

Course Name - Operations and Revenue Analytics

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Week - 05

Lecture - 25

Welcome, friends. As we are talking about the optimization of revenue or maximization of revenue, whenever we have a situation of capacity constraint—where supplies are limited—we have discussed that there are different types of issues when you have limited supply, and that supply is also perishable at a particular time. You may remember that we discussed the concept of run-out price, which is generally higher than the optimal price applicable in unconstrained supply situations. We also discussed the concept of marginal opportunity cost, which is applicable whenever you have limited capacity, as your marginal opportunity cost starts increasing. So, as you continuously reduce your capacity, your marginal opportunity cost decreases.

One simple answer is that we need to have more capacity so that our marginal opportunity cost is lower, and at the same time, you will have higher revenue based on the available capacity. But we all know that it may not be in our hands to decide the capacity. There are resource requirements for having capacity, and if you do not have those resources—maybe capital is one such resource— If you do not have enough capital, you will run short of the required capacity, and then you have to see how you can maximize the possible revenue from that available capacity. We have discussed some strategies, like differentiating customer categories, where for different customer categories, you may charge different prices, ultimately with the objective of revenue maximization.

You may recall how we discussed the example of a stadium, where we considered two categories of customers: students of the university and the general public. That is how we

decided on a certain number of seats for Group A customers and a certain number for Group B customers. Similarly, airlines also use this kind of class division, and for a particular class, we can set a specific price label. You may recall that we discussed tactical pricing labels and how deciding all these different classes—along with the quota for each class—is the role of tactical planning. Then comes the role of booking control, where you must make decisions based on available capacity. If you have capacity and a request comes in, you will accept it; otherwise, you will reject it.

So, in this particular session, we will further discuss the implementation of the concept of booking control. We have already introduced another important concept, which we are discussing in this particular session: the concept of nesting. We discussed how it is possible and some drawbacks of booking control. So, with the concept of nesting, we are trying to overcome those limitations. So, with some numerical examples in this particular session, we will explore the concept of booking control and nesting.

So, we will talk in detail about booking control, allotment, the concept of nesting, and then dynamic nesting booking control. That is the most important contribution of this particular session, which we will cover. And then, there may also be cancellations. There are bookings happening, but at the same time, cancellations may also occur in different categories or classes. So, how are we going to handle those cancellations? Whenever a cancellation happens, how do we add this new availability into a particular category? That is to be seen.

Now, as we already discussed in our earlier classes, the concept of booking control is the real-time phase, the online phase of this concept of revenue management. Here, we will decide whether to accept or reject a particular booking request based on the criteria set at the technical level. Whenever a booking request is received, you will give a yes or no based on the capacity available for that particular class. Let us see with the help of some data, which we have also discussed, but just to provide context for this particular session, I am repeating the same example we discussed in our earlier class. For a B class, we have a request coming for 3 seats. There are different classes: A, B, and C.

These are the three different classes in a particular flight let us say BM 509 is the flight number and in that flight we have A, B, C. Now you are getting a request of three seat available capacity is to decision will be reject the request. Because, your available capacity is less than the requirement which is coming. Similarly, a request for 3 seats in C category is coming the available capacity is 4, you is going to accept this booking. So, this is how in real time you are going to do then the available capacity now in C is 1 seat and another request in C category is coming for 2 seats. Since, now the available capacity is less than the request you are going to reject.

So, all these things are happening in the real environment based on how things are processing and how much is the available capacity you have. Allotment is another term which we discussed in one of our earlier videos, where to manage bookings we have to divide the available capacity into the discrete classes. And for each class we are having a particular fare like in this example these are the classes ABC, these are the three different classes, if I go with this example of flight 137 we see that we are going to have these three classes B class, M class and Y class, B is the economy class you can say. M is a kind of a business class or executive class and this is the business class. Here, for each class you are assigning a particular fare level and how much you are assigning to a particular fare level is known as allotment or in our simple terms we can say that quota given to a particular fare level is allotment.

Class	Price	Allotted	Booked	Remaining
B Class	125	30	25	5
M Class	200	45	45	0
Y Class	560	25	10	15
Total		100	80	20

While the allotment approaches is very easy to understand, it has a major drawback that we are going to discuss. It does not work very well in particular it can result in a high fare customer being rejected, while a lower fare customer are still being accepted. As you can see in this particular example, now I have three fare categories 125, 200 and 560. I have

divided their capacity into three categories like 30, 45 and 25 respectively this is the total. Now, on a particular day, I checked as you can read on this slide that about two weeks before the actual departure of this particular flight we saw that 25 seats are booked in B class, 45 seats are booked in M class and 10 seats are booked in Y class and these are the remaining seats.

So, you can easily understand. Now, today if I go for asking a ticket for M class, I will be rejected my this request will not be accepted. While it will be funny thing that you may get a ticket at the lower fare class which is 5. So, which is B class. So, 5 seats are available here, but no seat is available for a higher fare class.

So, which is looking slightly disturbing, it is not natural justice that you are still ready to give tickets to those who are asking for lower fare class and you are rejecting the request of those who are coming for the higher fare class. So, that is a limitation of this approach of allotment. So, therefore, a concept which is known as nesting we already discussed very briefly in one of the earlier video about the concept of nesting. Now, this concept is basically developed to avoid the rejection of high fare booking in favour of a low fare booking, wherever possible you should always accept the natural thing looks that first you should close the booking of B class, then you should close the booking of M class and then you should close the booking of Y class. If for example, it becomes even more funny that you are not giving Y class seats, but still you are giving B and M class seats.

So, therefore, this concept of nesting says that it is a situation which can be avoided that you are rejecting the high fare customers and you are accepting the low fare customers. So, each time every fare class has access to all of the inventory available to lower fare classes. So, if I am coming for Y class the available inventory for B and M class is also available to me. If I am coming for M class, inventory available for M and B class is also available to me and obviously, B class is the lowest fare class. So, all the if I am coming only for B class, so the inventory available for B class only will be available to me.

So, this concept is stated in the form of mathematical inequality like this that you are numbering the fare classes, so that 1 is the highest fare class and N is the lowest fare class. Like in this case, we have B, M and Y, B, M and Y. So, this will be known as first,

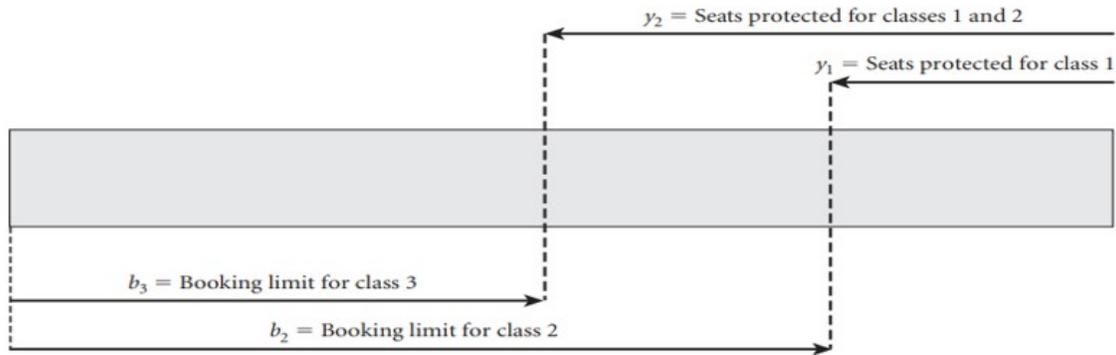
this is second and this is actually N which is third. So, inventory available booking limit for class I is I, let us say is 1. So, booking available for 1, b1 is all the inventories up to bn. For example, in this case it is b1, b2 and b3. If I talk of I equals to 2, the b2 available for b3 also.

So, you can say that if b1 is my total inventory, when I am talking of I equals to 1 that means the limits which are available for if I talk in terms of b this is b1, this is b2, this is b3. So, in the beginning of this particular allotment process b1 is equals to 100, b2 is equals to 75 and b3 is equals to 30. So, that is how the total inventory which is available for B is available for Y class. Inventory available for B and M class is available for M class and for inventory for B class whatever is available for that is available. So, this is the concept of nesting and now you can easily understand that this helps us in avoiding the rejection of high fare booking in favour of low fare booking.

So, if I see this concept in a graphical manner. So, this concept looks like this image. Here, you see in this image that you have a different concept also which we will be discussing that is the concept of protection level. Now, let me talk that this is b2 which is the booking limit for class 2, b3 is the booking limit for class 3. Now, here from this particular point to this particular point, this is our total inventory, total capacity of the service system b1.

Out of that, only these many seats are protected for class 1. Like, if I go back to this diagram, this table B1 is the total number of seats, but out of this total number of seats, 25 seats are protected for class 1. Here, you can add Y also. It will be confusing, so we will add there only. So, my total seats are like 30, 45, and 25; this is for 3, 2, 1.

So, if I talk about them in terms of booking limits, in terms of B—sorry, not B—this is B3, that is 330; B2 is 75, and B1 is 100, which is equivalent to the total capacity available in the system, that is what we just studied. Now, another concept which we are introducing is the protection level, that is, how many seats, how much capacity is available only for that particular class. So, if I see that 25 seats—if you see this diagram from left to right—you are seeing the booking controls.

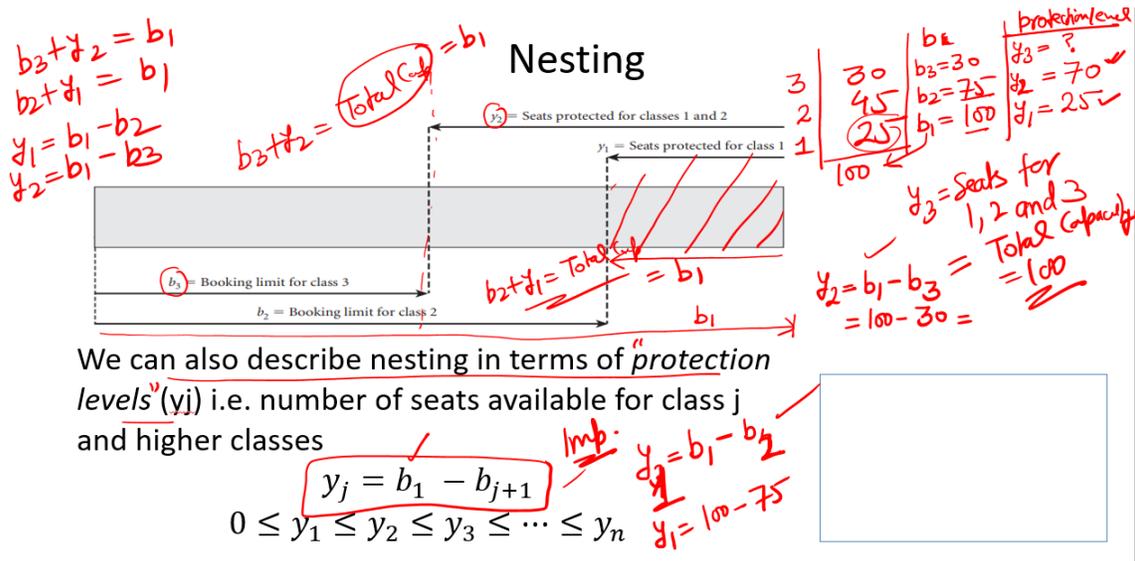


But, when you see this diagram from right to left, you see how many seats are protected for a particular class. So, first is the protection level which is coming for class 1, and this is Y_1 , and Y_1 is 25 seats available for category 1. Similarly, if I see for category 2 and category 1, that is Y_2 , that is how many seats are protected for 1 and 2 together.

And 1 and 2 together, the total seats which are protected—that you see coming from the right-hand side—it will be seats which you can see up to this point. How many seats are there? These are a total of 70 seats which are protected for class 1 and class 2, and then comes Y_3 , where you see how many seats are protected for class 1, 2, 3—that means all, which actually has no meaning, because a total of 100 seats are there, and all these 100 seats are basically protected for 1, 2, 3. So, practically, this has no meaning; Y_3 is seats for 1, 2, and 3, and that is basically total capacity equals to 100. Now, if you see the relation between booking control—that is, the booking limits for different classes—and the protection level, you are getting a very interesting relation.

This is a very important relation between production levels and the booking limits. What does the relation tell? y_j equals b_1 minus b_j plus 1. For example, if I want to calculate, let us say y_3 , y_3 will be b_1 minus b_j plus 1, which means b_j is 3 plus 1—4. Now, in this case, you see that b_4 is—sorry, b_2 —this will come y_j plus 2, because we are talking about the protection level for j_1 as the first. So, y_1 equals b_1 minus b_2 , and b_1 is 100, b_2 is 75, and you directly get y_1 equals 25.

Similarly, if I want to calculate y_2 . Now, y_2 will be b_1 minus b_j plus 1, which is b_3 , that is 100 minus 30, and you get y_2 equals 70. So, in this way, using this mathematical expression itself, you can calculate how many seats are reserved for a particular class and how many seats are total available up to a particular class. So, this concept of nesting is very interesting—that these many seats you are guaranteeing for this particular class, but otherwise, the total inventory available up to that class is also a very good idea. Therefore, if you see this diagram, b_3 plus y_2 . If you see these two things, b_3 plus y_2 is the total capacity.



Similarly, you see this further diagram: b_2 plus y_1 is the total capacity. So, you can directly get the total capacity, which we know is how much? b_1 ? So, the total capacity can be replaced by b_1 , by b_1 . So, in this way, b_3 plus y_2 equals b_1 , or we can write b_2 plus y_1 equals b_1 , because b_1 represents our total capacity. That is how this particular formula is generated—whether you are talking about y_1 equals b_1 minus b_2 . Or y_2

equals b_1 minus b_3 , and so on. You can develop your different particular expressions for this.

But the genesis of this formula we just discussed. Going further, we also have a concept which is very important to understand, that is dynamic nested booking control. You can update your booking limits in a dynamic environment. For example, you have 5 fare classes, and these are the booking limits: b_1 , b_2 , b_3 , b_4 , b_5 . Now, with this, you can easily understand that the total seats available in this flight are 100; b_1 is 100.

Now, this flight is currently not accepting any bookings for class 5; b_5 is 0. So, no bookings for class 5. Any booking class with a limit of 0 is said to be closed; that is, in general, you can understand. We can derive the corresponding protection levels y_1 , y_2 , y_3 , y_4 , y_5 like this. For example, you can use this expression again: how b_3 plus y_2 equals y_1 , or you can say y_1 equals b_1 minus b_2 .

So, y_1 equals b_1 minus b_2 , or y_2 equals b_1 minus b_3 . Similarly, you can derive y_3 equals b_1 minus b_4 , y_4 equals b_1 minus b_5 . So, in this way, you can develop the protection levels for all the classes, and you can just check whether the protection levels given here match these calculations or not. For example, b_1 is 100 minus b_2 , which is 73, that comes out to be 27. So, y_1 is 27; 100 minus b_3 , where b_3 is 12, that is 88, which is y_2 ; 100 minus b_4 , which is 4, that is 96, which is y_3 ; y_4 is 100 minus b_5 , which is 0, so 100; and y_5 is the total capacity, which equals 100.

$$\begin{aligned} y_1 &= b_1 - b_2 = 100 - 73 = 27 \\ y_2 &= b_1 - b_3 = 100 - 12 = 88 \\ y_3 &= b_1 - b_4 = 100 - 4 = 96 \\ y_4 &= b_1 - b_5 = 100 - 0 = 100 \\ y_5 &= 100 \end{aligned}$$

So, you can see how we can calculate that as our booking limits change, our corresponding protection levels will also change accordingly.

Dynamics of booking limits and protection levels

	BOOKING LIMITS					PROTECTION LEVELS					Request	Action
	1	2	3	4	5	1	2	3	4	5		
1	100	73	12	4	0	27	88	96	100	100	2 seats in Class 5	Reject
2	100	73	12	4	0	27	88	96	100	100	5 seats in Class 2	Accept
3	95	68	7	0	0	27	88	95	95	95	1 seat in Class 2	Accept
4	94	67	6	0	0	27	88	94	94	94	1 seat in Class 4	Reject
5	94	67	6	0	0	27	88	94	94	94	3 seats in Class 3	Accept
6	91	64	3	0	0	27	88	91	91	91	4 seats in Class 3	Reject
7	91	64	3	0	0	27	88	91	91	91	2 seats in Class 3	Accept
8	89	62	1	0	0	27	88	89	89	89	4 seats in Class 3	Reject
9	89	62	1	0	0	27	88	89	89	89	1 seat in Class 3	Accept
10	88	61	0	0	0	27	88	88	88	88	2 seats in Class 2	Accept
11	86	59	0	0	0	27	86	86	86	86	2 seats in Class 3	Reject

Now, you have this simple diagram table, which helps you understand the booking limits and the protection levels, and then what type of booking control system is happening with this dynamic of booking limits and protection levels. So, here the initial booking limits are given to us: 100, 73, 12, 4, 0, as we know that we are not accepting any bookings in class 5. And accordingly, the initial protection levels are given to you. We just calculated 27, 88, 96, 100, 100. So, 27, 88, 96, 100, 100—these are the protection levels. Now, you see that I will only explain one or two rows in this case, and then we will not explain further. Since this slide will be available to you as part of our homework, let us say I request you to develop these rows on your own based on the requests which are coming in this part.

So, these are the requests. I have this request for all of you: please do not see—do it on your own. By doing it on your own, you will be able to develop this table and this table as well. So, you have to use these requests, and now continuously, you have to decide whether you are going to accept or reject these requests, and you have to dynamically update the booking limits and the protection levels. Let us see how things are happening. For the first three rows, at least, I will demonstrate to you. Now, there is a request coming for two seats in class 5.

Now, two seats in class 5. Class 5 booking limit is 0. So, I am not accepting any requests in class 5. I will reject this particular request, and therefore, there is no change in this particular case. 100, 73, 12, 4, 0, 27, 88, 96, 100, 100 remain as they are. The second request comes: 5 seats in class 2.

Now, class 2 has 73 seats. The booking limit which is available at class 2 is 73. So, I will book 5 seats therefore, I will accept this request and as a result of that I will book 5 seats in the class 2 and therefore, the inventory of all the classes will reduce by 5 seats. But, since this column 5 has no meaning at the moment. The seats from this will be reduced by 555 everywhere. So, out of first also 5 seats are reduced because, now the total inventory available in the system are 95.

5 seats are given to class 2. 73 out of that also 5 seats are given. 68 seats will be total available booking limit for class 2 now. Out of class 3 also 5 seats are given to somebody, total 7 seats are remaining and in class 4, 4 seats were available in the beginning. So, only 0 seats are remaining because 5 is lost, 5 is already booked. Now, let us see the protection level because this may look slightly confusing.

So, you see that in the protection level, we use the same formula y_1 equals to b_1 minus b_2 and without applying too much of thought you can use this formula that y_1 equals to b_1 minus b_2 , y_2 equals to b_1 minus b_3 , y_3 equals to b_1 minus b_4 and y_4 equals to b_1 minus b_5 . And using this concept you see that y_1 equals to b_1 95 minus 68 that is 27 seats are still protected for, still protected for class number 1 because you are booking in class 2. So, the overall inventory has reduced for 5 seats from 100 it became 95, but it is not affecting, it is not affecting the quota which is available the protected seats, which are exclusively for class 1 these are remaining unaffected. Let us see y_2 , b_1 is 95 minus b_3 that is 7 that means 88 seats. So, protection level for b_2 is 88, protection label for y_3 that is 95 minus b_4 that is 0, 95 and similarly 95 minus 0 is 95.

$$\begin{array}{l} y_1 = b_1 - b_2 = 95 - 68 = 27 \\ y_2 = b_1 - b_3 = 95 - 7 = 88 \\ y_3 = b_1 - b_4 = 95 - 0 = 95 \\ y_4 = b_1 - b_5 = 95 - 0 = 95 \end{array}$$

So, you see that you got these new levels 95, 95, 95. So, what it means that earlier you had the protection of 96 seats for class 3, now you have only 95 seats protected for class 3 which are exclusively available up to class 1, 2, 3. And you have only 95 seats for class 4 up to class 4 and total 95 seats up to class 5 also. In fact, class 5 has no meaning at the moment because you are not accepting any seat in that, but earlier up to class 4 you were having 100 seats available, but now up to class 4 you have only 95 seats available. For up to class 3, you were having earlier 96 seats available but now you have only 95 seats available.

But you are still having the interesting thing is you still have 88 seats available for class 1 and class 2, class 1 and class 2 you still have 88 seats available and therefore you can understand that where you are reducing your inventory. You are now have reducing 5 seats overall up to stage 4 and you are reducing 1 seat up to stage 3. So, that is the meaning of dynamic booking limits and protection labels because with every new request coming your booking limits are also changing and your protection labels are also changing. But ,when you will do this completion, you will see that how you are getting the request for other categories, other classes and these other classes are not affecting though the overall booking limit is changing as you are getting bookings in different classes. So, the total seats are reducing.

So, therefore, booking limits are reducing, but the protection label the seeds which are available for class 1 are not being affected all these are remaining 27, 27, 27. But the protection level, the seeds which are available for class 1 are not being affected all these are remaining 27, 27, 27. So, this is a very interesting concept which ensures that you have always inventory available for higher class and whenever a booking request come for higher class it cannot be rejected. in favour of a lower class when you follow the concept of nested systems of booking control and you have to dynamically update your booking limits. You can understand that this type of system is very easy to implement when you use computer algorithms.

So, these days you have requests coming online, and when you are online, the booking limits are automatically updated. Protection levels are also always updated, and based on the booking limits, you will decide whether you are accepting or rejecting. So, all these tables which you are seeing on your screen can be very easily automated with the help of computer programming. So, with this, we come to the end of this particular session, and we will be talking about one important concept of cancellation that we will complete in our next class. Thank you very much.