

Project Management

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Week: 4

Lecture 20 : Critical Path Method (CPM)

Dear students, in the previous lecture I have discussed about different quantitative techniques for analyzing the project risk management. Now, we are entering into the next topic that is project scheduling. So, in the project scheduling one way to schedule the project is critical path method that we are going to discuss in this lecture. So, this slide as usual it says about previously we discussed about quantitative assessment, now we are going to discuss critical path method. So, the agenda for this lecture is introduction to scheduling. Then what are the terminologies for network model, then project scheduling based on the expected activity times, then we will say what is a critical path, then I will explain what is the forward pass, backward pass, then I will explain what is the meaning of slack.

Part-II

Project Planning

Traditional project activity planning
Agile project planning
Coordination through integration management
Project feasibility analysis
Estimating project budgets
Project risk management
Quantitative risk assessment methodologies
Critical path method (CPM)
Programme evaluation and review technique (PERT)
Risk analysis with simulation for scheduling
Scheduling with scrum
Crashing a project
Resource loading
Resource levelling
Goldratt's critical chain

Course outline



Agenda

- Introduction
- Terminologies
- Project Scheduling Based on Expected Activity Times
 - Critical Path
 - Forward Pass
 - Backward Pass
 - Slack
- Contributions of PERT/CPM



Then what are the benefits of this critical path method, there is another technique that in detail we will cover in the next lecture that is called the program evaluation and review technique. Scheduling, scheduling is the conversion of a project work breakdown structure into operating timetable. Already we have work breakdown structure that we have to convert in the form of timetable, when it should be started, when should be finished that is the nothing but the scheduling. As such it serves as the basis for monitoring and controlling project activity and taken together with the plan and budget is probably the major tool for management of projects.

Scheduling

- A schedule is the conversion of a project work breakdown structure (WBS) into an operating timetable.
- As such, it serves as the basis for monitoring and controlling project activity and, taken together with the plan and budget, is probably the major tool for the management of projects.



Scheduling

- In a project environment, the scheduling function is more critical than it would be in an ongoing operation because projects lack the continuity of day-to-day operations and often present much more complex problems of coordination.
- Indeed, project scheduling is so important that a detailed schedule is sometimes a funder-specified requirement.



In a project environment the scheduling function is more critical than it would be in an ongoing operation because project lack the continuity of day to day operations and often present much more complex problem of coordination. Indeed project scheduling is so important that a detailed schedule is sometime as a funder specified requirement. A properly designed detailed schedule can also serve as the key input in establishing the monitoring and control system of the project. In general the schedule is developed down to the work packages level, but in a very large project the schedule of the project manager may only be the two or three levels deep with the supplement schedules for each major sub project. Many time the projects are so large or complex that a manager cannot possibly remember all the information pertaining to the plan schedule and progress of the project.

Scheduling

- A properly designed, detailed schedule can also serve as a key input in establishing the monitoring and control systems for the project.
- In general, the schedule is developed down to the work package level, but in very large projects, the schedule for the project manager (PM) may only be two or three levels deep, with supplemental schedules for each major subproject.



Scheduling

- Projects are often so large or complex that the manager cannot possibly remember all the information pertaining to the plan, schedule, and progress of the project.
- In such situations the program evaluation and review technique (PERT) and the critical path method (CPM) have proven to be extremely valuable.



In such situation this PERT program evaluation review technique and the critical path method have proven to extremely valuable. Now we will discuss some of the common application of this PERT and CPM. PERT and CPM can be used to plan schedule and control a wide variety of projects. Some common applications include research and development of new products and process, construction of plant building and highways, maintenance of large and complex equipment, design and installation of new system. The various jobs or activities required to be done to ensure the entire project is completed on time could often be interconnected with each other.

Common Applications

PERT and CPM can be used to plan, schedule, and control a wide variety of projects.

Common applications include:

- Research and development of new products and processes.
- Construction of plants, buildings, and highways
- Maintenance of large and complex equipment
- Design and installation of new systems



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PERT/CPM

- The various jobs or activities required to be done to ensure the entire project is completed on time could often be interconnected with each other.
- For example, some activities depend on the completion of other activities before they can be started.
- Because projects may comprise as many as several thousand activities, PERT and CPM help project managers answer various questions.



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Typical Questions

What is the total time to complete the project?

What are the scheduled start and finish dates for each specific activity?

Which activities are “critical” and must be completed exactly as scheduled to keep the project on schedule?

How long can “noncritical” activities be delayed before they cause an increase in the total project completion time?



For example, some activities depend on the completion of other activities before they can be started because project may comprise as many as several thousand activities. PERT and CPM help project managers answers various questions like what is the total time to complete the project is a very basic and important question. Second one is what are the scheduled start and finish date for each project activity. Then we should know which activities are critical and must be completed exactly as scheduled to keep the project on schedule. Then how long can non-critical activities can be delayed before they cause an increase in the total project completion time.

Origin & History

- Initially, CPM and PERT were two different approaches
 - CPM used deterministic time estimates and allowed project crunching
 - PERT used probabilistic time estimates
- CPM was developed originally by DuPont and Remington Rand primarily for industrial projects for which activity times were certain and variability was not a concern.



So, these questions can be answered with the help of scheduling. Initially critical path method and PERT were two different approaches. CPM used deterministic time estimate and allowed for project crunching. PERT used probabilistic time estimate. So, critical

path method was developed by DuPont and the Remington Rand primarily for industrial projects for which activity times were certain and the variability was not a concern.

Origin & History

- CPM offered the option of reducing activity times by adding more workers and/or resources, usually at an increased cost.
- Thus, a distinguishing feature of CPM was that it identified trade-offs between time and cost for various project activities.

CPM offered the option of reducing activity times by adding more workers or resources usually at increased cost. Thus a distinguishing feature of critical path method was that it identified a trade-off between time and cost for various project activities. Today's computerized version of PERT and CPM combine the best features of both approaches. So, MS project have blended critical path method and PERT into one approach. Thus the distinction between them is no longer necessary.

Modern Interpretation

- Today's computerized versions of PERT and CPM combine the best features of both approaches.
- Microsoft Project (and others) have blended CPM and PERT into one approach
- Thus, the distinction between them is no longer necessary.



Terminologies

- **Activity** - A specific task or set of tasks that are required by the project, use up resources, and take time to complete
- **Event** - The result of completing one or more activities
- **Network** - The combination of all activities and events that define a project
 - Drawn left-to-right
 - Connections represent predecessors



Now we will see some of the terminology which are used in the scheduling. One is what is activity? A specific task or set of task that are required by a project use of resources and take time to complete that is an activity. Activity is something that will consume time. Event is the result of completing one or more activities that will not use the resources but that will give an indication when the activity started when the activity is finished. Then the another term is network that is the combination of all the activities and events that define your project.

So, network is drawn left to right and the arrow represents the predecessors. Then what is a path? A series of connected activities is called a path. Then what is a critical path? An activity or event or path which if delayed will delay the completion of the project that is called critical path or it may be critical activity. So, critical path is the path through the project where if any activity is delayed the project is delayed. So, there is always a

critical path there can be more than one critical path.

Terminologies

- **Path** - A series of connected activities
- **Critical** - An activity, event, or path which, if delayed, will delay the completion of the project
- **Critical Path** - The path through the project where, if any activity is delayed, the project is delayed
 - There is always a critical path
 - There can be more than one critical path



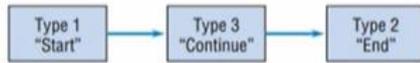
Terminologies

- **Sequential Activities** - One activity must be completed before the next one can begin
- **Parallel Activities** - The activities can take place at the same time
- **Immediate Predecessor** - That activity that must be completed just before a particular activity can begin

Then sequential activities one activity must be completed before the next one can begin. Then there may be a parallel activities. Activities can take place at the same time. Then another term called immediate predecessor the activity that must be completed just before a particular activity can begin. Now, I brought the example of sequential activities say start continue end and another important thing is there are two way to draw a network activity on node and activity and arrow format.

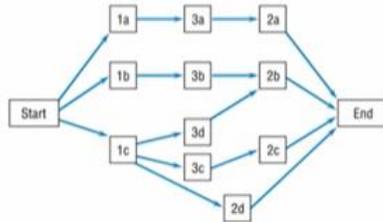
Terminologies

- **Sequential Activities**

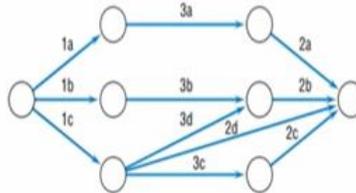


Three sequential activities, AON format.

- **AON and AOA Format**



Activity network, AON format



Activity network, AOA format



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Project Scheduling Based on Expected Activity Times

Example

- The owner of a Shopping Center plans to modernize and expand the current 32-business shopping centre complex.
- The project is expected to provide room for 8 to 10 new businesses.
- The first step in the PERT/CPM scheduling process is to develop a list of the activities that make up the project.



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So, the left hand side you see this activity on node. So, here node represents activity and the right hand side this arrow represent the activity, activity on arrow. So, there are activity on node activity on arrow. Now, I will take a sample problem then I will explain how to do the scheduling of that project. So, the sample problem is a owner of a shopping center plan to modernize and expand the current 32 business shopping centers complex.

Activity	Activity Description	Immediate Predecessor	Expected Activity Time
A	Prepare Architectural Drawings	-	5
B	Identify potential new tenants	-	6
C	Develop prospectus for tenants	A	4
D	Select Contractor	A	3
E	Prepare building permits	A	1
F	Obtain approval for building permits	E	4
G	Perform construction	D, F	14
H	Finalize contracts with tenants	B, C	12
I	Tenants move in	G, H	2
		Total	51

Table – 1
List of activities



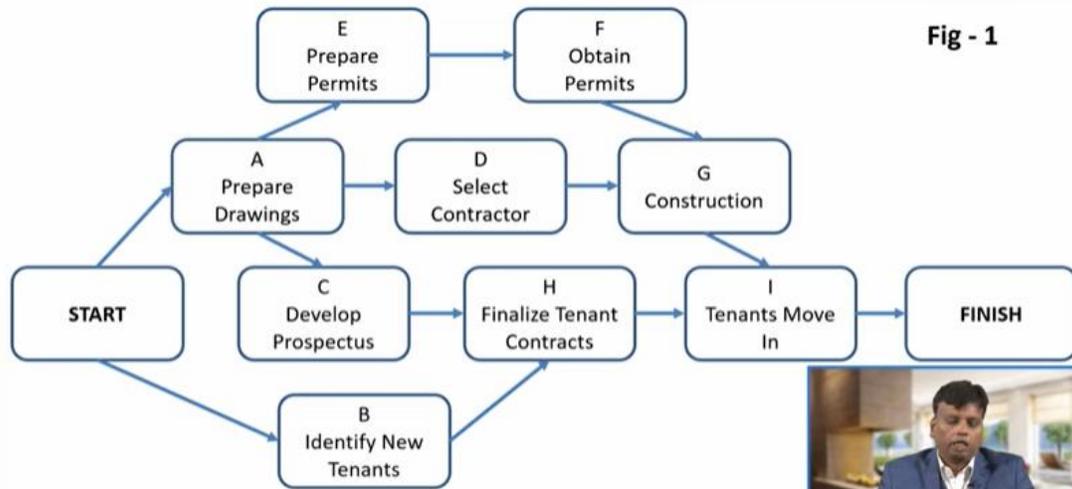
The project is expected to provide room for 8 to 10 new businesses. The first step in the PERT CPM scheduling process is to develop a list of activities that make up the project. So, these are the list of activities. We have activity A, B, C, D, E, activity description is given, immediate predecessor is given and the expected activity duration also given. The sum of the expected activity is 51.

Project Scheduling Based on Expected Activity Times

- The sum of expected activity times is 51.
- As a result, you may think that the total time required to complete the project is 51 weeks.
- However, two or more activities often may be scheduled concurrently (assuming sufficient availability of other required resources), thus shortening the completion time for the project.



As a result you may think that the total time required to complete the project is 51 weeks. However, two or more activities often may be scheduled concurrently assuming sufficient availability of other required resources. So, shortening the completion time for the project. If you do it some activities parallelly that will reduce the project duration. The activities correspond to the node of the network drawn we are drawn the rectangle here and the arc the line with arrows shows the precedence relationship among the activities.



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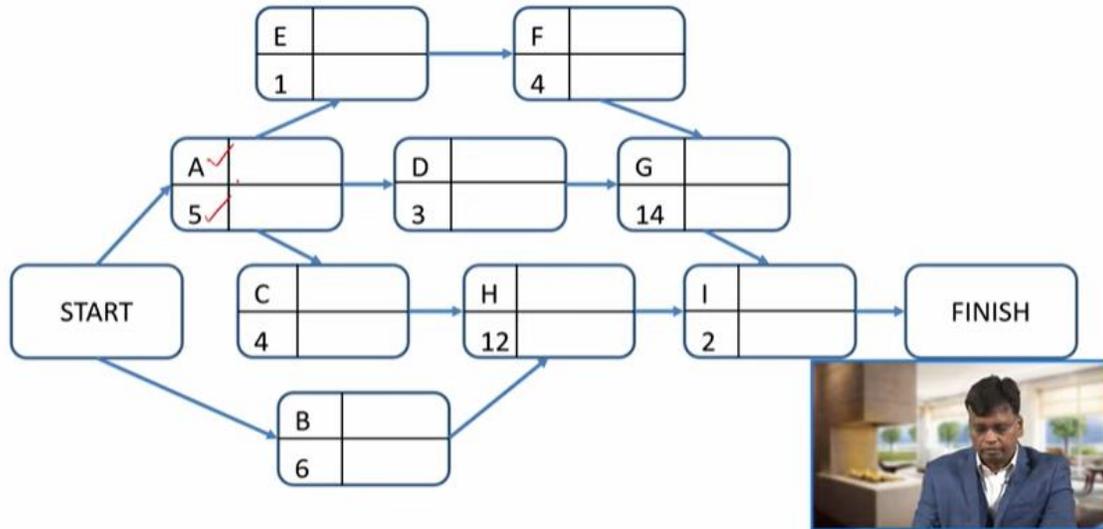
In addition node have been added to the network to denote start and finish of the project. We have to add two node one is a start another one is a finish. A project network will help a manager visualize the activity relationship and provide a basis for carrying out POD CPM computations. So, previously the activities are given in the table. Now, what I have done by considering the precedence and the duration of that activity I have constructed in the network form.

Project Scheduling Based on Expected Activity Times

- To facilitate the PERT/CPM computations, we shall modify the project network as shown in the next slide.
- Note that the upper left-hand corner of each node contains the corresponding activity letter.
- The activity time appears immediately below the letter.



So, note that I have added two new things one is start and finish then activity A, C, B, D, E, F, G, H, I all I have drawn by considering the precedence given in the table. To facilitate POD CPM computations we shall modify the project network as shown in the next slide. Note that the upper left hand corner of each node contains the corresponding activity letter. The activity time is appear immediately below the letter it is like this. For example, A represents activity name the 5 represents here.



This represent activity name this is the duration of the activity. First we will see how to find out the critical path. To determine the project completion time we have to analyze the network and identify what is called critical path of the network. A path is a sequence of connected nodes that leads from the start node to the finishing node. All path in the network must be traversed in order to complete the project.

Critical Path

- To determine the project completion time, we have to analyze the network and identify what is called the critical path for the network.
- A path is a sequence of connected nodes that leads from the Start node to the Finish node.
- All paths in the network must be traversed in order to complete the project, so we will look for the path that requires the **greatest time**.

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So, we will look for the path with requires greatest time. Because all the path are shorter in duration the longest path determine the total time required to complete the project. If activities on the longest path are delayed the entire project will be delayed. Thus the longest path is called critical path. Activities on the critical path are referred to as the critical activities of the project.

Critical Path

- Because all other paths are shorter in duration, this longest path determines the total time required to complete the project.
- If activities on the longest path are delayed, the entire project will be delayed.
- Thus, **the longest path is the critical path.**
- **Activities on the critical path are referred to as the critical activities for the project.**



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Now, we will use some notation to find out earliest time of an activity. So, here E S represent earliest starting time for example, it is like this. Suppose I have activity like this. So, this is earliest starting time this is earliest finishing time.

Earliest time for any activity

Let:

- ES = earliest start time for an activity
- EF = earliest finish time for an activity
- t = expected activity time



The earliest time for any activity is:

- $EF = ES + t$



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So, this is the duration. So, if you want to know the earliest finishing time this is the earliest starting time plus the duration of the activity. So, the activity A can start as soon as the project starts. So, we set the earliest start time for activity A is equal to 0. So, earliest finishing time will be earliest starting time 0 plus duration of activity 5.

Forward Pass

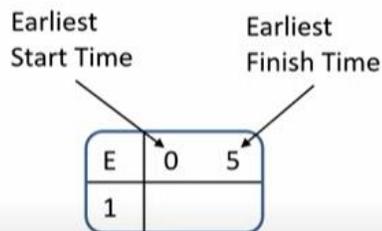
- Activity A can start as soon as the project starts, so we set the earliest start time for activity A equal to 0.
- With an expected activity time of 5 weeks, the earliest finish time for activity A is :
- $EF = ES + t = 0 + 5 = 5$.
- We will write the earliest start and earliest finish times in the node to the right of the activity letter.



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Forward Pass

- Using activity A as an example, we have :



Activity	Activity Description	Immediate Predecessor	Expected Activity Time
A	Prepare Architectural Drawings	-	5
B	Identify potential new tenants	-	6
C	Develop prospectus for tenants	A	4
D	Select Contractor	A	3
E	Prepare building permits	A	1
F	Obtain approval for building permits	E	4
G	Perform construction	D, F	14
H	Finalize contracts with tenants	B, C	12
I	Tenants move in	G, H	2
	Total		51



So, 0 plus 5 equal to 5. So, we will write the earliest start and earliest finishing time in the node right of the activity letter like this. So, here 0 represents earliest starting time 5 represents the earliest finishing time. Because an activity cannot be started until all immediately preceding activities have been finished. The following rule can be used to determine the earliest starting time for each activity that rule is called forward pass. So, what is that rule says the earliest start time of an activity is equal to the largest of earliest finishing time for all its immediate predecessors.

Forward Pass

- Because an activity cannot be started until all immediately preceding activities have been finished, the following rule can be used to determine the earliest start time for each activity:

“The earliest start time for an activity is equal to the largest (i.e., latest) of the earliest finish times for all its immediate predecessors.”



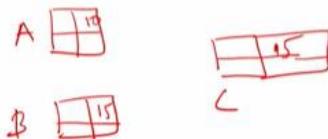
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So, what is the meaning of this one is suppose there is an activity. There is an activity there may be a assume that there is a two preceding activities. So, here the earliest finishing may be 10. So, here the earliest finishing is 15. So, this is activity assume that this is A this is B C.

Forward Pass

- Because an activity cannot be started until all immediately preceding activities have been finished, the following rule can be used to determine the earliest start time for each activity:

“The earliest start time for an activity is equal to the largest (i.e., latest) of the earliest finish times for all its immediate predecessors.”



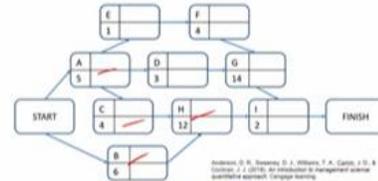
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So, if you if you want to start earliest starting time, you have to consider the largest value of earliest finishing time of preceding activity that is the earliest start time of activity this is equal to largest of earliest finishing time of all its immediate predecessors if you say 10 and 15 is there. So, 15 will be the earliest start time because out of 10 and 15, 15 is the largest. So, that is this logic of this forward pass. Let us apply earliest start time rule to the portion of the network involving nodes A B C H, you see A B C H, A B C H with the earliest start time of 0 and activity time of 6 for activity B see here.

Forward Pass

- Let us apply the earliest start time rule to the portion of the network involving nodes A, B, C, and H.
- With an earliest start time of 0 and an activity time of 6 for activity B, we show :
 $ES = 0$ and $EF = ES + t = 0 + 6 = 6$ in the node for activity B.

Activity	Activity Description	Immediate Predecessor	Expected Activity Time
A	Prepare Architectural Drawings	-	5
B	Identify potential new tenants	-	6
C	Develop prospectus for tenants	A	4
D	Select Contractor	A	3
E	Prepare building permits	A	1
F	Obtain approval for building permits	E	4
G	Perform construction	D, F	14
H	Finalize contracts with tenants	B, C	12
I	Tenants move in	G, H	2
Total			51



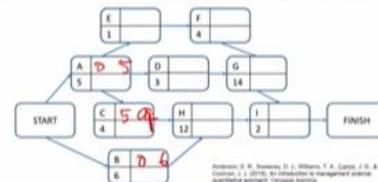
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So, earliest starting time is 0. So, finishing time will be 6 for activity B. Looking at the node C, we note that activity A is the only immediate predecessor of activity C here. The earliest finishing time of activity A is 5 because why because 0 and this is 5 here 0 here it is 6. So, earliest start time of activity C is 5 plus 4 = 9.

Forward Pass

- Looking at node C, we note that activity A is the only immediate predecessor for activity C.
- The earliest finish time for activity A is 5, so the earliest start time for activity C must be $ES = 5$.
- Thus, with an activity time of 4, the earliest finish time for activity C is $EF = ES + t = 5 + 4 = 9$.

Activity	Activity Description	Immediate Predecessor	Expected Activity Time
A	Prepare Architectural Drawings	-	5
B	Identify potential new tenants	-	6
C	Develop prospectus for tenants	A	4
D	Select Contractor	A	3
E	Prepare building permits	A	1
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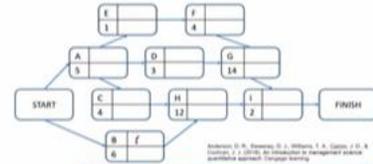
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So, 9. Now, for activity H, we have to find out what is earliest starting time. So, what is happening there are two earliest finishing time is a one is 9 another one is 6. So, after this which is larger 9 is larger. So, the 9 will be the earliest starting time of activity H. So, moving to activity H with the both activities B and C as immediate predecessors, the earliest start time of activity H must be equal to the largest of the earliest finishing time of B and C.

Forward Pass

- Moving on to activity H, with both activities B and C as immediate predecessors, the earliest start time for activity H must be equal to the largest of the earliest finish times for activities B and C.
- Thus, with EF = 6 for activity B and EF = 9 for activity C, we select the largest value, 9, as the earliest start time for activity H.
- With an activity time of 12 as shown in the node for activity H, the earliest finish time is : $EF = ES + t = 9 + 12 = 21$.

Activity	Activity Description	Immediate Predecessor	Expected Activity Time
A	Prepare Architectural Drawings	-	5
B	Identify potential new tenants	-	6
C	Develop prospectus for tenants	A	4
D	Select Contractor	A	3
E	Prepare building permits	A	1
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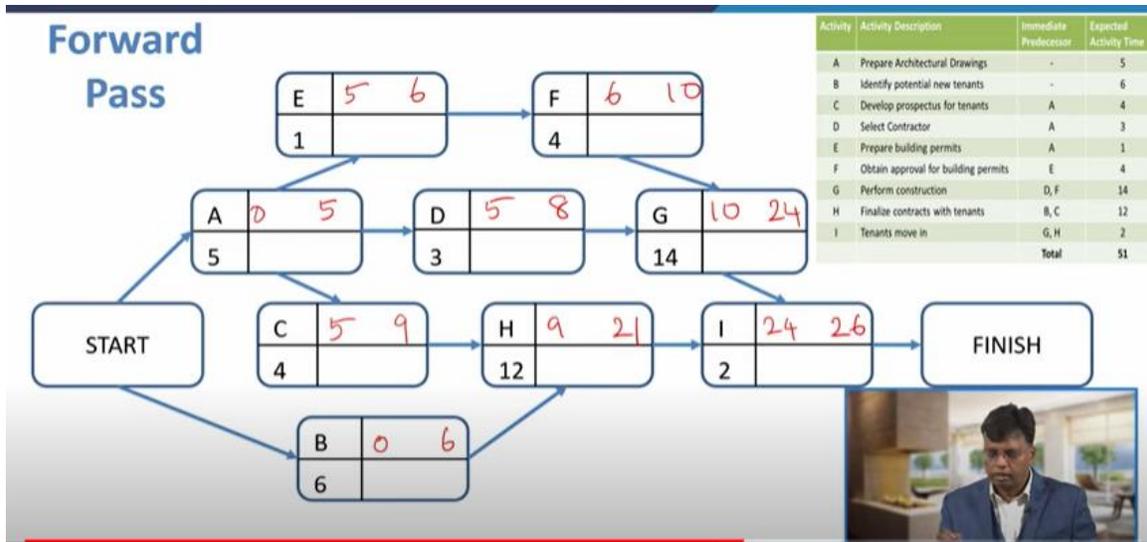
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So, for B it is 0, 6, this is 0, 5, this is 5, 9. So, the earliest finishing time equal to 6 for an activity B and the earliest finishing time 9 for activity C. So, we select the largest value 9 as the earliest starting time for activity H. So, we are selected 9.

So, the duration is 12. So, it will become 21. So, this is the rule of your forward pass. So, like that we can do it for all activities. So, I am going to start for everything.

So, it is 0, it is 5. So, 5, 9, 0, 6. So, 5, 6, 6, 10, 5, 8. So, 9, 21. Now, you for G there are two producers 8 and 10.

So, the largest one is 10. So, 10, 24. Now, for I, there are two options 21 is there 24, the largest is 24. So, 26. So, this is the forward pass. This I have shown continuing with this forward pass through the network, we can establish the earliest start time and earliest finishing time for each activity in the network. So, the earliest finish time for activity I, the last activity in the project is 26 weeks.



Forward Pass

- Continuing with this forward pass through the network, we can establish the earliest start time and the earliest finish time for each activity in the network.
- The earliest finish time for activity I, the last activity in the project, is 26 weeks.
- Therefore, we now know that the ***expected completion time for the entire project is 26 weeks.***



Therefore, we know that the expected completion time for entire project is 26 weeks. Now, we will discuss about the backward pass. Let us now continue the algorithm for finding the critical path by making backward pass through the network. Because the expected completion time for the entire project is 26 weeks, we begin backward pass with the latest finishing time of 26 for activity I. So, what is the notation here, Ls is the latest starting time, Lf is the latest finishing time.

Backward Pass

- Let us now continue the algorithm for finding the critical path by making a backward pass through the network.
- Because the expected completion time for the entire project is 26 weeks, we begin the backward pass with a latest finish time of 26 for activity I.



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Backward Pass

Let

- LS = latest start time for an activity
- LF = latest finish time for an activity
- $LS = LF - t$



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So, we get latest starting time by subtracting t from our latest finishing time. So, what we do here, we know latest finishing time, we know duration. From this we find latest starting time. Beginning the backward pass with activity I, we know that the latest finishing time is 26 and the activity duration is 2.

Backward Pass

- Beginning the backward pass with activity I, we know that the latest finish time is $LF = 26$ and that the activity time is $t = 2$.
- Thus, the latest start time for activity I is :

$$LS = LF - t = 26 - 2 = 24$$

So, the latest starting time is 24. So, what is the backward pass rule? The following rule can be used to determine the latest finish time of activity. What the rule says, the latest finish time of activity is the smallest of the latest start time of all activities that immediately follow the activities. Logically, this rule states that latest time of an activity can be finished equals the earliest that is the smallest value for the latest start time of following activity. So, what this rule says, assume that again there are two latest start time, here 25 say 23. So, the latest finish time for the preceding activity should be the smallest value the 23.

Backward Pass

- The following rule can be used to determine the latest finish time for each activity in the network:

“The latest finish time for an activity is the smallest (i.e., earliest) of the latest start times for all activities that immediately follow the activity.”

- Logically, this rule states that the latest time an activity can be finished equals the earliest (smallest) value for the latest start time of following activities.

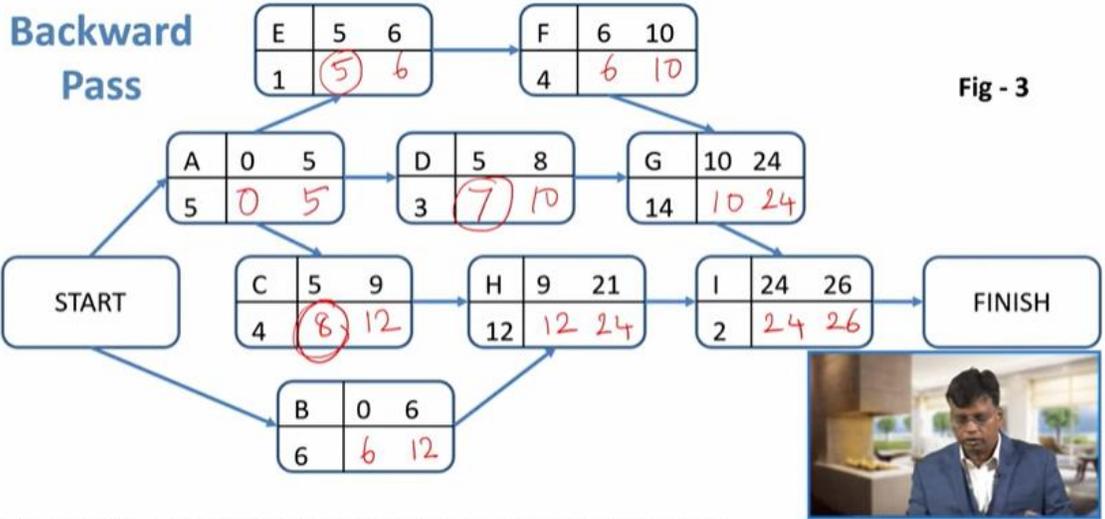


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So, there are two activity the latest start time of this activity is 25, this is 23. Since we are coming the backward, so we have to see which is the smallest. So, the smallest will be the latest finish time of that activity. So, we I will explain how to go for this backward pass.

So, start from 26. So, 26 minus 2 is 24. So, this 24, 24 minus 14, 10. So, here also will be 24, 24 minus 12, it is 12. So, here 10, 10 minus 4, 6.

So, 10 minus 3, 7. Now, it is 12 minus 6, 6. Here 12 minus 4 is 8, here 6 minus 1 is 5. Here it is 8 is there, here it is 7 is there. For activity A, so activity E, D, C is there. So, here the latest start time is 5, here the latest start time is 7, here the latest start time is 8.

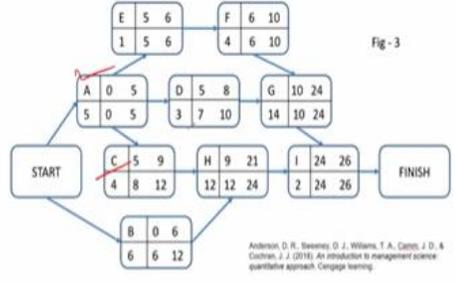


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So, we have to find out the minimum value. So, minimum is 5. So, 5 minus 5 is 0. So, this is the logic of the backward pass. So, what it says that when you come from backward, so we have to choose the latest start time as the latest finishing time of preceding activity. Here this we have find that. So, activity A require more involved application of the latest start time rule, this activity.

Backward Pass

- Activity A requires a more involved application of the latest start time rule.
- First, note that three activities (C, D, and E) immediately follow activity A.
- The latest start times for activities C, D, and E are 8, 7, and 5 respectively.



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Note that 3 activities C, D, E, the latest start time of activity C, D, E is 8, 7, 5. So, the latest finish time rule for activity A states that the latest finish time of activity A is the smallest of the latest start time of succeeding activities. So, with the smallest value is 5.

So, we say latest finish time of activity A is 5.

Slack

- After we complete the forward and backward passes, we can determine the amount of slack associated with each activity.
- Slack is the length of time an activity can be **delayed** without increasing the project completion time.
- The amount of slack for an activity is computed as follows:

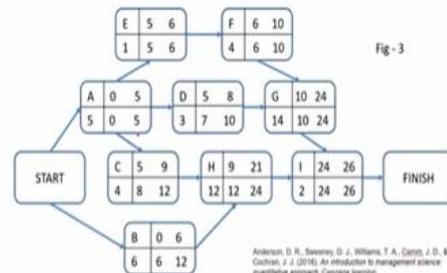
$$\text{Slack} = \text{LS} - \text{ES} = \text{LF} - \text{EF}$$



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Backward Pass

- The latest finish time rule for activity A states that the LF for activity A is the smallest of the latest start times for activities C, D, and E.
- With the smallest value being 5 for activity E, we set the latest finish time for activity A to LF = 5



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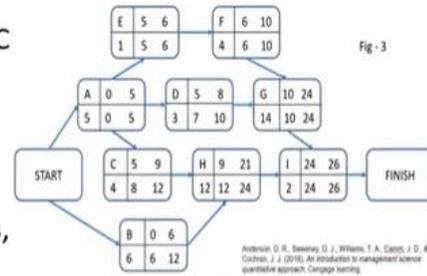
So, the duration is 5, then it will be 0. Now, after we complete the forward and backward passes, we can determine the amount of slack associated with each activity. What is a slack? Slack is the length of the time activity that can be delayed without increasing the project completion time. So, the amount of slack of an activity can be computed as latest starting time minus earliest starting time. So, for example, earliest starting time, earliest finishing time, suppose this latest starting time, latest finishing time.

Slack

- For example, the slack associated with activity C is :

$$LS - ES = 8 - 5 = 3 \text{ weeks}$$

- Hence, activity C can be delayed up to 3 weeks, and the entire project can still be completed in 26 weeks.
- In this sense, activity C is not critical to the completion of the entire project in 26 weeks.

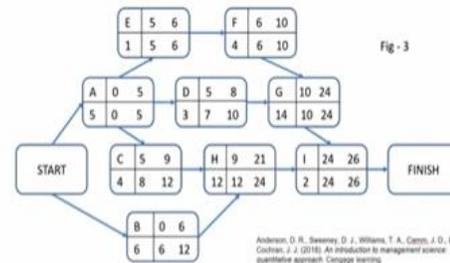


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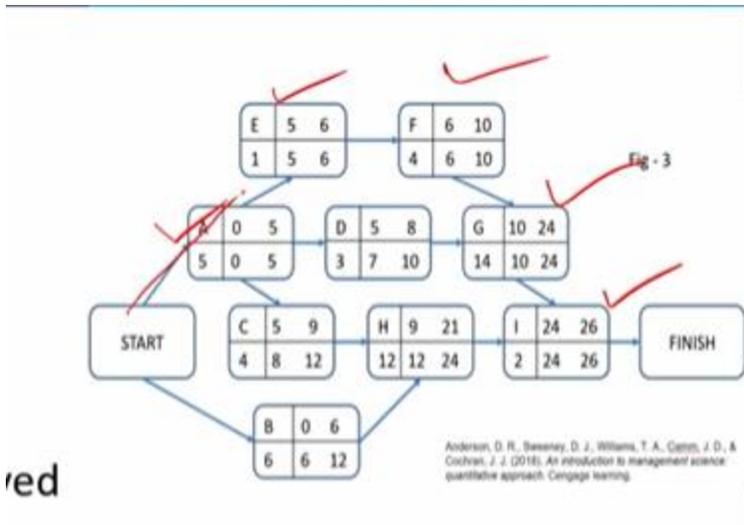
So, this difference is called your slack. For example, the slack for activity C is here 8 minus 5 is 3. Hence, activity C can be delayed up to 3 weeks and the entire project still be completed in 26 weeks. In this case, the activity C is not critical.

Slack

- Next, we consider activity E.
- The slack is $LS - ES = 5 - 5 = 0$.
- Thus, activity E has zero, or no, slack.
- Consequently, this activity cannot be delayed without increasing the completion time for the entire project.



Next, we consider activity E. Here, the slack is 0. So, this activity cannot be delayed without increasing the completion time for entire project. So, this is a critical activity. So, wherever the slack is 0, here it is 0, here 0, here 0, here 0, here 0. So, this route is your critical path. In other words, completing activity E exactly as scheduled is critical in terms of keeping the project on schedule.



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So, exactly E is a critical activity. So, more important point is the critical activities are the activities with 0 slack. So, for each from the previous table, I have constructed a table. Earliest starting time, latest start time, earliest finishing time. So, this is this way, earliest starting time, earliest finishing time, latest start time, latest finishing time, then slack.

Activity	ES	LS	EF	LF	Slack (LS – ES)	Critical Path?
A	0	0	5	5	0	YES
B	0	6	6	12	6	
C	5	8	9	12	3	
D	5	7	8	10	2	
E	5	5	6	6	0	YES
F	6	6	10	10	0	YES
G	10	10	24	24	0	YES
H	9	12	21	24	3	
I	24	24	26	26	0	YES

Table - 2
Activity
Schedule



Contributions of PERT/CPM

- We previously stated that project managers look for procedures that will help answer important questions regarding the planning, scheduling, and controlling of projects.
- Let us reconsider these questions in light of the information that the critical path calculations have given us.



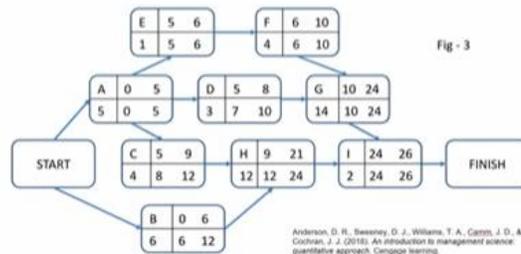
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If it is 0, there is a critical path. So, what is the critical path? A E F G I. So, we previously stated that the project manager look for procedures that will help answers important questions regarding the planning, scheduling and controlling of the project. Let us consider these questions in the light of the information that the critical path calculation have given us. So, the first question is how long the project take to complete? So, the answer is the project can be completed in 26 weeks if each activity is completed on schedule. Second question is what are the scheduled start time and completion time for each activity? So, from the table, we can find out earliest starting time, earliest finishing time in a cumulative scale.

Contributions of PERT/CPM

- **How long will the project take to complete?**

Answer: The project can be completed in 26 weeks if each activity is completed on schedule.



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Contributions of PERT/CPM

- **What are the scheduled start and completion times for each activity?**

Answer: The activity schedule (Table -2) shows the earliest start, latest start, earliest finish, and latest finish times for each activity.

Activity	ES	LS	EF	LF	Slack (LS - ES)	Critical Path?
A	0	0	5	5	0	YES
B	0	6	6	12	6	
C	5	8	9	12	3	
D	5	7	8	10	2	
E	5	5	6	6	0	YES
F	6	6	10	10	0	YES
G	10	10	24	24	0	YES
H	9	12	21	24	3	
I	24	24	26	26	0	YES



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Here the time is written in the cumulative scale. The third question, which activities are critical and must be completed exactly as scheduled to keep the project in schedule? So, wherever we have to look at wherever the slack is 0. So, activity A E F G I, these are the critical activities. So, how long can non-critical activities can be delayed before they cause an increase in the project completion time? So, here we have to look for the slack wherever there is a positive slack. So, that many days that non-critical activities can be delayed. Dear students, in this lecture, I have talked about introduction to the schedule and I have explained various terminologies.

Contributions of PERT/CPM

- Which activities are critical and must be completed exactly as scheduled to keep the project on schedule?

Answer: A, E, F, G, and I are the critical activities.

Activity	ES	LS	EF	LF	Slack (LS - ES)	Critical Path?
A	0	0	5	5	0	YES
B	0	6	6	12	6	
C	5	8	9	12	3	
D	5	7	8	10	2	
E	5	5	6	6	0	YES
F	6	6	10	10	0	YES
G	10	10	24	24	0	YES
H	9	12	21	24	3	
I	24	24	26	26	0	YES



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Contributions of PERT/CPM

- How long can noncritical activities be delayed before they cause an increase in the completion time for the project?

Answer: The activity schedule (Table - 2) shows the slack associated with each activity.

Activity	ES	LS	EF	LF	Slack (LS - ES)	Critical Path?
A	0	0	5	5	0	YES
B	0	6	6	12	6	
C	5	8	9	12	3	
D	5	7	8	10	2	
E	5	5	6	6	0	YES
F	6	6	10	10	0	YES
G	10	10	24	24	0	YES
H	9	12	21	24	3	
I	24	24	26	26	0	YES



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Then I have explained how to construct a network, then how to find out the critical path. There I have discussed about two important rule forward pass and backward pass. Then I have explained the physical meaning of this slack. Then I have explained what are the possible questions that can be answered by using this critical path network. Thank you. Thank you.