

Course Name - Operations and Revenue Analytics

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Lecture - 12

Welcome, friends. So, as we are discussing, there are multiple scenarios for handling inventory-related issues in our operations management case. And we see that a lot of analytics can be used in understanding how things were in the past because whatever uncertainties exist, these uncertainties arise from our historical data. We understand how things have happened in the past, and using that historical information, we try to make good decisions, and all those good decisions using different kinds of algorithms are part of predictive analytics. And in our previous sessions, we have discussed that there are uncertainties in demand, uncertainties in supply, uncertainties in lead time, possibilities of quantity discounts, and issues where suppliers are not ready to provide instant supplies, as the supplies are opening at a particular rate at which you are getting the replenishment.

All these are the different types of situations we have discussed in the case of inventory-related decisions. Now, moving further, we can also see that inventory decisions can be broadly classified into two categories, and these two categories are deterministic and stochastic. Wherever you have definitive ideas, these are the deterministic situations, but when you do not have complete information, whenever the information is in a possible mode, these are all stochastic situations. Now, in stochastic situations, you have different possibilities, such as possibility 1, possibility 2, possibility 3, possibility 4, and multiple possibilities are possible, and each possibility may have its own weight as well. So, how these uncertainties or the stochastic nature are to be handled is what we are going to discuss with the help of another very common tool, which is the Decision Tree Approach.

And we will have some kind of illustrations also in this particular session, explaining the use of decision trees in our decision-making activities. We all know that there are so many different types of decisions we make in operations management. Right now, we are talking in the context of inventory. If you stock one additional unit, for example, whenever we are talking about inventory and revenue optimization. We will see whether we should stock one more unit, and whether by stocking one more unit, our expected profit is going to increase or decrease.

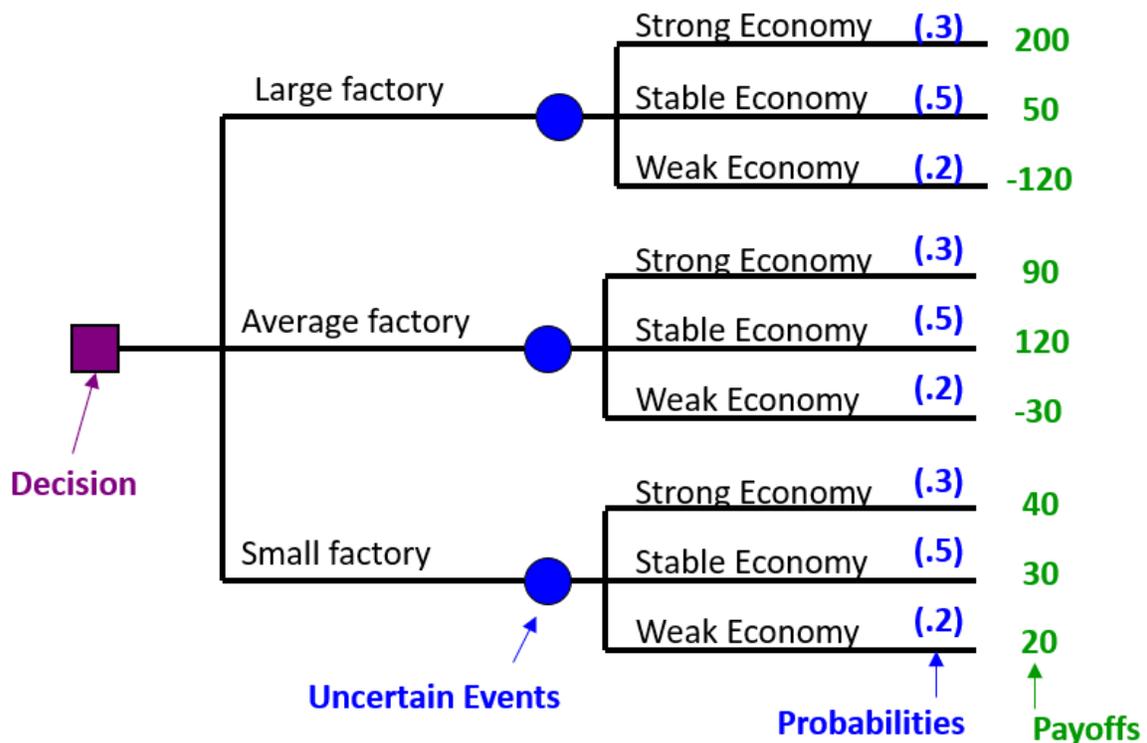
These kinds of decisions are very common, and with the help of this particular tool, which we are going to discuss in this class—decision tree—you can very conveniently make such kinds of decisions. Like investment-related decisions in plant size, what should be the capacity of your plant, whether you should have a plant of 10,000 units or 20,000 units. Depending upon what the possible demand should be, and then there are different possibilities of demand levels, whether you are going to use the capacity of this plant in a better way or that plant in a better way. You have answers available to you, and generally, now we have two types of uncertainties. The uncertainties which are for very short-term periods and uncertainties which are going to have a long-term impact also, for example, the size of the plant, location of the warehouse, the size of the warehouse.

These are the decisions where if you take any kind of wrong decision, the impact can be long-lasting. If you do not stock enough items in an order cycle, the impact is very small; only the particular order cycle is getting affected because of the wrong inventory decision. But because of the wrong decisions of the size, wrong decisions of the location, the impact is much more long-lasting. And therefore, we need to be more careful when we are taking decisions in the long term, and uncertainties are there in those long-term issues also. Where, for example, if I take the case of India, should I make a plant in Mumbai? Or should I make a plant in Delhi?

Now, you are not going to make a plant on a daily basis. So, if you take a wrong decision, Mumbai versus Delhi, this is going to have a permanent impact on your business. So, you need to see that uncertainty is there; neither are you clear about the Mumbai situation, how the business environment in Mumbai is going to fare, nor how the business environment in Delhi will be there in the future. But based on some kind of estimates,

you will see what the expected payoff is. If I take a decision in Mumbai or if I take a decision in Delhi's favor, what will be my expected payoff? And we always want to maximize our expected payoff, and for that purpose, we will see the application of our decision tree.

So, a decision tree is a very simple, you can say, pictorial representation of all the possibilities that may happen in the future. So, it begins with the initial decision and ends



with all possible outcomes and their respective payoffs. We are using two types of symbols in the decision tree as we progress. Whenever a square is present, it denotes a decision node, and a circle denotes uncertain events. Like this. So, this type of tree and its branches will keep developing after every decision node and every uncertain event.

Here, you see a sample decision tree diagram where all the important terminologies related to decision trees are mentioned. As I already mentioned, a square symbol represents the decision node. So, I take a particular decision; the decision can be either to open a large factory, a medium factory, or a small factory. I can take three decisions:

opening a large factory, a medium factory, or a small factory. Whether I open a large factory, a medium factory, or a small factory, there are three possibilities, which are uncertainties.

The economy may be very strong, the economy may be moderate, and the economy may be weak. You can say optimistic, pessimistic, and BAU; a stable economy is represented as BAU, which means business as usual. So, these are the three scenarios that are possible. I do not know which scenario will occur. Based on various estimates, I have the probability that the chances of a strong economy are 30 percent, a stable economy (business as usual) is 50 percent, and a weak economy is 20 percent.

And on the basis of that, if there is a large factory that I am opening, these are the different kinds of scenarios in the bracket in the blue color. These numbers are basically probabilities. Please note down that these probabilities are subject to 1 for every decision. Whatever all possible situations are coming, uncertainties are happening, the sum total of their probabilities is 1. Now, if I am opening a large factory and the economy remains strong, I may gain a profit of 200 rupees. If the economy is stable, I will gain a profit of 50 rupees. But if I open a large factory and the economy is declining,

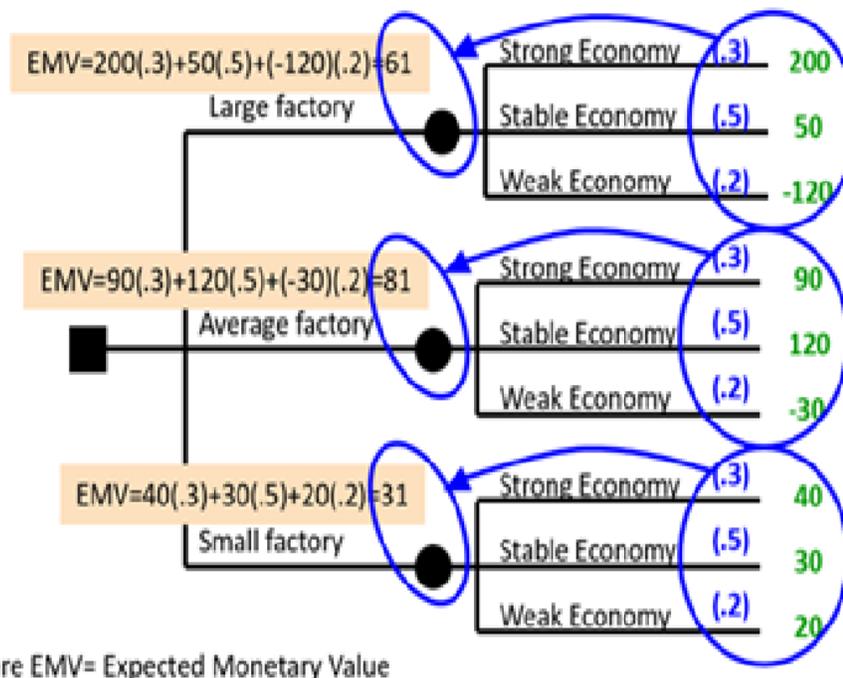
I may get a loss of 120 rupees, and so on. I have mentioned other scenarios also with respect to my decisions of opening an average factory, a moderate factory, or a small factory. So, here these are the payoffs. So, you have understood the decision node. These are the alternative decisions, and these are the alternative situations, scenarios, and these are in the bracket their probabilities, associated probabilities, and then these are the possible payoffs for all these decisions in a particular scenario.

Now, my objective here is to maximize the payoff. This is what I want, and for that purpose, if you see simply, I would like to open a large factory because if the economy is going to be strong, I will get the highest payoff of 200 rupees. But, what if the economy is weakened? Then probably I will take a loss of 120 rupees also. So, you can say that I am not sure whether the economy will be strong or the economy will be weak, but on one side, I may gain up to 200 rupees, and on the other side, I may have a loss of 120 rupees. On the other side, if I open a small factory, whether the economy is strong, stable, or

weak, I will always have a positive payoff, though this positive payoff is very small, 40, 30, 20, but still, I will never be in loss.

So, if I am a risk-averse person, I will not like to take risks. I will not like to go for losses. I may go for these kinds of scenarios. But on the other side, I am ready to take risks also. So, if I want to take risks, I will go for a large factory.

Fold Back the Tree: Add Probabilities and Payoffs



But, how to make more rational decisions in such scenarios, that is what we are going to discuss. So, here we can see that the payoff of opening a large factory, the payoff of opening a large factory, is calculated with the help of this term which is known as EMV.

EMV is mentioned at the bottom of the slide, that is expected monetary value. So, I am going to make three decisions: either I am going to open a large factory, a medium factory, or a small factory. I have to calculate the expected monetary value of these

decisions; the expected monetary value of opening a large factory, EMV large, will be the payoff multiplied by the associated probability.

So, in Excel, we can use the SUMPRODUCT function; I will recommend using Excel's SUMPRODUCT function for the payoff and associated probabilities, and this function will look like this way, and then you will calculate the associated EMVL as 61 rupees. The associated expected monetary value of a medium-sized factory is calculated by multiplying the respective probabilities and their payoffs; it will come to be 81, and the expected monetary value of a small factory will come out to be 31. Now, out of these three expected monetary values, it is now very easy for you to make a decision since the expected monetary value is highest for the medium-sized factory. We have written it as an average factory; you will always make a decision for this particular case; you have maximized your expected monetary value by opening the medium-sized factory. And this is how you can make decisions without much difficulty; in this case, the challenge is how we are determining the values of these probabilities.

So, many of you can say that efforts are there to predict these probabilities using our historical data. We can take the data of the last 10 or 15 years to see how the economy is going to be in the future, and based on that, these probabilities are assigned. Now, we can take a different case. This is how we understood the use of decision trees in our decision-making. Now, we are taking a case where we have some historical data, and that historical data is related to two important factors: how demand is going to increase from period to period. Based on that, I have to make some important decisions, maybe decisions related to capacity or decisions related to my facility location, etc.

So, here I am going to tell you how decision trees will be developed. Now, I am considering two variables in this example. In the previous example, we considered only one single variable, which was the total payoff. Now, here I am considering two things. One is price, and another is demand. Now, as we go from one period to another, that factor is either increasing or decreasing with a particular probability, and this is represented in this particular case using a specific probability distribution known as the binomial probability distribution.

So, for example, if I say only one particular factor, P , so the price in the factor period 1 is P at the beginning of the period. Now, the price can increase, and the price can also decrease. If it is increasing, it is up; I am saying it will increase by a factor of U . If it is decreasing, it is down by a factor of D , and then in the next period, it is again possible to increase or decrease. It is also possible to increase or decrease, and so on. From period to period, these kinds of changes are happening, and this is what we have represented in the lower part of the slide as well. Here, you can see with the help of a decision tree, in a graphical manner, how these probabilities are explained in a sequential manner.

So, let me use some space for that purpose. If in the beginning the price is P , and then it is increasing and decreasing with a particular probability. So, after period 1, this has become Pu or Pd . For our purpose, I am associating equal probabilities for both these branches that in period 1, there are 50 percent chances the price will go up by a factor of 10 percent, and the price may also decrease by a factor of 10 percent. U and D are plus minus 10 percent. Prices are increasing or decreasing by 10 percent, and the probabilities are 50 percent each. After period one, again in the second period, the price may increase with a probability of 50 percent, so this will become Pu square.

A 10 percent further increase in the price. There is a 50 percent possibility that the price may decrease. So, the price is decreased by again D percentage. So, Pu into D . Here also, it is possible that the price may increase. So, Pd into U , it becomes Pud , and the price may also decrease.

So, in that case, it becomes Pd square. This is what we had in period one: Pu , Pd . In the second period: Pu square, Pud , and Pd square. Then, in the third and fourth periods, you can create a similar table to extend everywhere. But it is very interesting and may be one of the questions also in our weekly quizzes that out of all these different branches, there may be some branches which are common. So, you will actually here, just by the diagram, it appears that there will be six different prices, but as we saw in the second period, though there may be four different prices, practically when we saw these, there are only three different prices. So, similarly, in the third period, theoretically, it appears that there will be six different prices, but actually, these are not six different prices. It will

be four different unique prices, and so on. As we have more number of periods, your tree will be bigger and bigger.

And in this way, we can always create a more generalized way of writing this particular thing that the type of U and D, which we are discussing, are multiplicative. Multiplicative means a price with me in period 0, initial price P naught is, let us say, 100 rupees, and U and D are 10 percent. So, in period 1, when I am saying it is P_u , that prices have increased by 10 percent, that is 110. Prices have decreased by 10 percent; it is 90 percent, 90 rupees. When I say in the second period, prices are further increasing by 10 percent.

So, P_u will become 121; prices have reduced by 10 percent. So, 110 minus 11, that is 99 P_{ud} . Prices have increased by 10 percent for P_d , 90 plus 9, and prices have reduced by 10 percent, P_d^2 , 90 minus 9, that is 81. So, that is the meaning of multiplicative distribution. And in this case, all possible outcomes can be determined for a particular period using this particular simple generic, you can say, formula. Now, there can be additive situations also.

In additive situations, if the initial price is 100 rupees, I will say that the price will increase or decrease by 10 rupees. So, in period 1, the price will become 110 or 90. Period 2, 110 plus 10, 120. 110 minus 10, 100. 90 plus 10, 100 and 90 minus 10, 80. 120 plus 10, 130. 120 minus 10, 110. 100 plus 10, 110. 100 minus 10, 90, 80 plus 10, 90 and 80 minus 10, 70. So, these are the cases of additive binomial distributions.

So, in this way, we are able to create our, let us say, decision tree, and in this decision tree, you also need to assign the probability. And these are all basically part of this particular part of our discussion: how, over a period of time, if you are opening a large factory, an average factory, or a small factory, and based on these particular scenarios, if you have this much of a price and these kinds of situations happen, you will earn a particular expected profit, expected monetary value, and accordingly, you may take a decision whether you should go for this kind of decision or not. So, how are you going to represent your different kinds of additive or multiplicative possibilities on the decision tree that we have discussed? Now, based on this, managers can make a very good choice between their long-term options, short-term options, and on the basis of these things, we

will see in our next session that we are able to make very interesting and very accurate decisions.

Whether option A is better or option B is better, we will see with the help of a facility location problem that there are multiple types of options and how, using the decision tree, we are able to decide which option is better. So, with this, we come to the end of this particular session, where we understood how we are going to construct the decision tree, and these decision trees are going to help us in making decisions that are very simple to analyze because everything is graphically available in front of our eyes. Thank you very much.