Quantitative Investment Management
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Lecture 60
Interest Rate Futures – 2

Welcome back. So, let us continue where we left off before the break.

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I shall be talking about the applications of short-term interest rate futures, for example, treasury bill futures.

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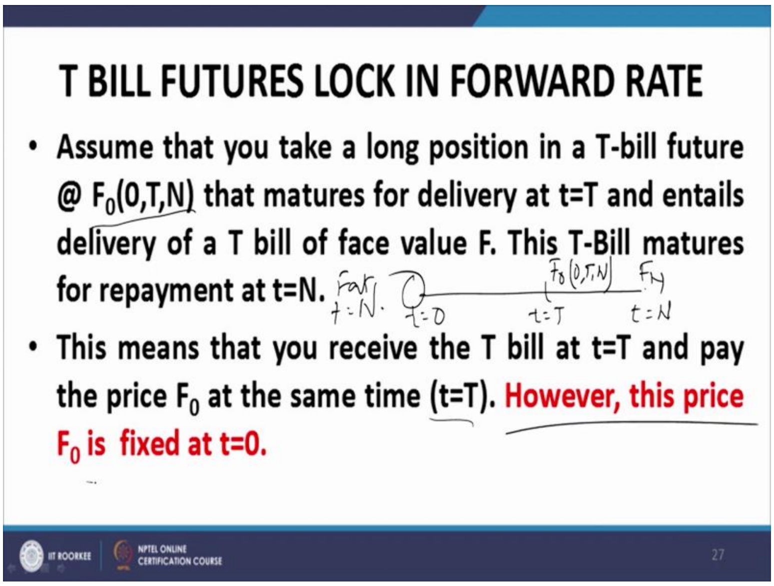

The first and the very obvious application is that the sT-bill futures lock in forward rate. How does it happen? It is quite simple. Let us say again, we will refer to our timeline today, let us say t equal to 0, let us say t equal to capital T is the maturity of the futures contract. And let us at t equal to N is the maturity of the underlying treasury bill. Now, what does it mean if I take a long position in the futures contract, what does it mean?

It means that I will receive a T-bill at t equal to capital T, which will mature at t equal to capital N. And the important part is that the price that I am going to pay for this bill will be settled at t equal to 0. So, the price is agreed at t equal to 0, the price is agreed as of today and the actual settlement of the bill will take plus at t equal to capital T and the settlement of the proceeds of the bill the majority of the bill that is will take place at t equal to N.

Let me repeat. The settlement of the futures contract by the delivery of the bill will take place at t equal to capital T. The settlement of the bill by payment proceeds from by the issuer will take place at t equal to N. Now, the price is known at t equal to 0 and that will be paid at t equal to capital T. So, let us call this F0 0 comma T comma N. And the price at which the bill is going to be liquidated the bill is going to be settled by credit by the issuer of the face value of the bill at t equal to N let us call it FN.

Now, the important part here is that both is quantities FN that is the face value of the bill and F0, which is the price at which the bill is going to be delivered to the parties long in the future are known at t equal to 0. And because both things like cash inflow and the cash outflow or vice versa is known at t equal to 0. The return or the interest rate over the period from t equal to capital T to T equal to N is known and fixed at t equal to 0.

So, that is how the treasury bill futures lock in the forward rate. Let me read it out for you. Assume that you take a long position in a T-bill future at the rate of F0 0 comma t comma N that matures for delivery at t equal to capital T and entails delivery of a T-bill of face value F, the t T-bill matures or repayment at t equal to N, so you will receive F an amount of cash flow of f at t equal to N.

And the price that you have paid that is F0 this will occur at t equal to capital T. But both these quantities F0 and F are known at t equal to 0. This means that you receive the T-bill at t equal to capital T and pay the price F0 at the same time that is T equal to capital T. However, this price is fixed at t equal to 0. This is the catch, this is the important part.

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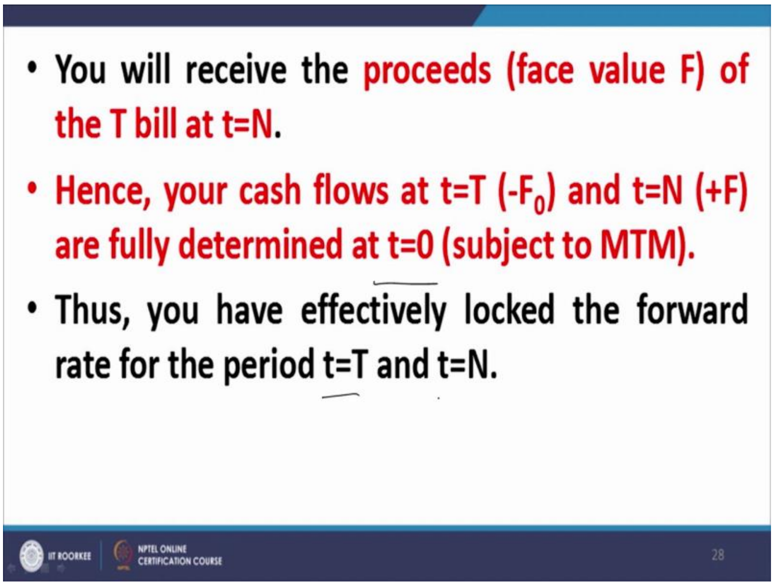

You will receive the proceeds the face value of the bill at t equal to N. Your cash flows at t equal to capital T that is minus F0 and t equal to N that is plus F are fully determined at t equal to 0 subject to mark into market. Thus, you have effectively locked in the forward rate for the period from t equal to T to t equal to N. So, this is quite simple this is a rather for forthright application of the T-bill futures. You lock in the forward interest rates over the life of the T-bill that forms the substratum of the futures contract.

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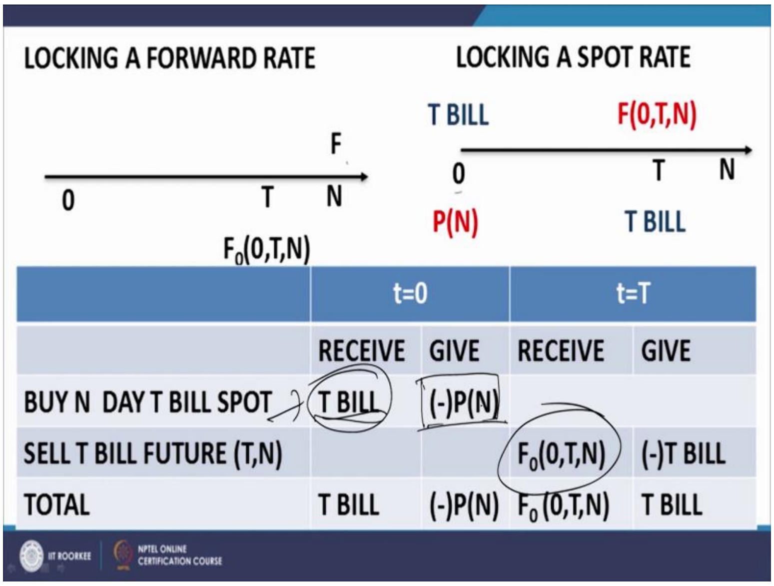

Now, you can also lock in a spot rate for a maturity which is different from the maturity for which traditional spot rates are available in the financial or money markers. How can you do that? That is shown in the right-hand side panel. What you do is at t equal to 0, you buy an N day T-bill in the spot market and you sell a T-bill future which has a maturity at t equal to capital T. And entails the delivery of a T-bill, which matures the t equal to capital N.

Let me repeat. You take a long position in an N day T-bill, which matures at t equal to capital N at which point you will get the face value of the T-bill from the issuer of the instrument. And you also take a short position in a T-bill future, which matures for delivery at t equal to capital T and this futures contract is such that the underlying the T-bill that will be delivered under this futures contract will mature at t equal to capital N.

The majority of the T-bill which you have taken a long position in and the T-bill which will be delivered under the futures contract is the same date t equal to capital N. So, what happens at t equal to 0 you pay the current price of the T-bill that is minus PN. And as far as the futures is concerned there is no transaction.

At t equal to capital T what will happen the T-bill that you receive in the first leg of the transaction can be delivered against your obligation of the short obligation in the T-bill futures contract because the greater maturity of this T-bill that you have received under the purchase is the same as the majority of the T-bill that is to be delivered against the short futures position that you have undertaken.

So, this T-bill that you have here can be redeemed or can be used for satisfying your delivery obligation at t equal to capital T against the futures contract. And what will you receive, you will receive the futures price F0 0 T N which you have worked out which you have agreed at t equal to 0. So, your cash outflow at t equal to 0 is known minus PN and your cash inflow is known at t equal to capital T. And in other words, you have logged in the spot rate between t equal to 0 and t equal to capital T.

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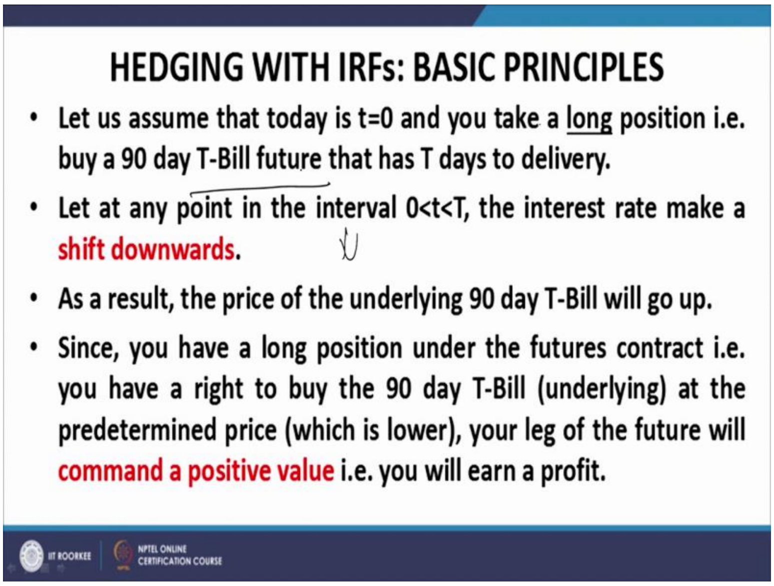

Now, we talk about hedging with interest rate futures, what are the basic principles. Let us assume that today is t equal to 0 and you take a long position that is you buy a 90-day T-bill future that is T days to delivery. Let at any point in the interval 0 less than small t less than capital T that is any arbitrary point between 0 and capital T the interest rate make a drift downwards, the interest rate goes down.

If the interest rate goes down, what happens to the price of the T-bill? The price of the underlying T-bill will go up in this spot market. And because of the positive correlation that usually substance between the spot market and the futures market, the price of the T-bill future will also increase. In other words, if the interest rate has gone down, the price of the futures T-bill futures is likely to go up.

Now, because you have a long position in the T-bill futures, this one, you have a long position in the T-bill futures, what will happen, you will make a profit on this you will gain. Therefore, if the original position is such that downward a fall in the interest rates, it results in a loss to you then you can create a hedge by taking a long position in the T-bill future because the interest rates when they fall down, they go down the effect is spotted T-bill prices, which also affects the futures T-bill prices, which increases.

And as a result of this impact you make a profit on your long position in the T-bill future. So, if your original position your investment position is such that a downward decline in the interest rates results in a loss to you, you can cover it by a long position in the T-bill futures. So, let me read it out for you. Since you have a long position under the futures contracts, that is you have right to buy the 90-day T-bill underline at the predetermined price which is lower, your leg up the futures will command a futures value that is you will earn a profit.

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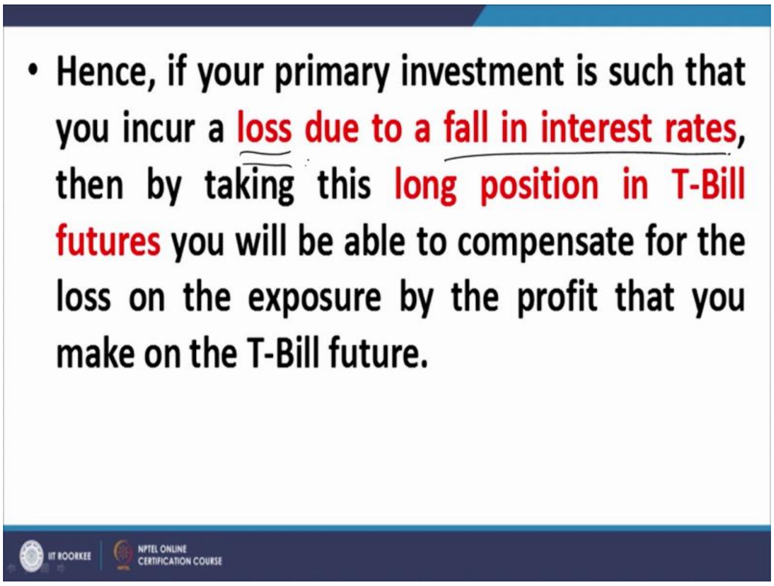

Hence, if your primary investment is such that you incur a loss due to decline in interest rates, you incur a loss due to declining interest rates then by taking this long position and T-bill futures, you will be able to compensate for the loss on the exposure by the profit that you make on the T-bill futures.

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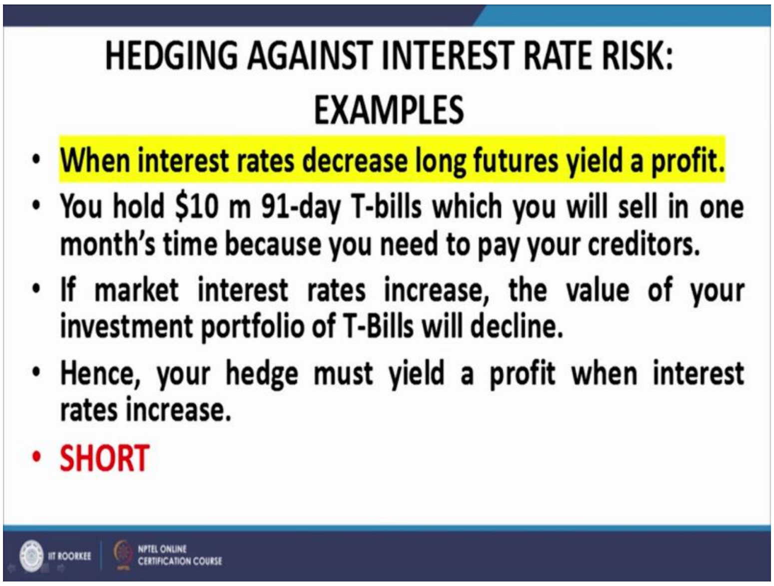

So, let us do some examples. Please recall I have highlighted in yellow, when interest rates decrease long futures increase your value, long futures yield your profit. So, let us take this example, you hold a US dollar 10 million 91-day T-bill, which you will sell in one month's time, because of your need to pay your creditors. Now, if the market interest rates increase, what will happen to the value of your portfolio, value of your portfolio will decline.

And if you want to hedge against a decline in value due to interest rates increase, what position will you take in the futures market, you will take a short position in the futures market, because if you are short in the futures market, and there is an increase in interest rates, the market prices of T-bills will fall in the spot market and correspondingly in the futures market as well. And because you have a short position in the hedge, you will make a profit on the hedge.

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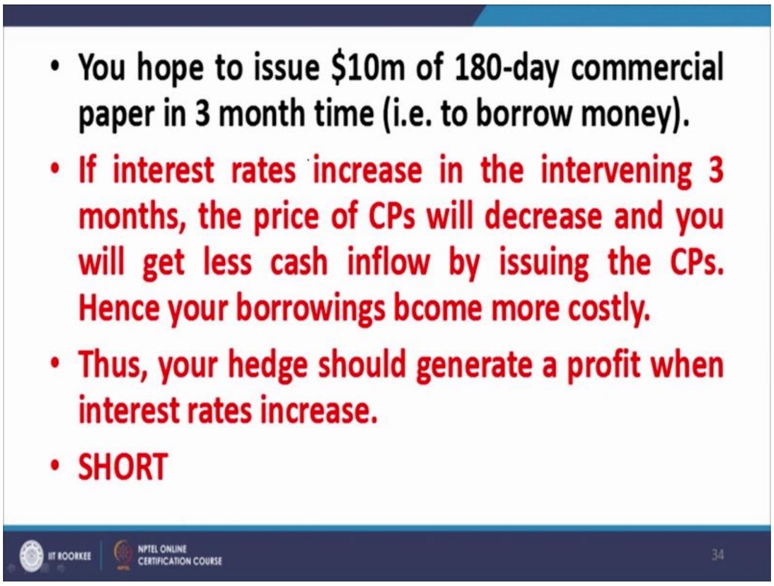

Now, another example, you will receive dollars 10 million in 6 months’ time which you want to invest in T-bills for 90 days. If interest rates decline in the intervening 6 months, the price of T-bill increase and you will get less face value or T-bills for your investment. For the same amount of money, you will be able to buy lesser face value of T-bills, thus your return will decrease if the interest rates decline. Now, if the interest rates decline that means, your hedge should be such that if the interest rates decline, you should be able to make a profit that is in other words, we will take a long position in T-bill futures.

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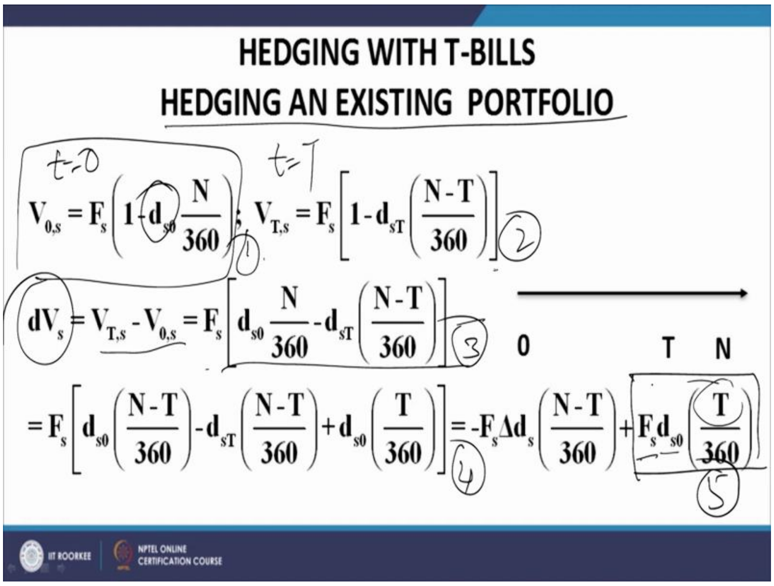

You ought to issue dollar 10 million of 180-day commercial paper in 3 months' time to borrow money. If interest rates increase what will happen? If the interest rates increase in the intervening period, the price of the CPs at issue will decline. And in other words, their cost of servicing will increase.

Your cash flow that you receive at the point of selling or of the commercial paper will be lesser and this may adversely affect your projects also due to paucity or scarcity of funds, and it will also increase the cost of servicing because the repayment has to be made at the face value. Now, in order to protect yourself against all this, what happens you take a short position in T-bill futures whereby if the interest rates increase, you make a profit on the short position.

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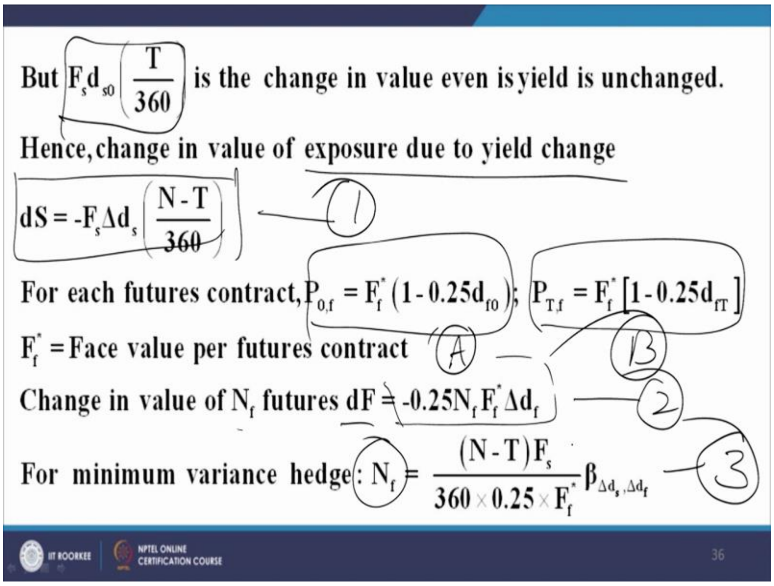

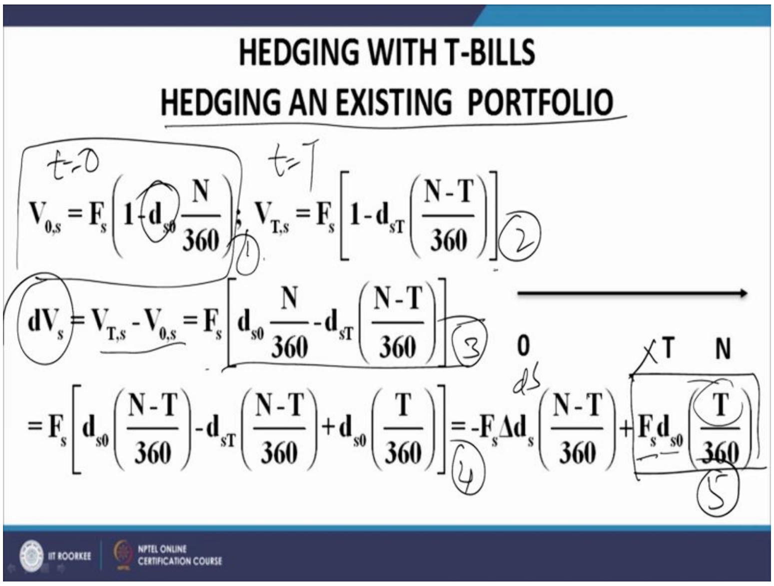

Now hedging with T-bills. Let us look at the quantitative treatment. Hedging with T-bills. Hedging an existing portfolio. You have an existing portfolio of T-bills and you want to hedge the T-bills portfolio against fluctuations in value due to changes in interest rates discount yields. So, the V 0 comma s is the value of your portfolio t equal to 0, Fs is the face value of your investment in T-bills, Fs is the face value, V 0 comma s is the market value of your investment at t equal to 0, market value of the investment at t equal to 0 is V 0 comma S.

Fs is the face value of your investment and d 0 comma s comma 0 is the spot discount yield at t equal to 0. And N is the maturity of the T-bills that constitute your portfolio. Then we have this expression for V 0 comma s that is equal to Fs into 1 minus d s 0. The d s 0 is the discount. Please note. Not interested discounted that is prevailing at t equal to 0 on T-bills with a maturity equal to capital N.

And the corresponding equation at t, this is the equation at t equal to 0. The corresponding equation at t equal to capital T which is the investment horizon or the point at which you are going to lift the hedge, the point at which you are going to liquidate the investment. And this is the corresponding equation that you have.

Please note. N upon 360 is replaced by N minus t upon 360 because the remaining life of the T-bills, please note the life of the T-bills capital N, your holding period is capital T, so the remaining life of the T-bills at the point at which you are going to liquidate them is equal to N minus t upon 360. And d sT is the discount yield that is to prevail or that is expected to prevail at t equal to capital T.

Therefore, the change in value that is V T s minus V 0 s is equal to this expression. This is straightforward. And you can write this, let us call this equation number 1, let us call this 2, let us call this 3, Let us call this 4. We can write equation number 3 in the form of equation number 4. The purpose of writing this as equation number 4 will be apparent in the next slide.

But for the matter, there is only an algebraic rearrangement, nothing more. And using that algebraic rearrangement we are reorganizing equation number 3 to equation number 4, which can be written as equation number 5. Now, if you look at equation number 5, that this factor of equation number 5, the factors that have enclosed in the box, this factor depends on the initial yield, which is known to you, which is not a random variable, which is unknown parameter unknown quantity, which is prevailing at t equal to 0.

Therefore, this change in value, or this component of dV, this component of the change in value is redundant insofar aging is concerned, because this is a fix component. Fs is known to you, d S0 is known to you, capital T is known to you, so obviously, this factor is fixed at t equal to 0. And therefore, it does not form any ingredient insofar as the hedging process is concerned.

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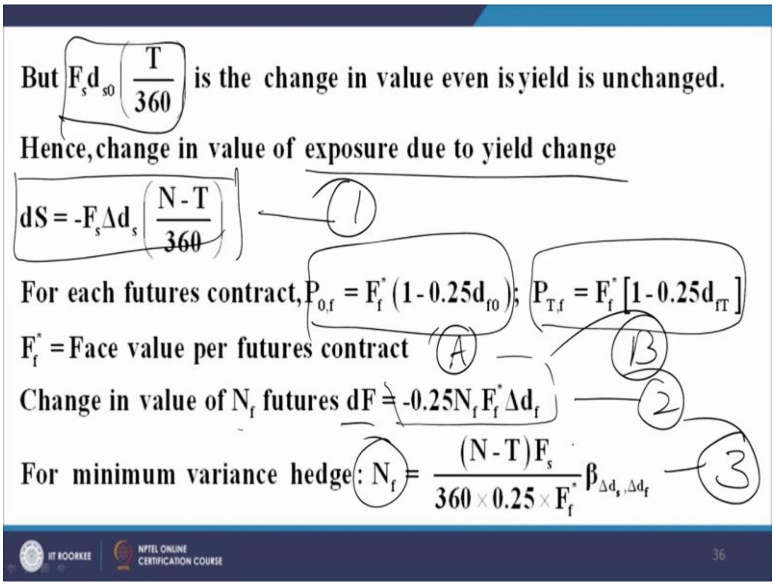

So, this factor that I highlighted in the last slide is the change in value even if the yield is unchanged. Hence, the change in value of exposure due to the yield change, which is the factor, which is the quantum that is required that is relevant insofar as hedging is concerned is given by this expression, let us call it dS, this is the first component of equation number 5. You can see here, this is dS and this we dropout.

The first factor we rename as dS and the second factor will dropout. Now, for each futures contract what happens P 0 comma f that is the price at the futures at t equal to 0 is given by this expression, we are all familiar with this. And the price at t equal to capital T is given by the corresponding expression which I have also enclosed within the box. So, F star f is the face value per futures contract.

Therefore, change in value of Nf futures dF is equal to this expression that is obtained by subtracting A from B. So, dF is equal to minus 0.25 into Nf into F star f into delta df, where delta df is equal to dS T minus dS 0. Now, we have two equations, this is equation number 1, this is equation number 2. We have got an equation for the change net change or the relevant change insofar as our portfolio value is concerned, we have got equation number 2, which gives us the change in value of the future.

So, for minimum hedge ratio, we can follow the process that we followed for the minimum variance ratio that we have encountered a number of times earlier. And what we end up with is, on simplifying the process what we end up with is the number of futures contracts is given by equation number 3. Where what is beta? Beta is the regression coefficient between delta dS as a dependent variable and delta df as the independent variable.

N minus T is the number of days remaining after the lifting of the hedge. Number of days tenure to maturity of the T-bills after lifting of the hedge, Fs is the face value of your investment and F star is the face value per futures contract, face value of T-bills covered by one futures contract. So, this gives you the expression for the number of futures contract that you need to take a position in in order to optimize your hedge.

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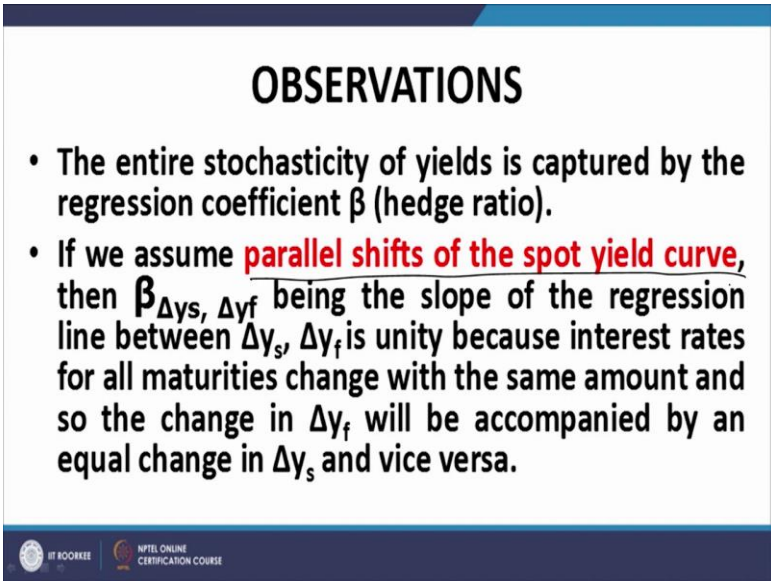

Observations, the entire stochasticity of yields is captured by the regression coefficient beta. There is no other, please note, there is no other term which is random in this expression except for beta. Beta is the term that captures the randomness or captures the fluctuations in this spot and futures, which are interacting for the hedging objective.

If we assume parallel shifts of the spot yield curve and beta delta ys comma delta yf being the slope of the regression between delta ys comma delta yf is unity, because interest rates is for all maturities will change with the same amount. And so, the change in delta yf will be accompanied by an equal change in delta ys and vice versa.

So, this is the special situation, where beta turns out to be 1 in this special situation, where we assume that the spot yield curve shifts parallel to itself.

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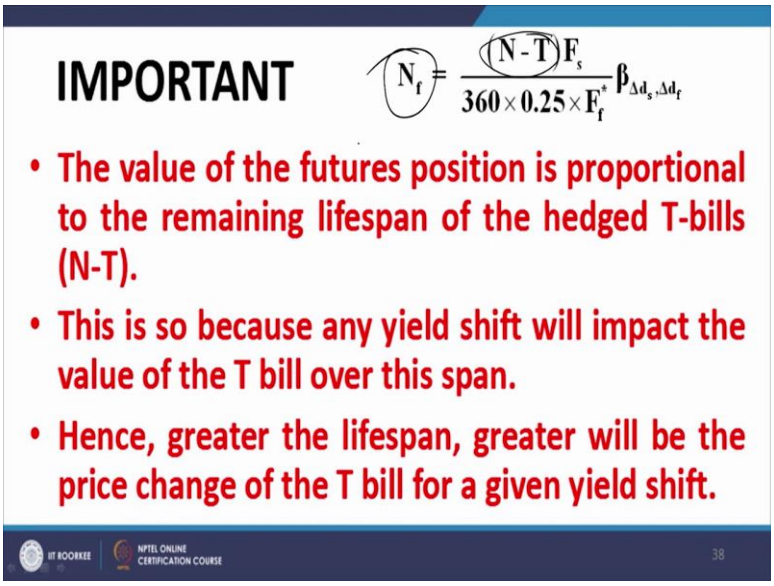

The value of the futures position is proportional to the remaining lifespan. You can see here this Nf is proportional to N minus T. So, the value of the futures position is proportional to the remaining lifespan of the hedge T-bills, why is that, because, any yield shift will impact the value of the T-bills over the span. If there is a yield shift, it will affect the T-bills prices with reference to the remaining life of the T-bills, not what has already happened.

So, if there is a change in the interest rates, change in the yields, it will affect the value of the T-bills with reference to the remaining life of the T-bills not with reference to what has already elapsed. hence, greater this lifespan, greater the remaining life of the T-bills, greater will be the price change of the T-bills for a given yield shift.

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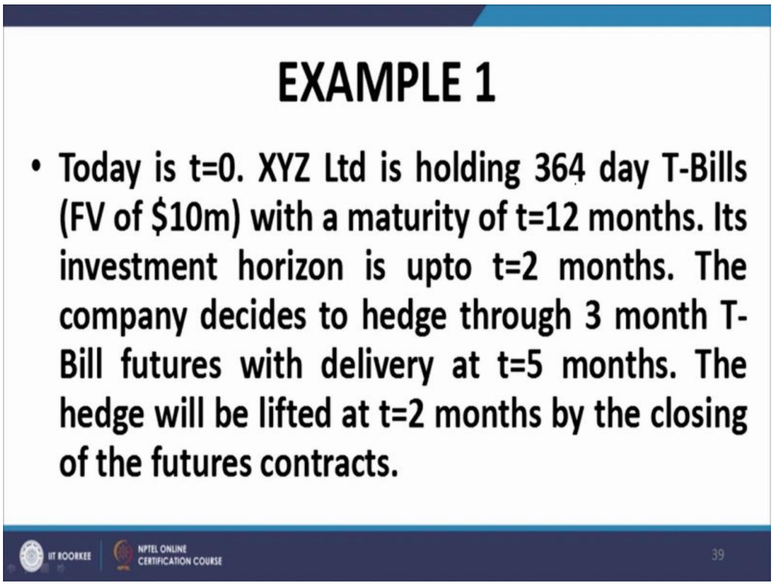

Let us do an example. Today is t equal to 0. XYZ Limited is holding 364-day let us call it 1-year T-bills of face value of US dollars 10 million with a maturity of 12 months, that is 1-year. Its investment horizon is up to t equal to 12-month. So, although N is equal to 12 months, t is equal to only 2 months.

The company decides to hedge through 3-month T-bill futures with delivery at t equal to 5 months. So, at t equal to 5 months, the futures entails delivery, but obviously, they will be liquidated at equal to 2 months when the hedge will be lifted. The hedge will be lifted at equal to 2 months by the closing of the futures contracts or futures position.

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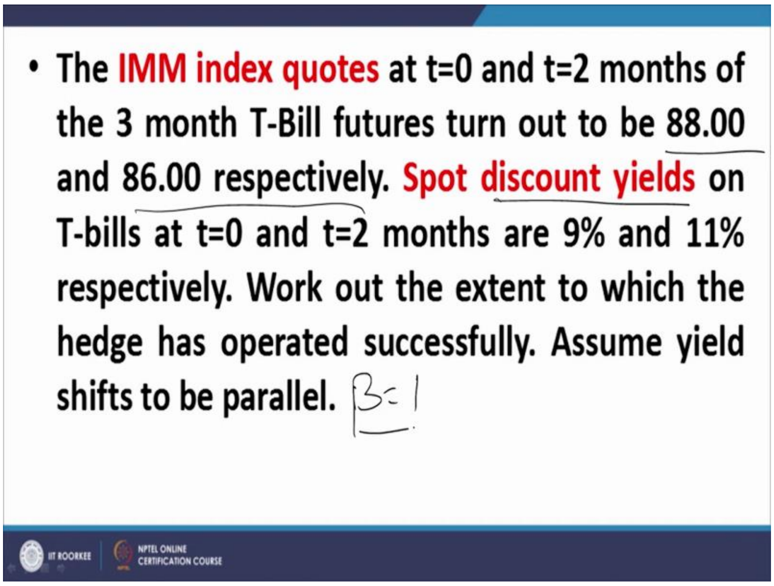

The IMM index course at t equal to 0 and t equal to 2 months of the three-month T-bill futures turned out to be 88 and 86 respectively. Spot discount yields on T-bills at t equal to 0 and t equal to 2 months. Please note. These are discount yields. So, spot discount yields on T-bills at t equal to 0 and t equal to 2 months are 9 percent and 11 percent respectively. Work out the extent to which the hedge has operated successfully. Assume yield shift to be parallel that means beta is equal to.

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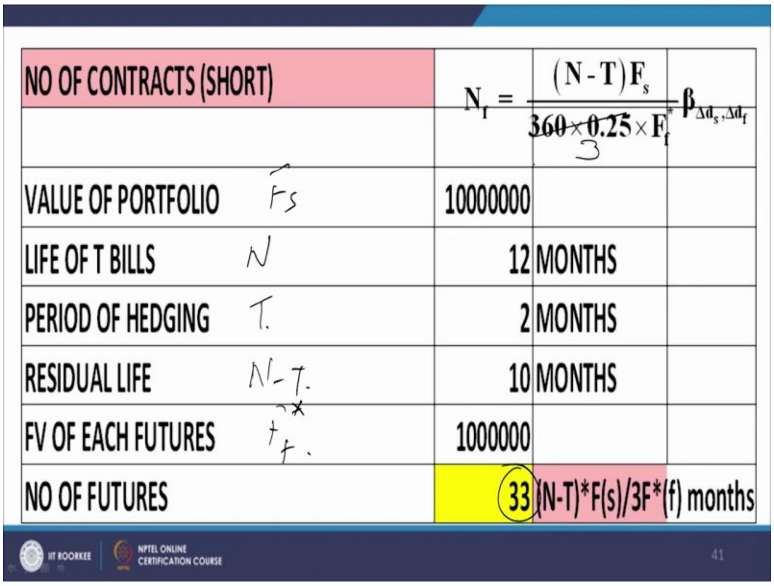

NO OF CONTRACTS (SHORT)	$N_f =$	$\frac{(N-T)F_s}{\cancel{360\times 0.25}\times F_f^*}$ 3	$\beta_{\Delta d_s,\Delta d_f}$
VALUE OF PORTFOLIO F_s	10000000		
LIFE OF T BILLS N	12	MONTHS	
PERIOD OF HEDGING T.	2	MONTHS	
RESIDUAL LIFE $N-T$.	10	MONTHS	
FV OF EACH FUTURES F_f^*.	1000000		
NO OF FUTURES	33	(N-T)*F(s)/3F*(f) months	

Now, this is the formula for calculating the number of contracts that you have in the right-hand top panel, we have got all the values here value of the portfolio, we have got this as Fs life of the T-bill, this is N period of aging, this is capital T residual live, this is N minus T. So, and the face value of the future this is F star f. So, using all these expressions, what we find is that the number of futures is equal to 33.

Please note everything is given in terms of months. So, the formula is modified instead of the 316 into 0.25 we write it as 90, 90 corresponds to three months. So, if you transfer all the quantities from years to months, you will have a factor of 3 years. That is precisely what is used and we end up with 33 contracts for aging.

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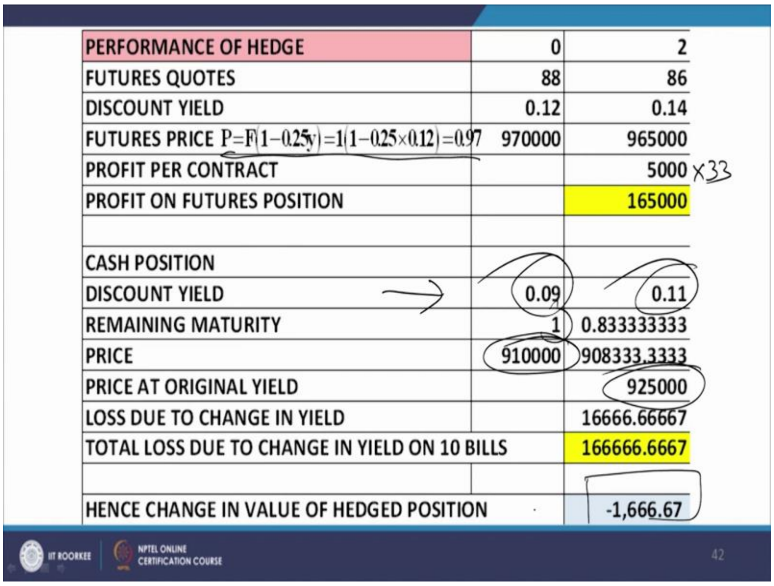

PERFORMANCE OF HEDGE	0	2
FUTURES QUOTES	88	86
DISCOUNT YIELD	0.12	0.14
FUTURES PRICE $P=F(1-0.25y)=1(1-0.25\times0.12)=0.97$	970000	965000
PROFIT PER CONTRACT		5000 x33
PROFIT ON FUTURES POSITION		165000
CASH POSITION		
DISCOUNT YIELD	0.09	0.11
REMAINING MATURITY	1	0.833333333
PRICE	910000	908333.3333
PRICE AT ORIGINAL YIELD		925000
LOSS DUE TO CHANGE IN YIELD		16666.66667
TOTAL LOSS DUE TO CHANGE IN YIELD ON 10 BILLS		166666.6667
HENCE CHANGE IN VALUE OF HEDGED POSITION		-1,666.67

Now, let us look at the performance of the hedge. The futures positions is quite simple. The future quote at t equal to 0 was 88. The futures quote at t equal to 2 months was 86. The discount yield is therefore 12 percent at t equal to 0 or 0.12. And the discount yield at t equal to 2 months was 14 percent or 0.14.

The futures price worked out as per the standard formula that I have shown here, I have underlined now, works out to 970000 per 1 million value face value of futures at t equal to 0 and 965000 per million of face value of futures at t equal to 2 months. Therefore, the profit per contract because, please note you will have a short position, profit per contract is 5000. And because there are 33 futures contracts, so 5000 into 33, that is 165,000.

Let us now look at the cash position. The discount yield at t equal to 0 is 9 percent. And the discount yield at t equal to 2 months is 11 percent. The time remaining to maturity of the T-bills at t equal to 0 is 1 year. And the time remaining to maturity of the T-bills after the investment horizon of 2 months, is equal to 10 months, that is equal to 0.833 of the year.

Using these quantities and the standard expression for the discount yield, what we have is, the price of the T-bill at t equal to 0 was 910000 per 1 million of T-bill, the price of the T-bill at t equal to 2 months per 1 million of face value is equal to 908333. Now, if we work out the price of the T-bill, now, this is the important factor. If we work out the price of the T-bill for 10 months, for a maturity of 10 months, with the original yield of 9 percent with the original yield of 9, I get 925000.

Therefore, due to the increase in yield, the damage that determined that my portfolio has suffered is equal to 925000 minus 908333 per 1 million of T-bills that is equal to 1666 per 1 million of T-bills. Our investment value is 10 million face value of T-bills. And therefore, the loss that we suffered on account of this increase in discount yield is equal to 166666.6667.

So, the future hedge has compensated us to the extent of 165000, the actual loss was 166666.67. Therefore, the net loss on the hedged position is equal to only 1666.67 US dollars on an investment value of US dollars 10 million.

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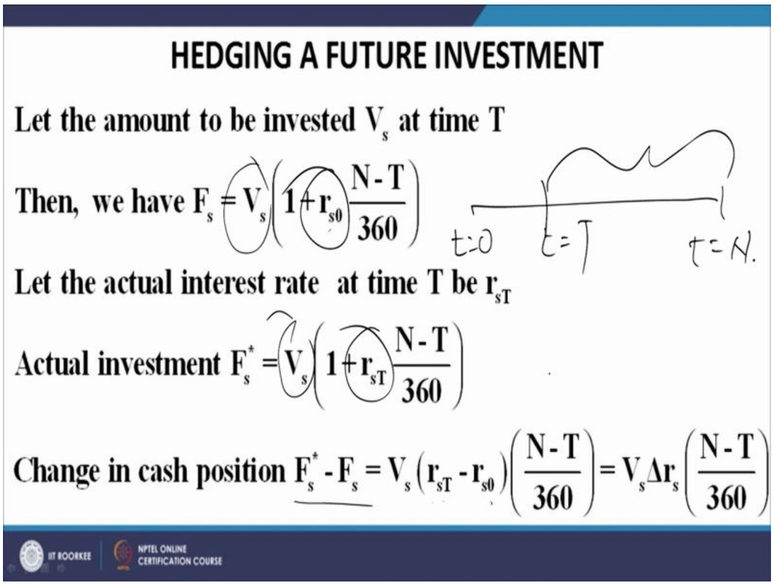

Now, we will talk about hedging of a futures investment. In the previous case what we did, we talked about hedging of a existing portfolio of T-bills, we have a portfolio of T-bills and we take a position in order to hedge yourself. Here, we are planning to make an investment and it is the planned investment that we want to hedge against fluctuations in interest rates. The process is pretty much similar.

However, the difference is that here we have used interest rates instead of discount yields. So, Fs is equal to Vs into 1 plus rs0. What is rs0? rs0 is the interest rate prevailing at t equal to 0. And rsT is the interest rate that is prevailing at t equal to capital T. Therefore, corresponding to rs0 with an investment value of Vs, I could make a face value investment as equal to Fs.

Now, because the interest rates are changed from rs0 to rs capital T, the face value of my investment or the money that I have used to that is Vs can be used to have an investment of face value F star s. So, the damage or the detriment that has occurred due to a change in interest rates is equal to df which I can write as F star s minus Fs that is equal to Vs into rsT minus rs0 and that can into the time factor that is N minus T upon 360.

Because your investment is going to be made at, today you are at t equal to 0 your investment is going to be made at t equal to capital T and the life of the investment is t equal to N. So, it is this factor which is relevant insofar as calculating the face value is concerned.

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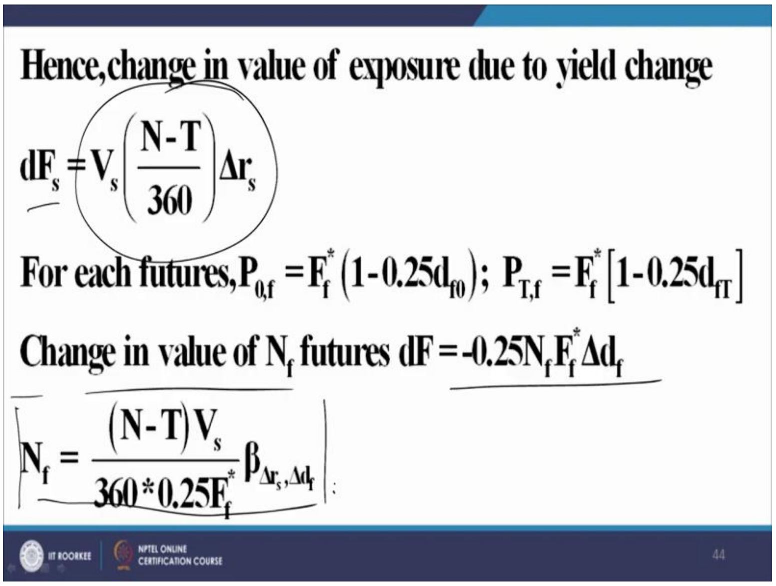

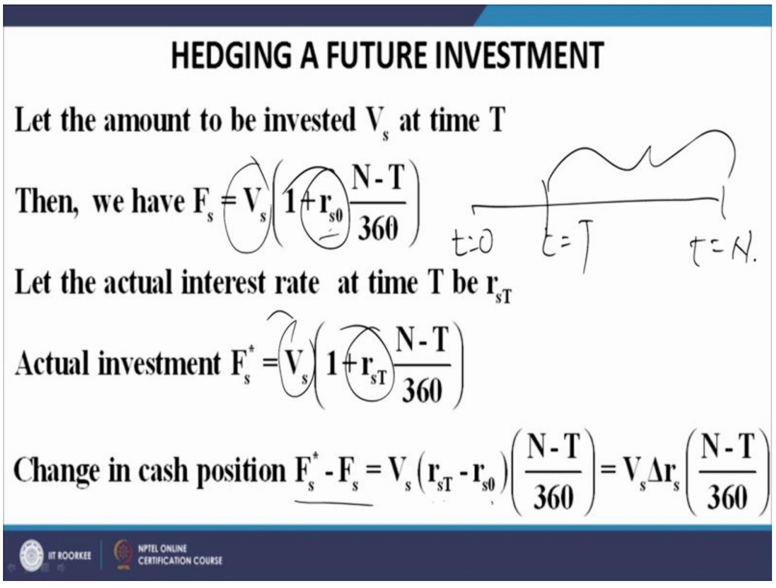

As well as the future, therefore, we have dFs is equal to this expression with which I have encircled. As far as the futures is concerned it is pretty much the same situation same thing, we continues to use, futures discount yeild and we end up with the expression for dFs the same as we had in the previous example, in the previous derivation.

And when we try to arrive at the optimal number of contracts, we get the expression that we have in the box here. So, this is the expression that I have, when we are trying to hedge prospective investment intervals, we are expecting to receive surplus funds and we want to make any investment in T-bills. Please note in this derivation the one important part that we have used is that we have used interest rates instead of discount factors.

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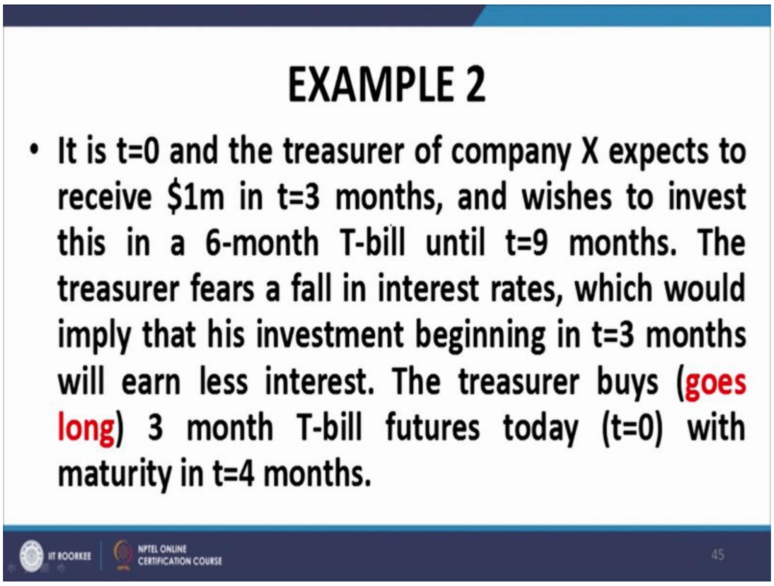

On the basis of the derivation that we have done there, we have an example here, it is t equal to 0 and the treasurer of company X expects to receive dollars 1 million in t equal to 3 months, and wishes to invest this in a 6-month T-bill until t equal to 9 months. The treasurer fears of falling interest rates, which would imply that is investment beginning in t equal to 3 months will earn less interest.

Because if there is a falling interest rates, naturally, the value or the face value of that you can get by using the same amount of money will be lesser because prices will increase. The treasury buys because long 3-month T-bill futures today, t equal to 0, then maturity in t equal to 4 month.

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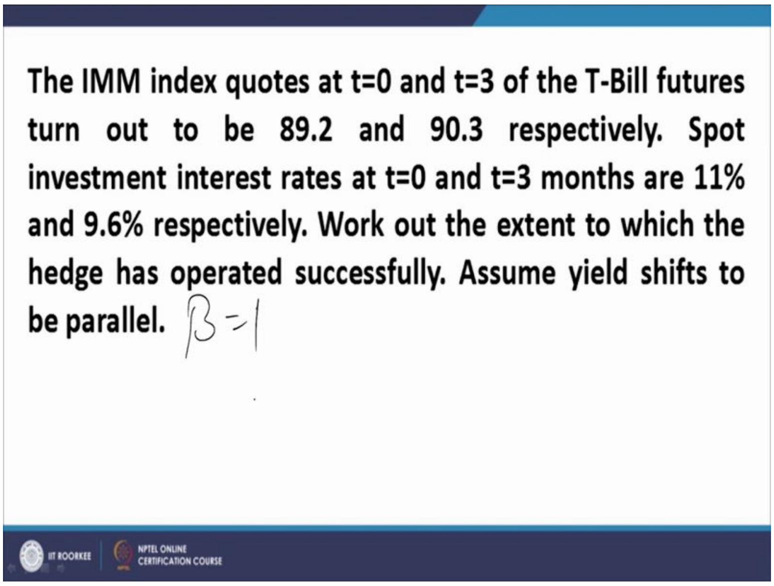

The IMA index quotes at t equal to 0 and t equal to 3 of the T-bill futures turned out to be 89.2 and 90.3. Thus, investment interest rates at t, also spot investment rates at t equal to 0 and t equal to 3 months are 11 percent and 9.6 percent. Work out the extent to which the hedge has operated successfully. Assume yields shifts to be parallel. So, we can assume beta is equal to 1.

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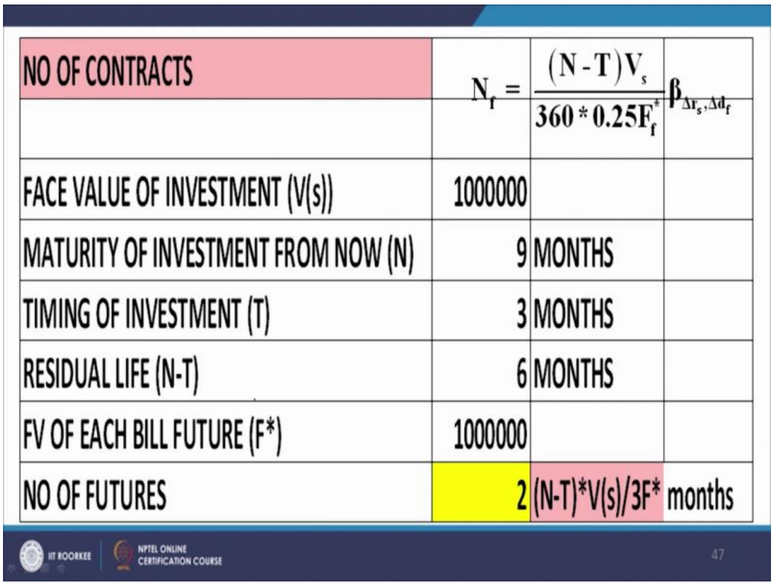

NO OF CONTRACTS	$N_f =$	$\frac{(N-T)V_s}{360*0.25F_f^*}$	$\beta_{\Delta r_s,\Delta d_f}$
FACE VALUE OF INVESTMENT (V(s))	1000000		
MATURITY OF INVESTMENT FROM NOW (N)	9	MONTHS	
TIMING OF INVESTMENT (T)	3	MONTHS	
RESIDUAL LIFE (N-T)	6	MONTHS	
FV OF EACH BILL FUTURE (F*)	1000000		
NO OF FUTURES	2	(N-T)*V(s)/3F*	months

This is the solution. It is quite straightforward, absolutely on the same lines, as we did in the previous example. So, I will not devote time to this.

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PERFORMANCE OF HEDGE	0	3
FUTURES IMM QUOTES	89.2	90.3
FUTURES DISCOUNT YIELD	0.108	0.097
FUTURES PRICE	973000	975750
PROFIT PER CONTRACT (LONG)		2750
PROFIT ON FUTURES POSITION		5500
CASH POSITION		
INTEREST RATE	0.11	0.096
MATURITY	0.5	0.5
INTEREST INCOME	55000	48000
LOSS DUE TO CHANGE IN INTT RATE		-7000
PV OF THE LOSS		-6686.406299
NET LOSS		-1186.406299

With this we come to the end of this course. I hope it has been an enjoyable learning experience for all the learners. Now, before I sign off, an important thing, there are certain topics which have either been left out or have been dealt with marginally. I would request the learners I would sincerely request the learners to refer to the video lectures on financial derivatives and on security analysis and portfolio management.

The links to which are already there in the first presentation that would help the learners significantly in understanding in having a holistic view of the entire subject for preparing for various professional examinations. Thank you and all the best.