PRINCIPLES OF BEHAVIORAL ECONOMICS

Prof. Sujata Kar

Department of Management Studies
IIT Roorkee

Week 21

Lecture 21

Hello everyone, this is lecture 21 of the course on Principles of Behavioral Economics. In this module, we are going to study prospect and risk attitude. As I previously mentioned, we have begun with the third topic of judgment and decision under risk and uncertainty. Under that, we first talked about various concepts of probability that we are going to utilize again going forward while discussing prospect theory. Using those concepts of probability, we first discussed Bayesian analysis, some Bayesian rules, and then Bayesian updating.

Now, moving further toward prospect theory, we are going to first define what prospects are and discuss risk attitude, how people perceive risk, and how that can be modeled using basic graphs and mathematics. That is the main purpose of this module. Expected utility theory, which we have already talked about previously in various modules, is also part of the neoclassical tradition. It has dominated the analysis of decision-making under risk.

It has been generally accepted as a normative model of rational choice and widely applied as a descriptive model of economic behavior. Thus, it is assumed that all reasonable people would wish to obey the axioms of the theory and that most people actually do so most of the time. So, basically, again the assumption of rationality prevails. Prospect theory by Kahneman and Tversky, primarily dealing with economic behavior under uncertainty, largely questioned the axioms of EUT and offered a more realistic modeling of behavior under uncertainty.

This module deals with one important aspect of EUT, namely the degree of risk aversion. First, we define prospect, and then, using the concept of prospect, we will define different risk attitudes. Decision-making under risk can be viewed as a choice between prospects or gambles. A prospect XP1, XP2. x1, p1, xn, pn is a contract that yields outcome xi with probability pi where p1 to pn adds up to 1.

A prospect is like a combination of outcomes associated with probabilities. So, if I go back to the simple probabilities or examples we were considering in the previous module, then, say, if I flip a fair coin, then the prospects are: heads will appear with a probability of half or 0.5, and the other prospect is tails with a probability of half. Similarly, when we roll a die, then the prospects are: 1 with a probability of 1 upon 6, 2 with a probability of 1 upon 6, and so on. Thus, The final one is 6.

The outcome is 6 with a probability of 1 upon 6. So, these are simple examples of prospects. To simplify notation, we omit null outcomes and use x, p to denote the prospect x, p; 0, 1-p. That yields x with probability p and 0 with probability 1- p. So, basically suppose somebody is going to give me 10 rupees. So, there is actually the probability of not getting the money is 0.

So, in that case, I will not be getting anything; that is, I will be getting 0. The probability is also 1 - p, that is, 0. So, as a result, we omit null outcomes. In that case, I will write this as 10, 1, and I am not going to get anything, that is, 0, with a probability of 0, which is not mentioned. So, null outcomes are basically omitted.

Similarly, if the prospect is like I am going to get 10 rupees with a probability of 0.5 and 0 with a probability of 0.5, then again, 0 with the probability of 0.5 can be omitted, and we can simply write it as I am going to get 10 rupees with the probability of 0.5. In fact, this is actually written as 10. So, here, since it is 1, the probability is 1, I do not need to even mention 1. As I just mentioned, the riskless prospect that yields X with certainty is denoted by X. So, if I am going to get 10 rupees with certainty, I will write it simply as X.

The present discussion is restricted to prospects with so-called objective or standard probabilities. So, most often, we would have situations where there are probability values between 0 and 1. If there is complete certainty, then there is no need for the discussion of probabilities, since this module is devoted to decision and judgment under risk and uncertainty. So, you would be dealing with some level of probabilistic events only. The application of expected utility theory to choices between prospects is based on the following three tenets: Expectation principle, asset integration, and risk aversion. These three also can be called the three main pillars or very important assumptions of the expected utility theory. Now, we are going to discuss each one of them.

Beginning with the expectation principle, a real-valued function with the domain of an outcome or probability space is a random variable. The realized values of X are denoted by X1, X2 to Xm or Xn. So, the thing is that this was also discussed in the previous module—the outcome space.

$$E(X) = P_1X_1 + \cdots + P_mX_m = \sum_{i=1}^{m} P_iX_i$$

We can also call it a probability space. So, for example, if I take the case of flipping a coin, then the outcome space would be H and T. Imagine that probabilities P1 to Pm are attached to each outcome or elementary event. So, these are called outcomes or elementary events. So, each one of them has a probability. So, we call it P1, we call it P2.

P1 takes a value of 0.5, P2 takes another value of 0.5, and so on. Then, the expected value of a random variable X is the expected value equal to P1 X1 plus P2 X2 and so on up to Pm Xm, and then we are summing it up. Now, just notice here one difference: here we are assuming H and T. Now, H and T cannot be here. So, most often in this kind of expression, we will be dealing with some monetary values.

For example, if you get an H, then you obtain 10 rupees. If you get a tail, then you have to pay 10 rupees. So, I can have two values: 10 and minus 10. In that case, as you understand, the expected value would be 0 because I will be having 0.5 multiplied by 10 plus 0.5 multiplied by minus 10. Done.

For example, imagine that in a game, you win $3 if a six-sided die is thrown and 1 or 2 comes up. And you win $12 if 3, 4, 5, or 6 comes up. An individual can get $3 with a probability of 1 by 3 because you are getting 1 or 2. So, you apply the OR rule. The possibility of 1 coming up is 1 upon 6.

The possibility of 2 coming up is 1 upon 6. I simply add them to arrive at 1 upon 3. In a similar fashion, the rest of the numbers—3, 4, 5, 6—if anything comes up, then you are going to get $12. By following the same principle, I can obtain the probability of getting any one of them. That is, between 3, 4, 5, and 6, it is 2 by 3.

So, how do we write the prospect? We write the prospect that I will get $3 if 1 or 2 appears. The probability of 1 or 2 appearing is 1 upon 3. I will get $12. If anything between 3, 4, 5, or 6 comes up, the probability is 2 by 3.

The expected value of this lottery will be simply 1 by 3 into 3. So, probability multiplied by the outcome and then probability multiplied by the outcome, it turns out to be $9. We just need to remember that. The outcome here is $9 because I might use it in the following slides. If you could choose between this bet and getting $9 for sure, which one would you choose?

Now again, here the short prospect is $9. So, I can write it as 9 or simply write it as 9. In economic experiments with such a choice, most people choose $9 for sure rather than a game with risk. Such experimental results are consistent with the fact that many people try to avoid risk in economic choices in real life, and we call them risk-averse individuals. Expected utility theory explains a person's risk aversion by predicting that she will choose a lottery that gives her the highest expected value of utility.

Rather than choosing a lottery that gives us the highest expected value of money prices. So, now we have started differentiating between money prices and utility. So, money prices in the previous example were $3 and $12. What we are trying to say is that it is not necessary that $3 will give me some utility which, if I multiply 3 by 4, then I will arrive at 12, right?

So, here the winning amount is 4 times higher than the winning amount when 1 or 2 comes up. But what it says is that when it comes to utility, it is not necessary that the utility from obtaining $3 is 4 times the utility of obtaining $12. So, this is not necessarily true. Nobody guarantees that this is going to happen. So now we are moving from money prices to utility, and people basically go for the highest expected value of utility rather than the highest expected value of money prices.

Consequently, the expectation principle considers the expected values of the utilities associated with the outcome and is written as something like this. So, now instead of having P1X1, I am having P1UX1, PNUXN. So, the probability of the nth outcome occurring and the utility associated with the nth outcome. Similarly, the probability of the first outcome occurring and the utility associated with the first outcome, and all of them are summed up to arrive at the total utility or expected utility.

This is the expectation principle. This is the expectation principle under the expected utility theory. Here, uxi refers to the utility from outcome i. Next, we'll talk about the second tenet, which is asset integration. Imagine that the individual holds E dollars as her initial endowment, and the additional gains from profit or losses from the stock price movements are denoted by X. Or it could be stock price movements, or it can be additional gains from any gambles, including the gambles we just considered in the previous slides. The individual's expected value of utility in that case will be the initial endowment plus whatever he or she gains. So, if there are several alternative outcomes possible, then again I have the initial endowment plus the first outcome, initial endowment plus the nth outcome.

So, now the utility is not only dependent on the additional gain but is actually a function of what you have and what is added to it. And then each outcome is multiplied by their respective probabilities and added up. We arrive at the expected value of the utility, which considers the initial endowment plus the outcome. And then, summing up, we write it like this. Now, suppose we have a specific form of utility function.

$$E\big(u(e+x)\big) = P_1 u(e+x_1) + \cdots + P_n u(e+x_n)$$
$$= \sum_{i=1}^{n} P_i u(e+x_i)$$

So, ux equals the logarithm of x. We specify it as the utility function and consider the previous example. Now suppose the initial endowment E is 10. The individual playing a gamble has $10 in his or her pocket. And then he is in a gamble.

Let's see what the prospects are that she has of winning or losing something. So if I continue with the above example, the previous example of throwing a dice with a prospect of 3, 1 upon 3, and 12 with a probability of 2 by 3. Then, because now he or she has a $10 initial endowment. So $10 plus 3 becomes $13. So, the logarithm of e plus x1 is the logarithm of 10 plus 3. Here, x1 is the first outcome, which is 3, and that turns out to be approximately 2.565. Similarly, the logarithm of e plus x2 is the logarithm of 10 plus 12, the alternative outcome, right? The second outcome, which is the logarithm of 22, is approximately equal to 3.091. The expected utility from this lottery is 1 upon 3, that is, 1 upon 3 multiplied by the logarithm of 13, then 2 upon 3 multiplied by the logarithm of 22, and that gives us roughly a value of 2.91. If compared with the prospect of getting $9 for sure, the expected utility will be the logarithm of 19 because my initial endowment is 10, and I am getting a sure gain of 9.

So, 10 plus 9, 19 makes it approximately equal to 2.94. Therefore, if the individual has the utility function of the logarithm of x, then she will prefer getting $9 for sure. to the lottery in the above example. So what we observe is that we claim that an individual is risk-averse if he goes for a sure gain of $9.

$$(1/3) \times ln(13) + (2/3) \times ln(22) \cong 2.91.$$

Now, a risk-averse individual will prefer going for a sure

gain, and he or she has a utility function which is like u equals the logarithm of x, right? So it is possible that the logarithm of x is a concave function. Next, we will be talking about that, but before that, I will formally define the asset integration concept. So formally, x1 P1 to xn Pn, that is, the prospect is acceptable at asset position w, which is basically the initial endowment, if and only if the utility from alternative prospects multiplied by their probabilities

is greater than the utility from the initial endowment. So, what this simply implies is that if the prospect basically provides additional value, as a result of which there would be additional utility, then only this prospect will be acceptable. So, this is the concept of asset integration. So, a prospect is acceptable if the utility resulting from integrating the prospect with one's assets exceeds the utility of those assets alone.

Thus, the domain of the utility function is its final states. Rather than gains or losses. Final state implies which includes one's asset position. Now imagine an individual with a $10 initial endowment who is thinking about either receiving a dollar amount for sure or receiving the bet having a prospect like this. So this is the previous example we are continuing with.

She will be indifferent between receiving the bet and receiving a certain amount of money for sure if the certain amount is about $8.45. Now, suppose I note that here I am talking about indifference. So, previously that individual preferred $9 over this prospect, right, which means $9 is strictly preferred to this prospect. Now, I am talking about an amount, say x, with which the individual is basically indifferent between this amount and the prospect, and this amount turns out to be $8.45. How?

This is simply the initial endowment, and if I plug in the value of $8.45, then I will be getting approximately a value of 2.92, which is equal to the expected value from the prospect. So, as a result, I can say that $8.45 gives approximately the same value as the prospect. So, the individual would be indifferent between having this amount and going for the prospect—indifferent in the sense that he would be as good as having the prospect or this amount, not $9. The certainty equivalent of a bet is the sure amount of money that an individual views as equally desirable as the bet.

Let us call it y. Then, the utility of the initial endowment plus the certainty equivalent is equal to the utility plus the initial endowment. Plus the amount that is coming from the prospect or expected value of the utility from the initial endowment plus the prospect. A risk premium is the difference between the expected value of the bet and its certainty equivalent. So, for example, in the previous case, our y was 8.45. So, all we tried to say is that the utility from the initial endowment 10 plus 8.45, that is 18.45,

was equal to the expected value of the utility from the initial endowment plus the prospect that we had. We just mathematically showed that these two numbers are the same. So, the individual is different. Now, the risk premium is the difference between the expected value of the bet and its certainty equivalent. The difference between the bet's expected value E(x) and its certainty equivalent

which is E(x) - y, is the risk premium. So, in the above example, if you remember, the expected value of the bet was 9, so here the risk premium is $0.55. Because the difference between 9 and the certainty equivalent 8.45 is 0.55. This is one way of finding out the risk premium in this context. Now, there are three possible kinds of attitudes towards risk for an individual. If the risk premium is positive, then she is risk-averse. In the sense, if I am looking for a positive risk premium, then I am a risk-averse individual.

If the risk premium is negative, then she is risk-loving. So basically, I am not looking for a risk premium. Here, by risk premium, I mean that, for example, we all know in investments where the risks are higher, then the returns are also higher. So the idea is that when I am going to take a risk, I want to be rewarded for that, and that reward comes through higher returns. Now, a risk-loving individual might not look for substantially higher returns because she or he has a substantial appetite for risk.

So, the risk premium is negative; the risk premium is zero when someone is willing to take a risk without looking for a reward. If the risk premium is zero, then the individual is risk-neutral. It is presumed that most people are risk-averse, and their risk premium for a lottery is positive. So, most people would look for a positive risk premium. A person is risk-averse if he prefers the certain prospect X to any risky prospect.

with expected value x. So, that is what we have been trying to discuss: if a certain prospect 9 is preferred to a risky prospect whose expected value is also 9, then this individual is basically risk-averse. In expected utility theory, risk-averse is equivalent to the concavity of the utility function. Concavity of U implies that the double derivative of the utility function is 0.

The prevalence of risk aversion is perhaps the best-known generalization regarding risky choices. It led the early decision theorists of the 18th century to propose that utility is a concave function of money, and this idea has been retained in modern treatments. Now, I will present a simple example to show how the functional form of utility can actually imply concavity, convexity, or linearity of the utility function. Suppose you have 20,000 rupees in your bank account and you also have a car worth 50,000 rupees.

These are hypothetical numbers taken just to facilitate comparison, and I will be using only the numbers. The 'Ks' are that is why put in brackets. You are considering what kind of car insurance to go for. you have three alternatives, full coverage will cost you 7000 rupees, only third party will cost you 4000 rupees and you can also go for no insurance that is the third option you have but if you do not go for any insurance and suppose police finds you then

they will basically slap you with a fine of 20000 rupees. Suppose the probability of an accident or car theft is 0.05. In both cases what I am assuming here is that the entire value of your car will be gone if the car is theft, you lose 50,000; if it meets with an accident ,you lose complete 50,000 and the probability of one of this happening is 0.05 and police intervention is 0.2. There is 20% chance in case you do not have or 20% chance of police intervention in case you have an insurance of either type nothing happens. In case you do not have an insurance, then the police fines you of 20,000 rupees.

What is the full coverage aspect? The full coverage aspect is that, see, your total worth, initial endowment is 20 plus 50, which equals 70. If you are going for full coverage, then that is going to cost you 7,000 rupees. So, 70 minus 7. That gives you 63.

So here you are spending 63, which takes care of all possible uncertain events, such as car accidents, theft, or police intervention. You do not have to pay anything additional in case any of these happens. That's why it's a sure prospect of 63,000. What is the third-party prospect? The third-party prospect is when you basically pay 4,000 for your insurance. So first of all, you would be left with 70 minus 4, which is 66,000 rupees, and your total endowment in terms of the valuation of your car plus your bank balance and there is a 95% chance that nothing is going to happen to your car. Because the car theft or accident probability is only 0.05. So, nothing is going to happen to your car.

You just spend 4000 rupees, your total valuation remains at 66000. But in case there is an accident or car theft, now you see that since it is only third party, your car value is gone. So, there is 0.05% chance that you would be left with only 20000 minus the insurance amount that you have already paid, which is 4000. So 16000, right?

Police is not going to bother you. That's why that doesn't come into picture. And what is the no insurance prospect? In case you have not gone for insurance and none of these happens. So neither police intervenes, nor there is an accident, nor there is a theft.

Then you have total 70,000 left, which is the possibility of 75%. In case police intervenes, they will take your 20,000. You would be left with only 50,000 worth of car. And in case there is an accident or car theft, which the probability is 0.05, then you would be left with only 20,000 in your bank account and 50,000 would be gone because the car is gone. So, I have mentioned the prospects here again, and I have given them names like full coverage as option A.

Only third party is option B, and no insurance is option C. So, what are the expected payoffs from these three options? First of all, the expected value from option A is 63. The expected value from option B is 63.5. And the expected value from option C is also 63.5.

Utility function	Full coverage	Third party	No insurance	Predicted choice
Linear: $u(x) = x$	63	63.5	63.5	B~C
Concave: $u(x) = 10\sqrt{x}$	79.37	79.18	79.13	Full coverage (risk averse)
Convex: $u(x) = x^2/50$	79.38	83.02	83.90	No insurance (risk loving)

Next, we consider three alternative utility functions to map the relationship between utility functions and risk preferences. We are considering a linear function, a concave function, and a convex function. So, you can understand that a concave function, as mentioned, has a double derivative less than 0, and a convex function has a double derivative greater than 0. And, of course, a linear function has a double derivative equal to 0.

Now, for each functional form, I have full coverage, third-party, no-insurance values, and then what are the predicted choices. So, for the linear function, that is u(x) equal to x, of course, full coverage would be 63, third-party would be 63.5, and no insurance would also be 63.5. These are exactly the numbers that are equal to the expected value. In the case of concave functions, like $\sqrt{x}$, I will be plugging in different values of x.

Instead of having the value of X directly, I will be plugging in the value of X as it is given in the prospect. And then it turns out to be 79.37 for full coverage, 79.18 for third party, and 79.13 for no insurance. You can see that this is the largest number here. So, full coverage will be chosen here, as that is the maximum utility. Maximum utility is generated by the full coverage option, and that is why the individual would go for full coverage.

See, this is a concave function, and the individual turns out to be risk-averse. Now, if I go for a convex function, then the values are calculated as 79.38, 83.02, and 83.90. Now, 83.90 here is the largest value, which simply implies you should go for no insurance. The maximum value is generated by the option of no insurance.

So, no insurance implies that the individual is willing to take risks. So, we call them a risk-loving individual or call him or her a risk-loving individual. So, this again shows that when the functional form of utility is convex, then the person is a risk-loving individual. Next, I will be showing you the graphical forms of these utility functions.

But that would be done in or begin the next module with those analyses. I conclude this section here. These are the references. Thank you so much.