PRINCIPLES OF BEHAVIORAL ECONOMICS

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Week 22

Lecture 22

Hello everyone, this is lecture 22 of the course on Principles of Behavioral Economics. Hello everyone, this is lecture 22 of the course on Principles of Behavioral Economics. In the previous lecture or module, we studied risk attitude. So, I will continue or begin with risk attitude first and then discuss some of the axioms of expected utility theory. Alongside discussing the axioms, I will also talk about the violations observed by behavioralists in the literature. I discussed the three types of utility functions in the previous module.

Utility functions can be linear, concave, or convex. Now they are being portrayed here. As you can see, this is the linear functional form, which is risk-neutral. This one is for risk-averse, and this one is for risk-loving. So, this one we call a concave function, which is concave to the horizontal axis.

This one we call a convex function, which is convex to the horizontal axis. In prospect theory, we will also use these kinds of functional forms, and they will all be convex or concave with respect to the horizontal axis. Now, what does this curve signify, or why does it say that a risk-averse individual will have this kind of utility function? Now, I am going to explain this in the following slides.

Now here a concave function or concave curve is portrayed. Now suppose in a gamble one has a probability of 0.5 to win x1 and to win 0.5 again to win x2. x1 and x2 are amounts of money so they are measured on the horizontal axis. The expected payoff from this gamble, which is 0.5 into x1 plus 0.5 into x2 is exactly equidistant from x1 and x2. So, where does it lie?

This lies here and we are saying that the distance between this and this are the same. Note that on the vertical axis, we have expected value of or rather utility of expected value of x, which is utility of 0.5 x1 plus 0.5 x2. Basically, 0.5 x1 plus 0.5 x2 is the expected value. The utility associated with the expected value is actually greater than the expected value of the utility associated with that combination, right. So, which is basically 0.5 into ux1 plus 0.5 into ux2. Just there is minor difference. First, we are multiplying the outcomes with the probabilities and then summing them up, taking their utility. So, that is giving us the utility of expected value.

And when I am multiplying the utilities obtained from individual amounts or outcomes and then multiplying it their respective probabilities, that gives me the expected value of the utility associated with that combination. And what we observe is that E(u(x)) is actually lower than u(E(x)). So, E(u(x)) is basically exactly halfway through the two utilities associated with x 1 and x 2. So, this is the utility associated with x1, this is the utility associated with x2 and expected value of u(x) is exactly halfway between these two values.

But the utility of the expected value which is like this, is higher than the expected value of the utility. This means that the slope of the utility function between E(x) and x2 So, between E(x) and x2 is smaller than the one between x1 and E(x). So, that is evident, right? If we consider this stretch, where we have the expected value E(u(x))

is actually lying exactly in between then the slope here and the slope here are the same but here the slope is steeper and this is basically smaller. So, that's what is mentioned here, this means that the slope of the utility function between E(x) and x2 is smaller than between x1 and E(x), leading the midway between u(x1) and u(x2) to fall below u(E(x)). And this is happening because of diminishing marginal utility. So, as one has more and more wealth, possibly he or she becomes more and more risk-averse.

And that is why would prefer or associate lower amount of utility to an outcome like x2. This can also be alternatively seen as a situation that between u(x1) and E(x1), U(E(X)), the distance is larger than the distance between U(E(X)) and U(X2). What I am trying to say is that u expected value of x minus u expected value, sorry, u(x1) is greater than u(x2) minus u (expected value of x), right. So, this implies that

Utility is now increasing at a decreasing rate as we move from x1 to the middle point to the highest amount or a larger amount of x2. This is due to diminishing marginal utility of wealth. Further note that if xce is the certainty equivalent then u(xce) is equal to 0.5u(x1) + 0.5u(x2). So, this is exactly equivalent to this number expected value of ux.

$$u(E(x)) = u(0.5x_1 + 0.5x_2) > E(u(x)) = 0.5u(x_1) + 0.5u(x_2).$$

It yields the same level of utility as the expected utility of the gamble. So, this is how we define the certainty equivalent. The certainty equivalent falls below the expected value of the gamble. Again, the certainty equivalent is also lower than the expected value of the gamble, reflecting that the person is willing to give up some amount of money on average in return for certainty. In general, if a person is willing to take a reduced expected value to obtain certainty, we call the person risk averse.

Next, we start with the axioms of expected utility theory. In this module, we will first introduce some important axioms of EUT followed by their violations as observed by the behavioralists. The axioms that would be discussed in this and later modules are completeness, transitivity, continuity, independence, monotonicity and substitution axioms. So, we begin with completeness. Some of them have already been very briefly discussed in previous modules, but now we are going to deal with each one of them in detail.

Completeness says that for all A, B, either A is preferred to B or B is preferred to A, or A and B—the individual is indifferent between A and B. Now, these are the signs.

$\succ$: preferred to; $\sim$: as good as; $\succcurlyeq$: at least as good as

The next assumption is transitivity. For three prospects A, B, C, if A is weakly preferred to B, that is as good as B or preferred to B, and B is again weakly preferred to C, then A should be weakly preferred to C. This must hold.

These two axioms together are known as the order or ordering axiom. So, completeness basically tells you that I have a complete profile of my choices. If I am presented with 10 items, I can clearly say which one I prefer, or I can clearly rank all the items according to my preferences. And transitivity says that

moving from one end to another, there should not be any inconsistency. If I have placed one item to the extreme right and this is the most preferred item and the least preferred item is on the left, then it is not possible that something which is towards my right is not preferred to something which is on my left. So, the preference should be consistent. Ideally, it makes perfect sense and

We must be thinking that yes, that is how we should behave, and we might be behaving most of the time. But then there are so many evidences of intransitive preferences as well.

So we begin with an example of intransitive preference, which we also call preference reversal when we reverse our preferences. So these are done. Most of the examples or gambles in this module would involve monetary incentives or money values. So there are two alternative gambles, gamble A and gamble B. They are defined very clearly here.

That in gamble A there is 0.4 probability of winning $10, 0.6 probability of winning $3. Gamble B gives us 0.7 probability of winning $7.5 and 0.3 probability of winning $1. So one has to choose between either A or B. It was observed that 51% preferred B to A. So B is preferred to A. The larger percentage basically preferred B over A. Gamble C and D are defined as C: one probability, that is, there is complete certainty of winning $5. 0.7 probability of winning $7.5, 0.3 probability of winning $1.

And it was observed that 88% preferred C to D. So people mostly went for the sure gain. So here we observed C is preferred to D. Next, we have two more gambles, Gamble E and D. In E, there is 0.4 probability of winning $10 and 0.6 probability of winning $3, while Gamble F gives a sure probability of getting $5. So this is not actually winning but essentially getting $5. And 70% again observed to prefer E to F which implies that here people preferred E to F.

Now what is happening here is that gamble A is identically equal to E. You can see that Gamble A and Gamble E are identical. Gamble B and Gamble D are identical. Gamble C and Gamble F are identical, right? Now, if I write the preferences, then we observe that E was preferred to F. So, E, equal to A, was preferred to F. Now, F is equal to C. So, F and C are identical. So, F was equal to C. C was preferred to D.

Now, D and B are identical—D, B identical. So, D, B identical and B was preferred to A, which implies that if I combine all of them together, I will be finding A preferred to C, preferred to B, preferred to A. So, overall, this is an intransitive preference, or we call it preference reversal. This is one example of a violation of transitivity.

Next, we talk about the third assumption or axiom of continuity. Continuity says that if A is preferred to B, preferred to C, then there exists some probability r such that rA plus (1-r) C is identical to B or one individual would be indifferent between this combination and B. That is, there is indifference between the middle-ranked prospect B and the prospect A, r, C, 1-r. The latter is referred to as a compound prospect. This is called a compound prospect.

Now, the thing is that, so we are talking about some probability that exists such that the combination of A and C makes it as good as the combination B. Further, for any p which is greater than R, suppose R takes a value of 0.3. Then any p which is greater than r, say p is equal to 0.4. Then what it says is that pA + (1 - p) C should be strictly preferred to B. So as the probability increases of this compound prospect, earlier it was indifferent, but then it should not remain indifferent anymore.

It should be strictly preferred to B. Similarly, if there is any q which is lower than r, suppose q is equal to 0.2. Then, of course, as the probability here decreases of winning this amount, earlier it was indifferent, but it will not remain indifferent anymore. Now, B should be preferred to qA + (1-q)C. Continuity ideally implies that there is, first of all, this also implies that our preferences would remain consistent or transitive across different combinations of or different values of probabilities.

The second thing is that there is not really any jump that this is a combination that we prefer. Now with a change in probabilities, it is not the case that we are going to change our preferences drastically. So these are basically the understandings or the concepts behind the assumption of continuity. The fourth assumption or axiom is of independence. Independence is defined as for all A, B, C, if A is preferred to B, then (A, p; C, 1 - p) will be preferred to (B, p, C, 1-p) for all p.

It is almost like the axiom or the assumption of invariance, which ideally tries to tell us that something which is not determining the preferences between A and B. So A is preferred to B and that holds regardless of whether there is a third option which is C and the possibility of C happening is 1 - B. So this relationship remains valid despite bringing in a third factor like C. I can give you a very simple example: I like tea over coffee. Now C is the probability of rain on a particular day. So today, the probability of rain is 0.6.

Now, regardless of whether it rains or not, my preference for tea over coffee is not going to change. So even if it rains, I will prefer tea over coffee. Even if it does not rain, I will prefer tea over coffee. The preference is independent of another event, which is, say, rain. Now, this is an example given by Kahneman and Tversky.

If prospect A, which offers $3,000, is preferred to prospect B, which offers $4,000 with a probability of 0.8, then prospect A', which offers $3,000 with 25% probability, should be preferred to prospect B prime, which offers $4,000 with 0.2 probability. Now, see that between A and B and A prime and B prime, the outcomes are the same. The outcomes are

3,000, 3,000, 4,000, 4,000. What is changing here is the probabilities, and the probabilities are reduced in equal proportion.

So, something which was 1 earlier becomes 0.25. So, it is reduced to one fourth. And similarly, something was 0.8 earlier. This is also reduced by one fourth to 0.2, right?

So, this is why if A is preferred to B, then A prime should be preferred to B prime. The preference between A and B is independent of C. Here, C is basically 0 and p is 0.25. So, I can always, you know, write these prospects with a C in between. However, Kahneman and Tversky found that About 80% of those asked would choose gamble A over B. So A was preferred to B and about 65% would choose gamble B prime over A prime.

So even though they preferred A to B, they did not prefer A prime to B prime. This is a violation of the axiom of independence and expected utility theory. Another instance of violations of the axiom of independence is termed as the isolation effect. Consider the following two stage game. In the first stage, there is a probability of 0.75 to end the game without winning anything and a probability of 0.25 to move into the second stage.

If you reach the second stage, you have a choice between winning 4000 with a probability of 0.8 and 3000 with certainty. Now this is what is being shown here. So there is 7.75 or 3.4 chance of ending the game where no one is getting anything. If you reach the other stage with 0.25 probability or one fourth probability. Then you have two alternatives.

Either you win 3000 with certainty or you win 4000 with 0.8. And of course, the other remains at 0 with probability 1.5 or 0.2. Your choice must be made before the game starts. That is before the outcome of the first stage is known. So in this game, one has a chance between 0.5 and 0.8.

multiplied by 0.8 equals to 0.2 chance to win 4000 and 0.25 into 1 to win 3000. This is because there is three fourth of 0.5 chance of ending the game. So you would be winning these amounts only if you enter the game with 0.25 probability on the other side where these chances would appear. Now, this is called a chance mode, and this is called a decision mode.

Thus, in terms of final outcomes, the probabilities one faces are a choice between 4000 with 0.2 and 3000 with 0.25. So, though they appear like 3000 with certainty and 4000 with 0.8, since they In the overall game, you enter this side with 0.25 probability. Your chances are actually much reduced here. Of 141 students who answered, 78% chose the latter prospect, that is 3000 with 0.25.

This contradicts a previous gamble where 65% chose 4000 with 0.2 over 3000 with 0.25. So the gamble we just discussed previously that There were two options: 4000 with 0.8 and 3000 with certainty. People basically preferred this over this, but then when they became 4000 with 0.2 and 3000 with 0.25, Then people prefer this over this.

But again, here we observe that in this case, 78 percent actually preferred this one, which means a much lesser percentage preferred this. So this is a contradiction to what was observed previously. Evidently, people ignored the first stage of the game, whose outcomes are shared by both prospects, and considered it as a choice between 3000 and 4000, 0.8. So, the reduced percentages or probabilities are actually not considered by individuals, and as a result, we observed this contradiction. Choices are primarily determined by the probabilities of the final state so probabilities of the final state appeared like 3000 with certainty and 4000 with 0.8

and as a result of which that was the preference now another anomaly of this assumption is the common outcome effect consider the following prospect here this is one gamble and these are the expected payoff from each one of them And this is gamble C and D, another gamble. You have to either choose between A and B or between C and D. So, the gambles and their expected payoffs are mentioned here. Now, you notice that 0.89, or rather if I remove this third column, then of course, the option between

A and C that is the gamble A and gamble C are identical, gamble B and gamble D are identical. I repeat if I do not include the last column then gamble A and gamble C are identical with the probabilities and gamble B and gamble D are also identical along with the probabilities right and gamble between gamble A and gamble B are Of course, there is a chance of winning 500 in both cases with a probability of 0.89. Similarly, between gamble C and gamble D, there is a 0.89 chance of winning 0.

So, since these are common for both A, B and C, D, individuals must be ignoring this. So, the decisions between the first two options in both cases should be independent of the third. The decision between this and this should be independent of this. Rather, the decision between choosing A and B should be independent of this because this has the same probability and same amount. Similarly, the decision between C and D should be independent of this because again these are the same amount.

Now, that is why I am talking about removing the last column, which makes A and C identical, and B and D identical. But it is observed that A is preferred to B while D is preferred to C. The argument was that if A is preferred to B, then C should be preferred to

D because they are identical. But this has not been observed it should now be seen that when the third state of the world is ignored this is the third state of the world and if this is ignored then the choice between A and B is identical to the choice between C and D. Nevertheless we see that there are contradictions.

So, this is again the assumption of independence or the axiom of independence, which says that since the third state of the world is identical, it can be ignored, but that is actually not happening. People do not always make decisions according to what is stated under various axioms of expected utility theory. Next, we talk about monotonicity. We assume that there are several outcomes, like x1, x2 to xn, ordered from worst to best. A prospect q equals to x1, pq1, xn, pqn stochastically dominates another prospect like xpr1 to xn prn.

If the summation of all the probabilities under prospect q is greater than equal to summation of all the probabilities under prospect r. So, what does it imply? I will explain that with an example. Here, we need a strict inequality for at least one j. So, for this to hold, at least one pqj must be greater than one prj, and the rest can be equal.

$$\sum_{j=1}^{n} p_{qj} \geq \sum_{j=1}^{n} p_{rj}$$

Thus, this greater-than-or-equal condition holds. A clearly dominant gamble could be given as like this so here the case is that in an urn there are 90% white balls, 6% red balls, 1% green balls, 1% blue balls and 2% yellow balls. These informations are given to you along with the information, that if you pick up a white ball you get nothing. if you pick up a red ball you get $45 if you pick up a green ball you get $30.

If you pick a blue ball, you lose $15. If you pick a yellow ball, you lose $15. Similarly, the percentage of balls is the same here for Gamble B, but the amount of money is slightly different. Now, what you observe is that for white, the amounts are the same. For red, the amounts are the same. For green, the amount is higher here by $15.

For blue, the losing amount is basically smaller here. And for yellow, the losing amount is the same. So, clearly, gamble B would be preferred to gamble A. So, gamble B is strictly dominated by gamble B. Now, Kahneman and Tversky ask 124 people to choose between the following gambles. Now here, the original gamble I have reproduced is the gamble that Kahneman and Tversky asked 124 people to pick in an experiment.

Gamble C		Gamble D	
Marbles	Money	Marbles	Money
90% white	$0	90% white	$0
6% red	Win $45	7% red	Win $45
1% green	Win $30	1% green	Lose $10
3% yellow	Lose $15	2% yellow	Lose $15
xpected values:	$2.55		$2.75

So now just see that I have 90, I have white, red, green, and yellow balls. The gambles in terms of their percentages of balls also change slightly. The blue balls are dropped. The original strictly dominant gambles were B strictly dominates, A is also presented here just for reference. Now note that 90% white, 90% white remains the same where you pick up a white ball and you get nothing. Now 6% red and here it is 7% red. We have 1% green, you win $30; 1% green, you lose $10.

3% yellow, you lose $15; 2% yellow, you lose $15. Now, if I compare gamble C with gamble A, then you will see that here 1% blue ball is actually merged with 2% yellow. This 1% blue and 2% yellow are merged to obtain or arrive at 3% yellow. So, in both cases, we are losing $15; we are losing $15 here. Now, if I compare gamble D and gamble B, then here I have 7% red and here I had 7% red and 1% green.

In both cases, we had won $45. So here we have 7% red and $45 is the winning amount. 1% blue is replaced by 1% green where you are losing $10. 2% yellow losing $15, 2% yellow losing $15. They remain the same.

So these two gambles, like A, C and B, D, are very, very much comparable, right? So if B dominated A then D should clearly dominate C. Here the expected values are 2.55 and 2.75. From that perspective, D should be dominated by C. But nearly 60% were found to prefer C over D. People were attracted to the choice with fewer negative outcomes and more positive outcomes even though the choice with fewer negative outcomes was not as favorable so if i go back to the original table then you can see that here there are two

losses and one win here there are two wins and one loss and as a result of which people were attracted more towards option C than gamble D so that's what it's mentioned here this was due to a lack of transparency in presentation of the gamble that is what Kahneman and Tversky claimed with this, I conclude today's discussion on axioms of utility. One axiom remains, that is the axiom of substitution. We will continue from there in the next module. Thank you.