offers some improvement over the initial prospect theory that was proposed by Kahneman and Tversky in their 1979 paper. In 1992, they came up with a revised version. So, in order to discuss cumulative prospect theory, we mentioned two drawbacks of the initial prospect theory.

The first one was that it often did not fulfill the stochastic requirement of stochastic dominance. And the second thing is that it was not readily extendable to multiple prospects with multiple outcomes. So, we begin with the discussion of stochastic dominance. So first of all, this is how we define stochastic dominance where A and B, two prospects, are defined as X1 P1 to Xn Pn, Y1 P1 to Yn Pn. You can see that the probabilities associated with n outcomes are the same for A and B.

$$\text{Let } A = (x_1, p_1; \ldots; x_n, p_n) \text{ and}$$
$$B = (y_1, p_1; \ldots; y_n, p_n), \text{ then}$$

Now, B weakly stochastically dominates A if xi less than equal to yi for all i equals to 1 to n, which means that it is possible that all xi's are equal to yi, or it is possible that a few of them are different. But B strictly stochastically dominates A if xi is less than yi for all i equals 1 to n and some of the xi's are less than yi. Now, this is an example of stochastic dominance, which we had discussed in the context of the axiom of monotonicity.

So, first of all, we compared the two gambles, gamble A and gamble B, and we observed that gamble B strictly dominates gamble A. Because the winnings—see, the percentage of balls is the same here. The winning probabilities are the same here. This is higher. The loss amount is basically lower here, and the loss amount is the same here, right?

So, if we compare, then we can see that B strictly dominates. I also discussed that if we compare gamble C and gamble D we would see that if B dominated A then D should have dominated C, because, again, 90% white is the same thing. Then, 6% red—if I compare C with A, they would appear to be the same: 6% red, 1% green are here. Now, 1% blue and 2% yellow are clubbed together to make 3% yellow. The winning amount with red remains the same.

The winning amount with green remains the same. Here, it is a loss of $15 for both blue and yellow, which is again clubbed to a loss of $15 for 3% yellow. In a similar fashion if I compare gamble B and gamble D, then we would see that here again 1% green lose $10 can be equivalent to 1% blue lose $10. 2% yellow—lose $15.

It remains the same. So 6% red and 1% green is combined to have 7% red where the winning amount is again the same at $45. So that's why if B strictly dominates A, then D should strictly dominate C. But since loss appears twice here, people were observed to prefer C over D. So A is dominated by B but C is preferred to D by nearly 60% that was empirically observed and Kahneman and Tversky ascribed this preference to a lack of transparency

in the dominance in case of Gamble C and Gamble D. Now in the editing phase if there is not very clear dominance Then, if there is clear dominance, the prospects can be edited accordingly. But if there is a lack of clear stochastic dominance, then people can make such mistakes. And maybe in that case, their preferences would violate the principles of expected utility theory. Now, how to incorporate such behavior.

So that is why the concept of violation of stochastic dominance or the concept of stochastic dominance becomes important here. Now, there is another experiment where there are two gambles, Gamble A and Gamble B. So, between A and B, which one dominates? If you compare both of them, you would see that here it is 5% blue, and here it is 10% purple. The colors are completely different here. But then if we compare the percentages,

Then it is 10% purple $12 and 5% 5%, which is basically together $13 on average, right? And 90% red gives $96, 85% black or rather here 85% black, 5% yellow together makes it 90%, and this is actually $93. right so 73% preferred B to A now if we understand that if I go by expected utility theory then probably A should be preferred to B but then 73% preferred A to B. Now, in order to bring in more clarity, let us introduce gamble G. When gamble G is brought in, then we can show very clearly that They are comparable.

So 90% red $96, 90% red $96, and 5% white blue, 10% white. This is $13. This is $10. So what happens is that A dominates G very clearly. And in a similar fashion, we can also show that G dominates B. So A is dominant

or A dominates G and G dominates B, which implies A dominates B following the principle of transitivity. Therefore, A must dominate B, but that was not observed. Now, PT fails to explain such situations Dominated prospects are eliminated in the editing phase, but if there is no clear dominance, then people can make decisions which are in contradiction with the expected utility theory. Now cumulative prospect theory is a revised version of the prospect theory that incorporates the cumulative functional

and extends the theory to uncertain as well as risky prospects with any number of outcomes. it gives rise to different evaluations of gains and losses which are not distinguished in the standard cumulative model and it provides a unified treatment of both risk and uncertainty This revised theory addresses five major phenomena of choice which violate the standard model and sets a minimal challenge that must be met by any adequate descriptive theory of choice. The first one is the framing effect. Now I just need to tell you here is that I'm going to talk about five major phenomena, but it's not that these phenomena, all of them are not addressed in the prospect theory or the initial theory.

For example, the framing effect was addressed in the prospect theory, the original or initial prospect theory. Then we talk about nonlinear preferences. This is something that the cumulative prospect theory offers as an improvement. Source dependence is another factor that is offered as an improvement. Risk seeking was present in the initial or original prospect theory as well.

And then we have loss aversion, which was also addressed in the original prospect theory. I am going to briefly talk about the five of them, and then we can discuss a little more about the two aspects that are new to this cumulative prospect theory. Framing effects: the rational theory of choice assumes description invariance, meaning that equivalent formulations of a choice problem should give rise to the same preference order. Contrary to this assumption, there is much evidence that variations in the framing of options—for example, in terms of gains or losses—yield systematically different preferences. Then, the second point is nonlinear preferences.

According to the expectation principle, the utility of a risky prospect is linear in outcome probabilities. Allais's famous example challenged this principle by showing that the difference between probabilities of 0.99 and 1 has more impact on preferences than the difference between 0.1 and 0.11. So, basically, in both cases, there is a difference of 0.01. Now, when you move from 0.99 to 1.0, that is actually much more impactful compared to when you move from 0.1 to 0.11, or say 0.3 to 0.31, 0.6 to 0.61.

So these are much less impactful compared to this situation where you are actually moving from an uncertain situation to a certain one. In a similar fashion, moving from 0.00 to 0.01 is again much more impactful because this is something which is impossible not happening, and this is something which has a certain probability of happening. This is the most prominent difference between PT and CPT, which allows for different decision weights for gains and losses. Then, the third aspect is source dependence.

People's willingness to bet on an uncertain event depends not only on the degree of uncertainty but also on its source. Ellsberg observed that people prefer to bet on an urn containing equal numbers of red and green balls rather than on an urn that contains red and green balls in unknown proportions. So we are going to talk about something called the Ellsberg Paradox, which is basically an experiment conducted by Ellsberg. More recent evidence indicates that people often prefer a bet on an event in their area of competence over a bet on a matched chance event,

although the former probability is vague and the latter is clear. So this is a situation where we are saying that people would be preferring a situation where they have more confidence even though the probability of occurrence or the situation is much more uncertain Compared to a risky situation where the probabilities are more certain of certain things occurring, but since I do not have much information about that event, I would not prefer to bet on such an event.

Instead, I would prefer to go for something where I have more competence. So the thing is that source dependence does not actually, you know, give a straightforward rule that where people should bet or where people should not. So at times, uncertainties are preferred. At times, risky events are preferred.

So let's see what Ellsberg Paradox says. So in the Ellsberg paradox, we have again two alternative gambles. In gamble one, one individual has to choose one ball from either urn one or urn two. Now, Urn 1 contains a mix of red and blue balls in unknown proportions. in Urn 2 there are 50 red and 50 blue balls so here the difference is that we do not know what are the proportions of blue and red balls here we know that there are 50 blue and 50 red balls so when individual is asked to choose between Urn 1 and Urn 2 it is observed that say they go for Urn 2 now when they are asked to choose under gamble B one Urn, either Urn 1 or Urn 2 then again they choose Urn 2 so people generally preferred Urn 2 in both gambles A and B. Now in gamble A you are going to get 100 if you pick up a red ball and in gamble B you are going to get 100 if you pick up a blue ball so if you are choosing Urn 2 in gamble A, then it implies that you expect there are greater number of red balls in Urn 2.

The probability of picking up a red ball is actually higher if you choose Urn 2, where you know that in Urn 2, there are 50 red balls and 50 blue balls. Now, if you are choosing Urn 2 when there is actually 100, sorry, you are getting 100 for picking up a blue ball and again you choose Urn 2, that means you think that there is a greater probability of picking up a blue ball which means the probability of getting a blue ball is more than 50% or more than 0.5 so the probability of getting a red ball is more than 0.5 here

The probability of getting a blue ball is more than 0.5 here while we know that Urn 2 has 50 red and 50 blue balls. So what it basically shows, first of all, is that your preference is inconsistent if you are choosing urn 2 here. then you must choose Urn 1 here. Or if you are choosing Urn 2 here, you must choose Urn 1 here. But what it shows ideally is that people generally prefer that Urn where they are more certain about the number of blue balls and red balls.

Instead of choosing Urn 1 when they do not know actually the proportions of red and blue balls. So this phenomenon is known as ambiguity aversion, where there is uncertainty about the outcome's probability distribution. So when things are more uncertain, people tend to avoid that and preferably go for situations where they are more certain about it. Now, risk-seeking is the fourth aspect. Risk aversion is generally assumed in economic analysis of decisions under uncertainty.

However, risk-seeking choices are consistently observed in two classes of decision problems. First, people often prefer a small probability of winning a large prize over the expected value of that prospect. Second, risk-seeking is prevalent when people must choose between a sure loss and a substantial probability of a larger loss. So this basically refers to- these two situations refers to that gamble we just talked about in the previous module of winning 5000 with a probability of 0.001 against winning only 0.001 against winning probability winning only 5000.

with a probability of 1 and then again losing 5000 with a probability of 0.001 against losing minus 5 with a probability of 1 so these are the two situations and people prefer this over this and this over this. So these are the two instances of risk seeking and the fifth dimension is of loss aversion. One of the basic phenomena of choice under both risk and uncertainty is that losses loom larger than gains. So I must tell you that risk-seeking was covered in the original version of prospect theory. Similarly, loss aversion was also discussed in the original version of prospect theory.

The observed asymmetry between gains and losses is far too extreme to be explained by income effects or by decreasing risk aversion. In this context, let me remind you that we have this S-shaped utility function, where the slope of the function is much steeper in the domain of losses compared to the domain of gains. So this is what reflects the concept of loss aversion. Now, the new decision weights function introduced by CPT.

As discussed in the last module, the weighting scheme used in the original version of prospect theory encounters two problems. First, it does not always satisfy stochastic dominance. Second, it is not readily extended to prospects with a large number of outcomes. Both of these problems can be solved by the rank dependent or cumulative functional first proposed by Quiggin for decision under risk and by Schmeidler for decisions under uncertainty. Now, instead of transforming each probability separately, the revised model transforms the entire cumulative distribution function

and applies the cumulative functional separately to gains and losses. The development extends prospect theory to uncertain as well as risky prospects with any number of outcomes while preserving most of its essential features. Now consider a set of events S such that each state, denoted by a small s, belongs to capital S and has a consequence small x where small x belongs to capital X. So this is the outcome space. These are individual outcomes. An uncertain prospect f is presented as a sequence of pairs where fs

is equal to f xi Ai which yields xi if Ai occurs where xi greater than xj if and only if i greater than j, Aj is a partition of S. or AI is a partition of S. So, this basically shows that we are actually organizing the outcomes in, most often, an ascending order. So, there is an order of organizing the outcomes associated with individual states. If the prospect fs equals to f xi, Ai is given by a probability distribution pAi equals to pi, it can be viewed as a probabilistic or risky prospect given as xi minus pi. So, here I am not discussing the cases where we would be dealing with uncertainties.

Rather, I am going straight to the risky prospect, not dealing with the uncertain prospect. To define the cumulative functional, the outcomes of each prospect are arranged in increasing order. The value function now has two components: one is Vf plus, and another is Vf minus.

$$V(f) = V(f^+) + V(f^-)$$

So, Vf plus deals with all positive outcomes, and f minus handles all negative outcomes. Earlier, if you remember, we had this value function, which was simply a summation of pi into v xi right. Now first of all we are segregating them into positive and negative outcomes where f plus and f minus denote the positive and negative parts of a prospect such that f plus is fs if fs is greater than 0, and it takes value 0 for all fs less than or equal to 0. And similarly, f minus s equals to fs is fs is less than 0 and it is equals to 0 if fs greater than equal to 0. Again, s belongs to S is a state.

$f^+(s) = f(s)$ if $f(s) > 0$ and $f^+(s) = 0$ if $f(s) \leq 0$ and similarly,
$f^-(s) = f(s)$ if $f(s) < 0$ and $f^-(s) = 0$ if $f(s) \geq 0$, $s \in S$ is a state.

For a mixed or regular prospect, the positive and negative value functions are First of all, we also introduced strictly positive, strictly negative, and regular prospects. Regular prospect can be also called mixed prospect. So, we would be working primarily with mixed or regular prospects because they have both negative and positive outcomes. Strictly positive and strictly negative prospects have only positive or only negative outcomes, right?

So, we have V f plus. So, now first considering the positive outcomes, we can write it like this. So, as I just showed in the previous slide, previously we had only pi p i into vxi.

$$V(f^+) = \sum_{i=0}^{n} \pi_i^+ v(x_i)$$

Now that is replaced with specifically a plus sign here in order to indicate that we are considering only positive outcomes. And in a similar fashion vf minus would be written as pi i minus vxi again to indicate that we are considering only negative outcomes. So, since these are arranged in ascending order, we would be beginning with the most negative outcome and then would be moving towards the most positive outcome. So, if the most negative outcome has a number which is equals to minus m, this actually adds from minus m to 0 and this is from 0 to n.

$$V(f^-) = \sum_{i=-m}^{0} \pi_i^- v(x_i)$$

Now, the decision weights are defined as for any i between 0 and n minus 1 we have the positive part actually decision weights associated with the positive outcomes pi i plus which are given as w plus pi up to pn. So we are adding first of all all the positive outcomes or probabilities associated with all the positive outcomes. and then subtracting the one which is from the next one. So, p i, p i plus 1. So, we are now subtracting the weights associated with p i plus 1 to p n.

For $0 \leq i \leq n-1$

$$\pi_i^+ = w^+(p_i + \ldots + p_n) - w^+(p_{i+1} + \ldots + p_n)$$

Similarly, for i 1 minus m less than equal to i less than equal to 0 the decision weights associated with the negative prospects would be written as we begin from the lowest possible outcome and then go up to i the i which basically concerns us and then subtract you know, the outcome which is just prior to this. So, p i minus 1, that gives us pi i minus. This is because these functions are cumulative functions and that is why in order to arrive at an individual decision weight, we need to perform this kind of operations.

For $1-m \leq i \leq 0$

$$\pi_i^- = w^-(p_{-m} + \ldots + p_i) - w^-(p_{-m} + \ldots + p_{i-1})$$

For the highest outcome we denote it by pi n plus and that is simply w plus p n and the lowest outcome is pi minus minus m which is having the weight associated with it which is w minus p minus m. So, for the lowest one and the highest one it is only the associated probabilities but for any decision weights that lie in between, for probabilities that are in between, the decision weights are calculated following these two formulas for positive and negative outcomes. Now, we will be taking an example. But before that, the W plus and W minus are strictly increasing functions from the unit interval into itself, satisfying W plus 0 equals to W minus 0 equals to 0.

$$\pi_n^+ = w^+(p_n) \quad \text{and} \quad \pi_{-m}^- = w^-(p_{-m})$$

and W plus 1 equals to W minus 1 equals to 1. Now, we take an example where a die is rolled once and the results are x 1, 2, 3, 4, 5, 6, we all know. If x is even, you win x; and if x is odd, you pay or lose x. So, for example, if x—that is, you roll the die and get 1, 3, 5—then these are the amounts that you are going to lose. You have to pay the other person.

If you get 2, 4, 6, then they are basically your gains. That is the rule of the game. Therefore, f has the consequences. We are now arranging it in ascending order. Minus 5, minus 3, minus 1, 2, 4, 6, each with a probability of 1 by 6 for a fair die.

Now, further we can write the prospects as f plus: I have 0 with a probability of half, but 2 with 1, 6, 4 with 1, 6, 6 with 1, 6. Now, this 1, 6, 1, 6, 1, 6 add up to 1 by half, and that is why we need to add another 0, 1 by half. Similarly, f minus makes it minus 5, 1 by 6, minus 3, 1 by 6, minus 1, 1 by 6, and 0, 1 by 2.

$$f^+ = \left(0, \frac{1}{2}; 2, \frac{1}{6}; 4, \frac{1}{6}; 6, \frac{1}{6}\right)$$

$$f^- = \left(-5, \frac{1}{6}; -3, \frac{1}{6}; -1, \frac{1}{6}; 0, \frac{1}{2}\right)$$

Now, the value function—I have actually reproduced the functional forms of the decision weights so that we can compare them while calculating them. So, the value function again has two components.

$$\pi_i^+ = w^+(p_i + \ldots + p_n) - w^+(p_{i+1} + \ldots + p_n)$$
$$\pi_i^- = w^-(p_{-m} + \ldots + p_i) - w^-(p_{-m} + \ldots + p_{i-1})$$
$$\pi_n^+ = w^+(p_n) \quad \text{and} \quad \pi_{-m}^- = w^-(p_{-m})$$

So, first way, look at Vf plus. Vf plus, since it is actually arranged in ascending order, we will be first having V2. V2, let us consider i is equal to 2 here. Then, it would be P2. plus p3 plus, sorry, p4 and then p6.

$$V(f^+)$$
$$= v(2)\left[w^+\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right) - w^+\left(\frac{1}{6}+\frac{1}{6}\right)\right]$$
$$+ v(4)\left[w^+\left(\frac{1}{6}+\frac{1}{6}\right) - w^+\left(\frac{1}{6}\right)\right] + v(6)\left[w^+\left(\frac{1}{6}\right)\right]$$

That is the probability associated with getting a 2, getting a 4, getting a 6, which all are 1 upon 6, 1 upon 6, 1 upon 6. And then from there, I am supposed to subtract p i plus 1. So, here, pi plus 1 is p4 because 2, 4, 6 are the possibilities. So, the next probability is of p4 or getting a 4, And then the next one is getting a 6.

So again, they are 1 upon 6, 1 upon 6. They are subtracted, right? So this gives me the decision weight associated with probability of getting a 2. V4 multiplied by this, which is like probability of getting 4, probability of getting 6, minus probability of getting 6. So this gives me again pi plus the probability of getting a 4.

And this is pi plus p6 that is probability decision weight associated with getting a 6.

Similarly, I can have v minus vf minus where we begin with the lowest value which is v minus 5. You can see that for the lowest value it is simply its probability. So, I have w minus 1 by 6 plus v minus 3. So, I have to apply this.

$$
\begin{aligned}
&V(f^-)\\
&= v(-5)\left[w^-\left(\frac{1}{6}\right)\right] + v(-3)\left[w^-\left(\frac{1}{6}+\frac{1}{6}\right) - w^-\left(\frac{1}{6}\right)\right]\\
&+ v(-1)\left[w^-\left(\frac{1}{6}+\frac{1}{6}+\frac{1}{6}\right) - w^-\left(\frac{1}{6}+\frac{1}{6}\right)\right]
\end{aligned}
$$

I begin from the lowest value. So, 1 upon 6 refers to v minus 5. 1 plus 6 is v minus 3 minus again minus v minus 5. So, this is what I have. And then finally, v minus 1 is arrived in a similar fashion.

So, the value function vf, which is basically vf plus vf minus, we are going to now add these two that we have derived in the previous slide. So, that is how we have V2, then its decision weights, V4 its decision weights, V6 its decision weights and this is the decision weights. Then the V minus F, so V minus 5 decision width, V minus 3 decision width, V minus 1 decision width. As of now, I have not talked about how the W minus 1 upon 6 can be calculated. We have given a formula for pi p in the beginning, but then that pi p is going to undergo some kind of transformation. So, that is why I will be leaving the discussion of calculating these up to this point only. So, we are not going to derive the values of v minus 5, v minus 1, v 4, v 2, v minus 3, v 6, or anything. Neither are we going to derive the value of w plus 1 upon 3, w plus 1 upon 6, and so on.

$$
\begin{aligned}
&V(f)\\
&= v(2)\left[w^+\left(\frac{1}{2}\right) - w^+\left(\frac{1}{3}\right)\right] + v(4)\left[w^+\left(\frac{1}{3}\right) - w^+\left(\frac{1}{6}\right)\right]\\
&+ v(6)\left[w^+\left(\frac{1}{6}\right)\right] + v(-5)\left[w^-\left(\frac{1}{6}\right)\right]\\
&+ v(-3)\left[w^-\left(\frac{1}{3}\right) - w^-\left(\frac{1}{6}\right)\right] + v(-1)\left[w^-\left(\frac{1}{2}\right) - w^-\left(\frac{1}{3}\right)\right]
\end{aligned}
$$

I need to introduce a formula in order to calculate these values. So, I conclude this module with the discussion up to this point. In the next module, we will continue with the discussion on decision weights also, what are the functional form it takes, what are the empirical estimates or values of the different parameters and how do we utilize this information in order to calculate different value functions. I will also present examples to demonstrate them.

Thank you.