PRINCIPLES OF BEHAVIORAL ECONOMICS

Prof. Sujata Kar

Department of Management Studies
IIT Roorkee

Week 27

Lecture 27

Hello and this is the course on Principles of Behavioral Economics. This is Lecture 27, where we are going to discuss the weighting function. So, basically, now we are discussing prospect theory. In prospect theory, first we discussed the two phases: the editing and evaluation phases. After the evaluation phase, we are basically discussing the theory. Of course, evaluation and editing are both integral parts of the theory.

So, in the evaluation phase specifically, while evaluating the prospects, we need to use—or rather, we use—one value function. The value function has two components. One is a value scale, and another is a weighting scale. So, in the previous module, we talked about the value scale. In this module, we are going to discuss the weighting function.

In prospect theory, the value of each outcome is multiplied by a decision weight. So, we have already expressed the functional form of the value function, where each value scale—which is basically a functional form of each outcome—is multiplied by a decision weight. Decision weights are not probabilities. They do not obey the probability axioms, and they should not be interpreted as measures of degree or belief. Consider a gamble in which one can win 1000 or nothing depending on the toss of a fair coin where the probability of winning is 0.5.

So, the prospect is like you can win 1000 with a probability of 0.5 and then you win nothing with a probability of 0.5. However, the decision weight pi 0.5 which is derived from the choice is likely to be smaller than 0.5. That is pi 0.5 is less than 0.5. So all we want to say here is that probability value actually does not directly reflect our preferences or our choices. So here the probability of winning 1000 is 0.5.

But I still may not like this kind of a bet simply because my preferences for this kind of a bet or winning thousand rupees or thousand dollar is less. Alternatively, I attach less importance to the outcome of winning 1000 rupees with a probability of 0.5. So that is why 0.5 is greater than pi 0.5. So this is basically an expression of our preferences, our choices.

So this is what is called decision weight. There are a number of important characteristics of the waiting function that were observed by Kahneman and Tversky. Most obviously, pi is an increasing function of p with pi 0 equals to 0 and pi 1 equals to 1. So, when there is complete impossibility of something happening, then we have p equals to 0 and then pi also takes the value 0. Similarly, when there is complete certainty, then again the pi function does not have much role to play, and accordingly, pi p is equal to 1.

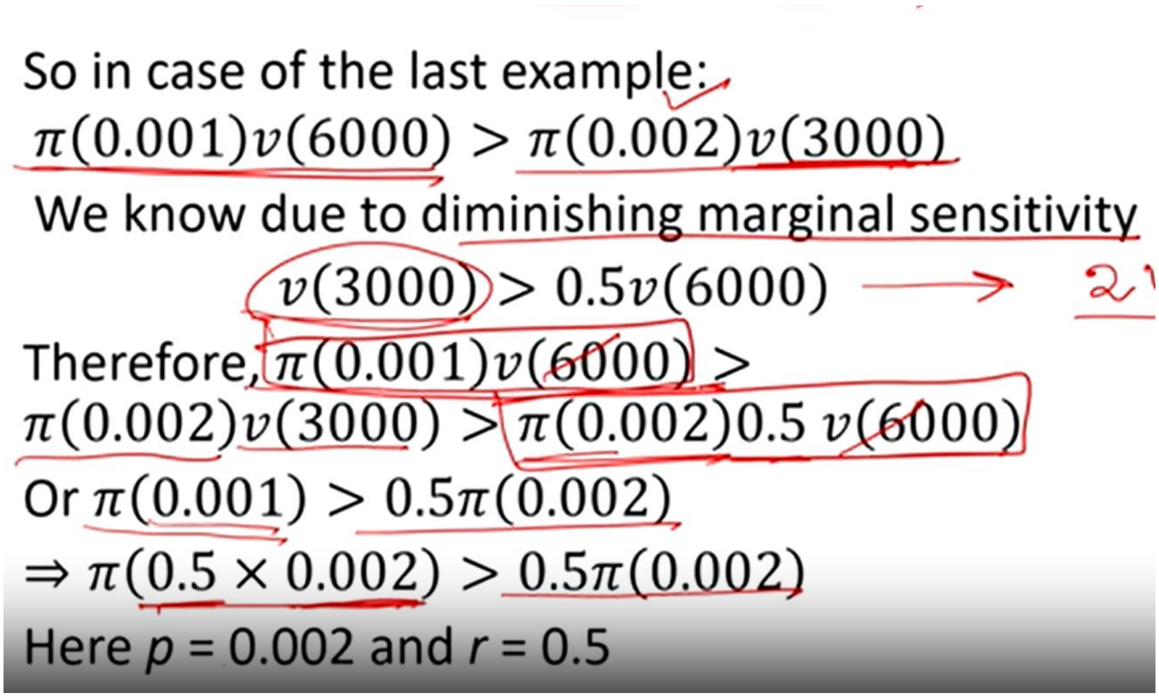
And pi is an increasing function, which implies that as probability increases, our preferences, as expressed by this decision weight, also tend to take higher values. So this implies that impossible events are ignored and the scale is normalized so that pi p is the ratio of the weight associated with the probability p to the weight associated with a certain event. Because, as you understand, if it is a ratio, then we will always have pi 1 in the denominator, which is actually equal to 1. And in the numerator, I will have pi p, where p is something less than 1. The two scales coincide—that is, pi p equals p—if the expectation principle holds, but not otherwise.

Of course, for any value which is greater than 0 and less than 1, because we already know that for p equals 0 and 1, pi p is definitely equal to p. But for any other value of p, the two scales become the same if only the expectation principle holds. In general, the decision weight attached to an event could be influenced by other factors, for example, ambiguity. And that's why the probabilities do not always reflect preferences. In addition, there are three important characteristics of pi that violate the normal probability axioms in expected utility theory, or in short, EUT.

They are sub-additivity, sub-certainty, and sub-proportionality. Now, we are going to discuss each one of these properties. We begin with the discussion on sub-additivity. This is a property of the weighting function for small probabilities. So, this property is valid only when p takes a very small value.

So, for small values of p. For small values of p, pi is a sub-additive function of p such that pi rp is greater than r pi p for any value of r which is between 0 and 1. So, r is also a fraction. Now, we need to see that when p is a fraction, that is, a probability, say p is equal to 0.3. Let r be another fraction, where r takes a value of 0.5.

then p r is actually another fraction which takes a value 0.15 and p r is always smaller than p and r. This comes from the conjunction rule. So, subadditivity implies that pi r p would be greater than r pi p. Let us take an example. In a gamble discussed previously, we have shown that winning 6000 with a probability of 0.001 was preferred to winning 3000 with a probability of 0.002 by 73% of the respondents. Now, first of all, note that these are very small probabilities.



So, first of all, you know, this is a context where we can actually check subadditivity. So, in the case of this example, we can always write that pi 0.001 V6000 is greater than pi 0.002 V3000 because that is the preference that people have expressed. We know due to diminishing marginal sensitivity, V3000 is greater than 0.5 V6000. What does it imply is that this can alternatively be written as 2V3000 is greater than V6000.

Now, what diminishing marginal sensitivity implies is broadly the fact that the utility function is a concave function. And we have already discussed this thing: suppose this is 3000, and so this is 6000. Now, 3000 and then two 3000 would be somewhere here. But 6000 is lower than that, right?

So, concavity implies that either 2V3000 is greater than V6000. Alternatively, we can write it as V3000 greater than 0.5V6000. Therefore, I simply incorporate this value V3000 into this expression. So, what I have is pi 0.002 V3000 pi.001 V6000 that is this one is greater than pi 0.002 I take it from here and then V3000 V3000 is now replaced with 0.5 V6000.

So if I now compare only this expression with this expression then you can see that we can always cancel out V6000 V6000 and then this implies pi 0.001 is greater than 0.5 pi 0.002 right which also implies that this 0.001 can be written as 0.5 multiplied by 0.002 and the right hand side remains as it is. So, this actually implies that pi r p is greater than r pi p where p takes a value of 0.002 here and r is equal to 0.5. So this proves sub-additivity for very small probabilities.

With larger probabilities, for example, in the same context, we previously showed that people preferred winning 3,000 with a probability of 0.9 over winning 6,000 with a probability of 0.5. So, if you try to prove sub-additivity in this context, you will not be able to because the p-values are not small; they are large. So, for small probabilities or small values of p, the sub-additivity relationship holds. This shows that for small p, normal risk aversion for gains and diminishing marginal sensitivity does not hold. It can be explained by a weighting function involving sub-additivity.

The overweighting of probabilities illustrated in the last slide also holds in the domain of losses. Again, where we have the probability of losing 3,000 with 0.002 and losing 6,000 with the probability of 0.001. So, again, when these probabilities are very small, we can easily prove that the principle of sub-additivity holds here as well. However, sub-additivity need not hold for large p, for example, where p is equal to 0.9. In the same context, there is something called overweighting small probabilities.

So, basically, what does sub-additivity imply? It implies that small probabilities are given larger weights. So, Kahneman and Tversky proposed that very low probabilities are generally overweighted, that is, pi p greater than p for small p. Consider the following choice problems for N equals 72. Here, there are first of all two gambles, A and B. A says that you can win 5000 with a probability of 0.001, and B gives you a certain amount of 5.

So, this is a certain win. Now, people preferred A to B. Only 28% like to have 5 rupees or 5 dollars. while majority preferred to have the other option where you can win 5000 with the probability of 0.001. other option, where you can win 5000 with a probability of 0.001. The other alternatives are like C and D. So again, the same individual or another set of individuals are asked to choose between C and D, where C says that you lose 5000 with a probability of 0.001, and your loss is certain by the amount of 5. So here, in this case, 83% preferred D to C, which means the smaller loss is preferred to the larger loss with a very small probability. Now, in A and B, people prefer a lottery ticket over the expected value of that ticket. So, this is how we can interpret the gambles A and B. These gambles are again put here for ready reference. So here, this is actually winning a lottery, right?

So, you can see that the expected value of this lottery is actually 5 rupees. In the sense, if you multiply 5000 by 0.001, then you will arrive at 5. So, people prefer a lottery ticket over its expected value. But in problems C and D, they prefer a small loss, which can be viewed as the payment of an insurance premium over a small probability of a larger loss. So in case you see that, for example, we go for life insurances, we go for health insurance, we go for

insurance against damages to our properties. So that is because we expect the cost to those damages or when there is health issues, medical issues or loss of life, then the damage is of much larger value. And relative to that, we are asked to pay a basically small premium. Now, the chances of those damages occurring is also very small. So, that can be compared with a gamble like C and we generally pay a small amount relative to the amount of the cost of damage as insurance premium.

So, this amount is equivalent to the insurance premium which is essentially again 0.001 multiplied by 5000. So, people prefer to pay an insurance premium as opposed to expecting a large loss, even though with a very small probability. Now, A prefer to B implies pi 0.001 V5000 is prefer to V5. Alternatively, we can write it as pi.001 greater than V5 divided by V5000 that is we are taking V5000 to the other side and this is expected to be greater than 0.001 for a concave function in the domain of gains.

Again by the same logic as I just shown in the previous slide when the utility function is a concave one then your 5000 is here. Your 5 is here and your 5000 could be here, which is basically a 1000 times. But then 5000 is actually would be much higher. That is, you know, 1000 times of 5 would be much higher as compared to where the 5000, V5000 actually is because of concavity.

And as a result of which we expect this ratio to be greater than 0.001 for a concave function in the domain of gains. Similarly the preferences to pay for insurance in gambles C and D implies the same conclusion assuming the value function for losses is convex. So this is what is basically implied by people's tendency to assign greater weights to small probabilities. See here the winning probability is very small, nevertheless people preferred

A to B and here basically the winning probability is again very small but still they expect the loss to take place. As a result of which they prefer to go for an insurance.

consider that this probability is very small and this is not going to happen, then I'll be willing to take the risk and I'll not actually be going for the premium. We would not go for premium unless and until we are really very convinced about the fact that there is a substantial possibility of some damages occurring, either in terms of our health issues or loss of lives or damages to our properties, etc. so as a result of which even though you know there are years when we do not use any medical facilities so health insurance go unused, nevertheless, every year we keep on renewing it because we assign much larger probability of a health problem and the associated damage.

So this is what is over weighting small probabilities. Next, we talk about the second property of subcertainty. The sub certainty principle is stated as pi p plus pi into 1 minus p less than 1 for all p which are between 0 and 1. So now, see that in the beginning, we emphasized that p are probabilities and pi p are actually an increasing function of probability, but it does not reflect any measure of probability.

So, as you understand that p plus 1 minus p is always equals to 1, but pi p plus pi into 1 minus p is less than 1 and it is not equal to 1. That has been empirically observed most often. So for this, we actually consider two gambles again with N equals to 72. Gamble 1 has a winning possibility of 2500 with a probability of 0.33, and the alternative is 2400 with a probability of 0.66.

Gamble B offers a sure gain of 2400 because you can see the probability is 1. And people prefer gamble B to gamble A. Now when we are asked to choose between gamble C and gamble D. C you can see you can win 2500 with the probability of 0.33 and in D you win 2400 with the probability of 0.34. Then people prefer gamble C over gamble D. Here the probability differences are very small and the winning amount are also very close but people prefer to go for the larger winning amount. So from gamble A and gamble B again it is evident that v2400 that is preferred to 0.66 v2400 plus pi 0.33 V2500.

$$v(2400) > \pi(0.66)v(2400) + \pi(0.33)v(2500)$$

Or $$[1-\pi(0.66)]v(2400) > \pi(0.33)v(2500)$$

Or this can be written as I take this term to the other side and write 1 minus pi 0.66 into V2400 which is greater than what remains on the right hand side that is pi 0.33 V2500. Now from Gamble C and Gamble D what we obtain? We see that pi 0.33 multiplied by V2500 is greater than pi 0.34 multiplied by V2400 now what I have written here 1 minus pi 0.66 V2400 greater than pi 0.33 V 2500.

Now, if I compare these two, then we understand that 1 minus pi 0.66 should be greater than 0.34 because we can always cancel 2400, 2400, 2500, 2500, right. So, this remains. Now, rearranging terms, we will see that pi 0.66 plus pi 0.34 is actually less than 1. So, this proves the sub-certainty principle that pi 1 minus p is actually less than 1. That has been observed empirically most of the time.

From Gamble C & D, we obtain

$$\pi(0.33)v(2500) > \pi(0.34)v(2400)$$

Therefore, $[1 - \pi(0.66)] > \pi(0.34)$

Or $\pi(0.66) + \pi(0.34) < 1$

See, these relations, when we use examples, then these examples are actually taken from real-life experiments. When people are asked to choose between A and B, they prefer B to A. And when they are asked to choose between C and D, they prefer C to D, right? And from these relationships, this principle is derived, or we arrive at this principle. So, these principles are empirically validated through experiments.

The slope of pi in the interval can be viewed as a measure of the sensitivity of preferences to changes in probability. Subcertainty entails that preferences are generally less sensitive to variations in probability than the expectation principle would indicate. In the expectation principle, of course, preferences are reflected by probability. So, as probability changes, preferences also change. But in the case of pi, when we use decision weights, they are less sensitive.

As a result, when p increases, pi p might not increase by an equal amount or rather it increases by a smaller amount. And that is why I might not have pi p plus pi. pi into 1 minus p equal to 1. This does not happen. Thus, subcertainty implies that the sum of the weights associated with complementary events is typically less than the weight associated with the certain event.

Now, the third principle is the principle of sub-proportionality. Recall this substitution axiom: if B is greater than A or B is preferred to A, then, for any probability mixture, B, p would be preferred to A, p. We can rewrite this alternatively as: if XP is equivalent to Y, PQ, then X, PR cannot be preferred to Y, PQR for any PQR that lies between 0 and 1. What does it imply?

Or $\pi(p)v(x) = \pi(pq)v(y)$ implies that

$$\pi(pr)v(x) \leq \pi(pqr)v(y)$$

Therefore, $$\frac{\pi(pq)}{\pi(p)} \leq \frac{\pi(pqr)}{\pi(pr)} \quad 0 < p, q, r \leq 1$$

So, I write it as pi pq into vy. Now, what the above statement says is that if this holds, then pi p r vx equals is less than pi PQR Vy. From these two expressions, we can always arrive at this expression where pi PQ by pi P is less than or equal to pi PQR upon pi PR. for all values of PQR that are greater than 0 and less than or equal to 1.

Now, this is known as the sub-proportionality principle. This implies that for a fixed ratio of probabilities, the ratio of corresponding decision weights is closer to unity when the probabilities are low than when they are high. Now, just notice that in this principle, if all PQR are less than 1, then PQ would be less than P. Similarly, PQR would be less than PQR. So, this is actually smaller than this.

This is smaller than this, right? This implies that the probabilities are actually getting smaller and smaller. So, on the right-hand side, the probabilities—that is, the values within the brackets are smaller than the probabilities which are there in the brackets on the left-hand side. We take an example; this is also a gamble we have already talked about in some other context. 80% preferred the certainty of 3000 to the uncertain prospect 4000, 0.8.

But when these outcomes had the probabilities reduced by a common factor of three-quarters, the situation was reversed. 65% preferred the prospect 4000, 0.2 to the prospect 3000, 0.25. Now here, R takes a value of 1 by 4. So what we are saying is that 3000 was preferred to 4000, 0.8 and in the next 4000 0.2 was preferred to 3000, 0.25.

Now, here you note that here the p takes a value of 1, and this is say 0.8. So, here the ratios are of, say, this is pq. If we call it pq, then this is the ratio of 0.8 by pi 1, and here this is 0.2 and this is 0.25. So, if we stick to our values of p as 1 and q as 0.8 then r would turn out to be 1.4.

Ideally in both cases we are actually having a ratio of 1 upon 4. This is also 1 upon 4, this is also 1 upon 4. In simpler terms people judge probabilities that are the same in relative terms like 1 to 0.8 and 0.25 to 0.2 to be more similar when probabilities are small. So 0.25 is judged more similar to 0.2 than 1 is to 0.8.

So, subproportionality together with the over weighting of small probabilities imply that pi is subadditive over 0.1. We are not going to get into the proof of this claim, but then these two together actually implies the very first principle that we discussed of subadditivity. The figure presents a hypothetical weighting function which satisfies over-weighting and sub-additivity for small values of p as well as sub-certainty and sub-proportionality. Pi p is less steep than the 45 degree line. This is the 45 degree line going through the center and this is the pi p line.

This is less steeper than the 45 degree line and discontinuous at both ends. We will offer some explanations for this kind of observation. The figure is reproduced here. The three properties discussed entail that pi is relatively shallow in the open interval and changes abruptly near the endpoints where pi 0 equals to 0 and pi 1 equals to 1. The sharp drops or apparent discontinuities of pi at the endpoints are consistent with the notion that there is a limit to how small a decision weight can be attached to an event.

If it is given any weight at all. A similar quantum of doubt could impose an upper limit on any decision weight that is less than unity. So basically, here these discontinuities are because of bounded rationality. We may not be able to assess properly how small a probability to attach to an event which is nearly impossible, or how big a probability we should attach to an event which is nearly certain.

Further, the simplification of prospects in the editing phase can lead the individual to discard events of extremely low probability and to treat events of extremely high probability as if they were certain. So the alternative is that for extremely uncertain outcomes or improbable outcomes, we can assign pi 0 equals to 0 straight away. even though the probability is not equal to 0. Alternatively, we can also for extremely certain event make pi p equals to 1 though p is not equals to 1.

Because people are limited in their ability to comprehend and evaluate extreme probabilities, highly unlikely events are either ignored or over-weighted, like this, and the difference between high probability and certainty is either neglected or exaggerated like this. Consequently, pi is not well-behaved near the endpoints. Now, decision weights are expressed as a function of stated probabilities. So pi p is essentially a function of probability where p is known to the respondents.

The most commonly used function is something like this. Gamma determines the curvature of the value function. We are not going to discuss this functional form much here because when we discuss later in cumulative prospect theory, then probably this will be discussed at length. The function is monotonic transformation of outcome probabilities that is if f is not equal to g and fs is greater than gs for all s belongs to s then f greater than g. than g. This simply implies that if I try to draw a monotonous function, I have say s and g here, I am measuring the functional form here.

Then a monotonically increasing function would say that if s is greater than g, then fs will also be greater than fg. So this is what it implies in pretty simple terms. There are two problems with this functional form though. It doesn't always satisfy stochastic dominance. And it is not readily extendable to prospects with a large number of outcomes.

So far, we have talked about prospects with only two outcomes, and this cannot be conveniently extended to more than two outcomes. And as a result of which in the next we are going to discuss or we are going to begin our discussions with stochastic dominance and then see what alternatives are suggested by Kahneman and Tversky in order to improve upon these drawbacks. These are the references. Thank you.