Course Name - Operations and Revenue Analytics

Professor Name - Prof. Rajat Agrawal

Department Name - Department of Management Studies

Institute Name - IIT, Roorkee

Week - 08

Lecture - 37

Welcome, friends. In our last session, we started discussions about pricing systems and how appropriate pricing can also maximize our revenue. We discussed different scenarios where you may have replenishment of stocks, and even without replenishment, there are certain situations. So, in that case, we saw that deterministic and stochastic situations are possible everywhere. In this particular session, we are going to discuss a specific scenario that is very similar to the cases we discussed when talking about capacity allocation.

This is a situation of single-product dynamic pricing without replenishment. Here, we will discuss the mathematical formulation of such a scenario and, with the help of an example, see how such cases are handled. As we already explained, there are many situations where single-product dynamic pricing without replenishment is applicable. For example, flights have extensive applications of dynamic pricing. There is no possibility of replenishment, and you have a single product, like these days where the entire flight is an economy-class flight.

So, that is economy class, and no replenishment is possible. This is a very classic case for this particular type of scenario. There may be other similar cases as well. In operations and inventory management courses, we discuss a very popular inventory management model called the Newsvendor model. The Newsvendor model is also similar, except that in this problem, we generally do not apply the concept of dynamic pricing.

But you can think of a situation that dynamic pricing may be applied that if you have a very specific type of newspaper where only some limited copies are available and you may charge to the customer as per the availability of the copies in the beginning when you have let us say 50 copies. You are charging 10 rupees per copy but after 25 copies you still expect more than 25 customers are possible to buy these newspapers. You may increase the price from 10 rupees to 12 rupees. So, something of that sort is possible in newspaper case also. So, there are variety of examples of this scenario and you may also like to add more number of scenarios for the single product dynamic pricing without replenishment case.

Now, in this case when we want to develop a mathematical model, we are considering only a single product and there is a single price at any particular time. So, only one price decision you have to take that okay what should be the price? Let us say if I consider from Monday to Saturday. So, what can be the price on Monday, what can be the price on Tuesday p1, p2, p3, p4, p5, p6 on different days of the day, what is the different price that is denoted by pt, p1, p2, p3, p4, p5, p6 like that. And when you have a particular level of price that particular level of price on a particular day will create a particular type of demand function which is represented by d(t, p) that based on that particular price you will have a particular demand function.

And this demand function may vary for different prices at price it is not that you just change the price and the demand function will remain same. The point which I am trying to say that let us say there is a demand function 100 plus p. So, now if demand is sorry if p is p1. So, D will be D1, if p is p2, D is D2 like that that is not the case. The case is that this demand function is changing based on the t value also. So, on for p1 on day 1, day 1 p1 then here the demand function will be for example 100 plus p1 and on day 2 when the price is p2 the demand will be let us say minus 50 plus 2p2.

So, that means on every day, if there are, let us say, six different times, and for six different times, on every time there will be a different price, p1 to p6. There will be six different types of demand rates. We call the demand rate a function of time as well as a unique price. So, these are the conventions which we are going to follow. Now, we introduce another important convention, that is J, which is a function of time and demand. Now, J is a function of time and demand, which basically represents marginal revenue. Marginal revenue is, at this level, on this particular time with this particular demand, if you sell one extra unit, what will be the marginal revenue at that level? So, this is r(t, d), and r(t, d) is basically the revenue multiplied by the demand. If you take the differentiation of that with respect to d, it will give you the marginal revenue, that is the J(t, d) function.

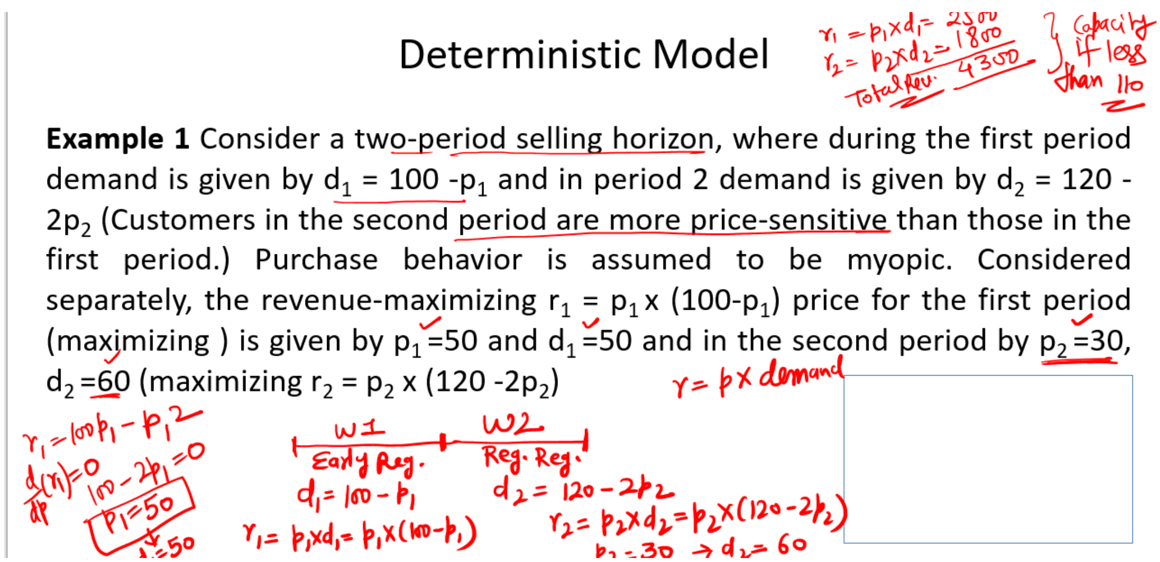
Single Product Dynamic Pricing without Replenishment

- Since we consider only a single product, there is a single (scalar) price decision at each time (t), denoted p(t), which induces a unique (scalar) demand rate d(t, p).
- $J(t, d) = \frac{\partial}{\partial d} r(t, d)$ denotes marginal revenue.

Now, going further, let us try to understand the working of this deterministic model with the help of this example. Now, here we have a two-period selling horizon, let us say week 1 and week 2. For example, whenever you are holding a particular research conference, in the research conference, week 1 and week 2 can be the early registration and regular registration. So, these are two periods of the selling horizon, and we want to keep different pricing for different time horizons. Now, for this first time horizon, our d1, that is going to be the demand of the first time horizon, is given by this demand rate: 100 minus p1.

p1 is the price for the first week, d2 is given as 120 minus 2p2. So, these are the demand rates given to us for two time horizons. Now, the purchase behavior is myopic. We understood what is myopic? That is the short-sighted.

The customer is going to see his benefit. Now, for revenue calculations, revenue is price multiplied by demand. So, r1 will be p1 multiplied by d1, or you can say p1 multiplied by 100 minus p1, because d1 is given as 100 minus p1. r2 will be p2 multiplied by d2, that is p2 multiplied by 120 minus 2p2. Now, going back to your basics, that will give you that r1 is 100p1 minus p1 squared, and to get the maximized value of this, you have to do the calculation that is r1 equals 100p1 minus p1 squared. Differentiate this with respect to p, set that differentiation (d/dp of r1) equal to 0, and when you set this equal to 0, you will solve it as 100 minus 2p1 equals 0, and you will get p1 equals 50.



And when you get p1 equals 50, the corresponding value of d1 will be 100 minus p1, which is 50. So, p1 is 50 and d1 is 50 for the maximized revenue in this case.

Similarly, for the second phase, you have p2 multiplied by 120 minus 2p2. You do the multiplication, differentiate this revenue function with respect to price, set that value equal to 0, and that will come as p2 equals 30. When you get p2 equals 30, the corresponding value of d2 will be 60, that is, p2 is 30 and d2 is 60. So, the maximum demand if we keep the price equal to 50 in d1, the first time period, you need to allocate 50 units. For the second time period, because in the second time period you already mentioned that customers are more price-sensitive,

and therefore, in the second time period, you are reducing the price from 50 to 30, and therefore, your demand is also increasing from 50 to 60 in the second period. So, in the first period, your revenue is p1 multiplied by d1, that is 50 multiplied by 50, which becomes 2500. The second period revenue is p2 multiplied by d2, that is 30 multiplied by 60, which is 1800. So, the total revenue is coming to be 4300. But have you observed one important thing in this? That important thing is that to get the maximum revenue of 4300 in this particular case, you should have a total capacity of 110: 50 for the first week and 60 for the second week. But, as we were talking of constrained supplies, let us try to understand it in this way.

If the capacity is 110, you will get a profit of 4300. If the capacity is more than 110, if the capacity is more than 110, then the profit is still only 4300. There will be some unused capacity in those weeks. But considering a scenario where the capacity is less than 110, if the capacity is less than 110, then how to distribute your capacity for these two time periods is an important issue, and for those capacities, what should be the pricing level also. Right now, if let us say 110 is the capacity, or you can say if there is no capacity constraint, your maximum revenue in this particular case will be 4300.

But there is a capacity constraint, and so let us see what we will do now. The effect, as I just told you, is coming when we have the capacity of 110, but if the capacity is less than 110, the effect of that capacity constraint is going to come on our revenue capabilities. We are considering a case where the capacity is 40. So now the question is, how are we going to divide the capacity between these two periods? So here again, we are going to write these marginal revenue functions J(1, d1). If you remember, we have already mentioned that J is (t, d), and t is time.

So, J(1, d1), J(2, d2) means the marginal revenue for the first and the marginal revenue for the second period. So, the marginal revenue equations—how did we develop those? We also saw that these are developed by differentiating this particular function r(t, d), and we have already seen how we have done this marginal value calculation in our previous discussions.

So, I am directly taking these J functions: J1 as minus 2d1 plus 100 and J2 for the second

$$\text{Here, } J(1, d_1) = -2d_1 + 100 \text{ and } J(2, d_2) = -d_2 + 60$$

period as minus d2 plus 60. Now, I would like to take you to our inventory management classes. In inventory management classes, we discussed that the cost of keeping the inventory will be minimal in our EOQ models where we are able to almost match our carrying cost and the ordering cost. Similarly, in this particular case also, when both periods are giving me almost equal marginal revenues, that is the point of optimization, and therefore, I need to see that the marginal revenue from this particular case and the marginal revenue from this particular case are almost equal.

So, we will show you with the help of this data that how we are going to allocate the capacity.

Allocations of capacity between periods 1 and 2 and the marginal values and total revenue.

d_1	d_2	$J(1, d_1)$	$J(2, d_2)$	r
22	18	56	42	2634
23	17	54	43	2646.5
24	16	52	44	2656
25	15	50	45	2662.5
26	14	48	46	2666
27	**13**	**46**	**47**	**2666.5**
28	12	44	48	2664
29	11	42	49	2658.5
30	10	40	50	2650
31	9	38	51	2638.5
32	8	36	52	2624
33	7	34	53	2606.5

Capacity is 40 and that we have to allocate betd2ween two functions J(1, d1) is minus 2d1 plus 100, minus 2d1 plus 100 and J(2, d2) is minus d2 plus 60. So, let us consider a scenario that I am giving these capacities equal. This is let us say 20 and this is also 20. So, if d1 is 20, d2 is 20, your J(1, d1) marginal revenue here is minus 2d1 plus 100 that is minus 40 plus 100 that is coming to be 60 and J(2, d2) is again minus d2 that is 20 plus 60 that is 40.

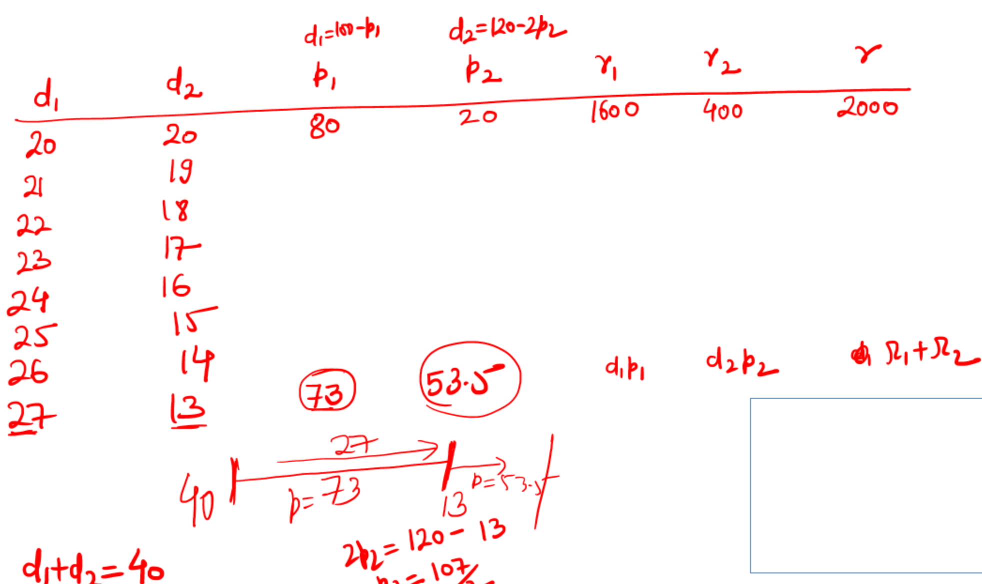
So, this calculation will come 60, 40. The value of r this is combination of r1 plus r2. So, for that we know that what is the function of r1 and r2 that is p1d1 and p2d2, r1 is p1d1 and r2 is p2d2. Now, what is my p1 and what is my p2 for that purpose let us again go back to this particular equation. d1 equals to 100 minus p1 and d2 equals to 120 minus 2p2, 100 minus p1 and d2 equals to 120 minus 2p2, yes.

So, if my d1 is given as 20, so my p1 is 80. d2 is given 20, so 120 minus 2p2, you will have 2p2 equals to 100 and p2 equals to 20. So, r1 comes out to be p1 that is 80 into 20 that is 1600, r2 equals to 20 into 20 - 400. So, the total revenue will be 2000. Total revenue which you are now getting is 2000.

Now, what we have done, which is ready-made for you, is that we have started adjusting the number of d1 and d2. We are increasing the number of d1, making it 21, then 19, then 22 and 18. And correspondingly, we have, in fact, these two columns may not be immediately needed. Rather, it is better that we have columns of, so for that purpose, I request all of you that we can make a table like this. d1, d2, p1, p2, r1, r2, and then r, the calculation which we just initiated: if it is 20, it is 20; p1 will come 80, and p2 will come, I think, 20.

So, r1 is p1d1, that is 1600; r2 is p2d2, that is 400; and total r will be 2000. So, this is our initialization of the calculation process. Then make it 21, make it 19, 22, 18, 23, 17, 24, 16, 25, 15, 26, 14, 27, 13. So, in this way, you can change the values of d1 and d2 with this constraint: that d1 plus d2 is 40. With this constraint, you will adjust the values of d1 and d2. I request all of you to complete this table, do the calculations for remaining p1 and p2, and that is also not very difficult because d1 equals to 100 minus p1, 100 minus p1, and d2 is 120 minus 2p2.

So, you can put your different values of d1, d2 in these two equations, in these two relations, and correspondingly, you will get the values of p1 and p2. For example, if I see that corresponding to 27, the value of p1 will be 73 approximately, and corresponding to 13d2, the value of p2 will be 120 minus 13 equals to 2p2. So, p2 will be 107 by 2, it is around 53.5. And with that, you multiply your d1 and p1, you will get r1; you multiply d2 and r2, you will get r2; and then r is d1, in fact, r1 plus r2.



So, the complete calculation, for your reference, ready reference is available in this table. We have already done this calculation for you, and here you see the different values of air as we are changing the combination of d1 and d2 - 2634, it is increasing, increasing, and going up to 2666.5.

And when you see this, the d1 is 27, d2 is 13, and here the marginal contribution for two periods, first and second period, is not exactly equal but almost equal. One contribution is 46, another contribution is 47. And this is the maximum revenue. This is the maximum revenue, and this maximum revenue is where these two marginal contributions are almost equal. So, this becomes a kind of theorem that at the optimal level, the two periods should give you equal contribution.

Allocations of capacity between periods 1 and 2 and the marginal values and total revenue.

d_1	d_2	$J(1, d_1)$	$J(2, d_2)$	r
22	18	56	42	2634
23	17	54	43	2646.5
24	16	52	44	2656
25	15	50	45	2662.5
26	14	48	46	2666
27	**13**	**46**	**47**	**2666.5**
28	12	44	48	2664
29	11	42	49	2658.5
30	10	40	50	2650
31	9	38	51	2638.5
32	8	36	52	2624
33	7	34	53	2606.5

For example, and how it can be practically used also. For example, if you are able to calculate—if you are not able to calculate the possible revenue for different combinations—but if you have this type of system where you have divided the capacity between two periods and you are able to calculate the marginal revenues from this capacity distribution.

If these two marginal revenues for two different periods are almost equal, then you can understand that you have actually divided the capacity for maximum benefit. But if there is a substantial difference between the marginal revenue of two periods, then you can say that there is scope for improvement. You are away from the marginal thing. You see that as our balance is moving away, here it is 46 and 47, a difference of just one. 46, 48, a difference of two.

44, 48, a difference of four. 45, 50, a difference of 5. As you are moving away from your optimal value, the difference in marginal revenue between the two is increasing, and therefore, you are moving away from these optimal values. So, you are therefore able to see that the maximum revenue is possible since you cannot distribute the capacity. Maybe, if there is a possibility to divide the capacity in fractions also, maybe we can have an allocation where both the marginal revenues are exactly the same and the difference is 0, and that will be, ideally, the theoretically maximum possible revenue. But because of practical reasons, you cannot go for the decimal distribution of the capacity.

So, therefore, these differences of 1 or 2 units out of all the differences between the marginal revenues of period 1 and 2, the minimum difference is of 1 unit. That is the allocation for which you are getting the maximum revenue. So, this is how you are going to decide two important things. One is your allocation of capacity for the two periods, and for this allocation, what should be the price? That is the second important thing. So, these are the two important things which we have seen—how in dynamic pricing, you know that in this case for the first period, you are keeping a very high price. That is how much? 73 rupees, and you know that when I am going close to the end of the sale, I will further reduce the price to 53.5, and I will keep this price of 73.

You can understand, not exactly in terms of week 1 and week 2. You can understand this in that way also—that I started with 40 in the beginning, and I am continuing with the price equal to 73. I am continuing till I use my 27 units, and when I am left with only 13 units, when only 13 are left, I will reduce the price to 53.5. And I will finish all my remaining inventory of 13 units with this lower price of 53.5. So, you must have seen in vegetable markets also—when fresh produce comes in the market in the morning, the price may be higher. But when he sees that, okay, now only 1 kg is left or 2 kg is left—whatever the limit he decides—the shop owner will reduce the price for finishing all the remaining inventory at that lower price.

So, this is the system of dynamic pricing without any replenishment. With this, we come to the end of this particular session, and we will continue with some more interesting cases of pricing for revenue maximization in our further session. Thank you very much.