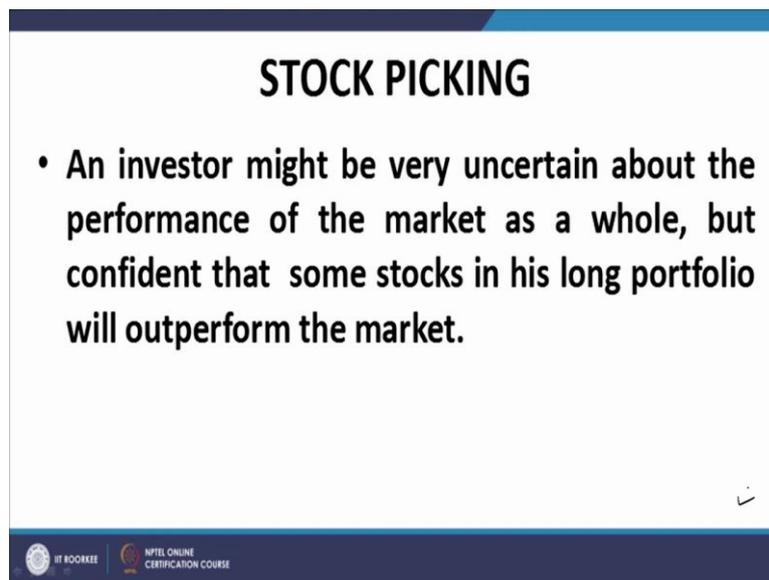


Quantitative Investment Management
Professor J. P. Singh
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Lecture 58
Stock Index Futures – 2

Welcome back. So, before the break, I was talking about the beta management of a portfolio using index futures.

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STOCK PICKING

- **An investor might be very uncertain about the performance of the market as a whole, but confident that some stocks in his long portfolio will outperform the market.**

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One of the situations or scenarios beta data management comes into very handy with index futures is when you are talking about or when you are looking at picking up of stocks, choosing stocks, in the context of having special information or special knowledge about certain stocks and their expected performance. This phenomenon is called stock picking.

An investor might be very uncertain about the performance of the market as a whole, but confident that some stocks in his long portfolio will outperform the market. So, while he is not sure about the behavior of the market is pretty mature, or relatively sure, at least, insofar as the performance of some of these talks that there in his portfolio are concerned.

(Refer Slide Time: 01:21)

STRATEGY

- A **short** position using index futures can be used to reduce the beta of the portfolio and thereby immunize the investor from the risk arising from market moves.
- It leaves the hedger exposed only to the intrinsic performance of the portfolio.

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So, what is the strategy that he adopts? He takes up a short position using index futures. By taking up a short position in index futures, what does it do, it reduces the beta of a portfolio, and therefore, by equivalently, he insulates his portfolio to changes in the market performance. If the market rises or falls or there is variation or fluctuations in the market, his portfolio will remain insulated, because his portfolio has a relatively very small beta and which is achieved by taking a short position in stock index futures.

So, a short position using index futures can be used to reduce the beta of the portfolio and thereby immunize the investor from the risk arising from market moves. It leaves the hedger, this is the important part, it leaves the hedger exposed only to the intrinsic performance of the portfolio. Because the beta is small, and if the beta is small, the impact of market fluctuations on the portfolio is minimal. And as a result of which it is the intrinsic performance of the stocks of his portfolio that would carry the day.

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EXAMPLE

- Consider an investor who in April holds 20,000 IBM shares, each worth \$100. The investor feels that the market will be very volatile over the next three months but that IBM has a good chance of outperforming the market. The investor decides to use the August futures contract on the S&P 500 to hedge the market's return during the three month period. The beta of IBM is estimated at 1.1.



Let us do an example on this. Consider an investor who in April holds 20,000 IBM shares each worth dollars 100. The investor feels that the market will be very volatile over the next 3 months but that IBM has a good chance of outperforming the market. The investor decides to use the August index futures on the S&P 500 to hedged the market's return during the three-month period. The beta of IBM is estimated at 1.10.

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- Suppose that the current futures price for the August contract on the S&P 500 is 900. Each contract is for delivery of \$250 times the index.
- The investor closes out the position in July. Suppose IBM falls to \$90 and the futures price of the S&P 500 falls to 750.
- Calculate the net profit/loss on the hedged portfolio.



Suppose the current futures price for the August contract on the S&P 500 is 900. Each contract is for delivery of 250 times the index, this is the contract multiple, dollars 250 times the index, this is the lot size or the contract multiple. The investor closes out the position in

July. Suppose IBM falls to dollar 90. And the futures price of the S&P 500 falls to 750. Calculate the net profit or the loss on the hedged portfolio and analyze it of course.

(Refer Slide Time: 03:54)

SOLUTION 1

- No of futures contracts shorted: $\beta V_s / z V_f$ **Why?**
- $= 1.10 * 20,000 * 100 / (250 * 900) = 9.78 = 10$ (rounded off)
- Net Loss on stock $= 20,000 * (90 - 100) = 200,000$
- Profit on futures $= 10 * 250 * (900 - 750) = 375,000$ *SHORT*
- Net profit on hedged portfolio $= 175,000$ due to stock picking.
- % loss on market $150 / 900 = 16.67\%$
- Beta of stock $= 1.1$
- Hence, loss due to systematic risk on his IBM portfolio $= 1.1 * 16.67\% = 18.33\% = 366,667$.




This is the solution. First of all, the number of futures contracts sorted is equal to beta into V_s divided by z into V_f . I will be explaining the symbols. In fact, you can keep this part pending which we will come back to it in a few minutes. Let us assume that we take a short position in 10 contracts on the S&P index. How this figure is arrived at?

I will come back to it. But for the moment take it that we take a short position in 10 contracts to manage our exposure. Net loss on the stock, you got 20,000 units of the stock, the price has gone down from 100 to 90, and therefore, the net loss on the stock is 200,000. Profit on the futures, please note we have got a short position, this is a short position, and because it is the short position and the prices have declined, you make a profit on your futures position.

10 is the number of contracts, 250 is the contract multiple and the futures have gone down from 900 to 750, so there is a profit of 375,000 dollars on the futures position. I reiterate, this is a profit because you have taken a short position the future. Although the prices have gone down, the futures have gone down from 900 to 750, 900 in April to 750 in July.

Notwithstanding that fact, we make a profit because you have taken a short position. Net profit on the hedged portfolio. Therefore, the net profit on the hedged portfolio is how much, profit on the future is 375, the loss on the stock is 200,000, so the net profit is 175,000. This is due to stock picking. Let us analyze it further.

The percentage loss on the market is 150 upon 900. How do we get this figure of 150 upon 900? The change in prices from 900 to 750. So, the percentage change is equal to 150 divided by the initial value that is 900 that is equal to 16.67 percent. The beta of the stock is 1.1. Therefore, the loss due to systematic risk on the IBM portfolio is equal to 1.1 into 16.67 that is 18.33 percent that is 366,667 dollars. So, this is the loss due to systematic risk.

We know that systematic risk is given in terms of beta and beta is 1.1 for the stock. The change in value of the market portfolio is 16.67 minus of course, and therefore, the change in value of the IBM portfolio stock should have been minus 18.33 percent and because their investment value is how much, investment value is 20,000 into 100, your loss that is 18.33 percent of 20,000 into 100 that is equal to 366,667.

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- Net loss on shares: 200,000
- Loss due to systematic risk: 366,667
- Therefore, profit due to unsys risk: 166,667
- Profit due to futures: 375,000
- Profit due to futures annuls loss due to sys risk.
Hence, profit due to unsystematic risk remains.

So, net loss on the shears is 2000 loss due to systematic risk was 366,667. What is happening? You see the loss on the share we have worked it out is 200,000. How do we get this 200,000? That is the loss is 10 dollars per share and you are holding 20,000 shares, so that is 200,000. Loss due to systematic risk should have been 366,667. Therefore, the profit due to unsystematic risk is 166,667.

And profit due to futures is 375,000. That means what, that means the profit due to the futures annuls last due to unsystematic risk. You can see this figure that approximately equal, this figure and this figure they are approximately equal. Why are they approximately equal? Because we have rounded off the number of contracts. So, had we not rounded off the number of contracts? These figures would have been converging even more.

And the point is that the profit due to the futures contract annuls the loss due to the systematic risk, and therefore, this loss that goes out of 366,667 and we end up with a profit of 166,667 or 175,000 because of the rounding of the contracts. The profit due to the future annuls loss due to systematic risk. Hence, profit due to unsystematic risk remains. This is the figure that I enclosed in the box.

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- **The superior return of the IBM stock could be attributed to some special knowledge, expertise available to the investor or access to some inside information at investment.**
 - **Since the market would not be aware of such information etc at $t=0$, it would not have been built into the price at the time of investment and hence, the under pricing.**
 - **Further, market will view this superior return as random return since the market did not have any information.**
- 

The superior return of the IBM stock could be attributed to some special knowledge, expertise available with the investor or access to some inside information at investment time. Since the market would not be aware, this is very important, this is the market would not be aware of such information etcetera at t equal to 0 at the point of investment. It would not have built this special information into the price at the time of investment and hence, the underpricing. This is important.

This is special, this is intrinsic to the investor. He has that special information. He believes that the IBM would outperform the market performing much better than the market. And that is why he has separated himself, he have insulated himself from the market and he has reaped the benefits there of. So, but what happened was that he had the special information market did not have that special information.

And because market did not have that special information market could not incorporate the worth of that special information into the pricing of the IBM shares. And as a result of it what happened, the IBM shares turned out to be underpriced and this particular investor could may take the advantage of that underpricing. Further, market will view the superior return this as

random return since the market did not have any information about this particular characteristic.

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BETA OF HEDGED PORTFOLIO

$$\Delta \tilde{I} = Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}$$

$$\tilde{R}_I = \frac{\Delta \tilde{I}}{I_0} = \frac{Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}}{I_0} = \frac{Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}}{I_{S,0}} = \frac{Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}}{Q_S \times S_0}$$

(since initial investment in futures is zero i.e. $I_{F,0} = 0$)

$$= \frac{\Delta \tilde{S}}{S_0} - \frac{Q_F F_0}{Q_S S_0 F_0} \Delta \tilde{F} = \tilde{R}_S - \frac{Q_F F_0}{I_{S,0}} \tilde{R}_F = \tilde{R}_S - \frac{V_{F,0}}{I_{S,0}} \tilde{R}_F \quad (1)$$

unhedged portfolio

So, let us look at beta of hedged portfolios. This is very important, we have been talking about beta management, let us quantify the strategy, we have already talked about the equation number 1 that we have here, this is the change in the value of the investment, this is given by the quantity of your stock at investment multiplied by the change in price. Now, we have a minus sign here.

Why do we have a minus sign? Because the hedged that is the deposition in the future has to be opposite to that of the investment. And that is why we are carrying a minus sign. You could as well work out the same thing with a plus sign, but then you in the net result you will end up with a minus sign, which will show that your hedged should be opposite to that of the primary investment.

So, delta I is equal to QS into delta S, where delta S is the change in price between t equal to 0 and t equal to capital T, which is the maturity of the hedged. The minus or plus as you may choose, we will go with minus because it is here on this slide. So, minus QF into delta F, QF is this position that you have taken in the futures market multiplied by delta F. What is delta F? Delta F is the change in the futures prices between t equal to 0 and the point at which you lift the hedged by closing your position in the futures market.

So, the return on investment, return on the hedged investment, please note that, the RI is the return on the hedged investment is equal to ΔI divided by I_0 where I_0 is the initial investment that is equal to the expression. When we replace ΔI by $Q_S \Delta S$ minus $Q_F \Delta F$, what we get is the second equation on this line.

And now I_0 because the futures do not entail any initial investment, the initial investment that we have made in this portfolio I that we are talking about now is equal to the investment in the portfolio itself and not in the hedged. The hedged comprises the futures and taking positions in futures do not entail any investment, of course, except margins, which we are not considering here.

So, ignoring margins, taking position in the futures for the purpose of hedging does not entail any investment, and as a result of it the investment in that portfolio is equal to the investment in the primary portfolio or the cash portfolio. So, I_0 is equal to I_{S0} , where I_{S0} is the investment on the spot or the investment itself and not the hedged.

We can write this as $Q_S I_{S0}$, where S_0 is the price of per unit of the underlying asset at t equal to 0 and Q_S is the value or is the number of units of the stock of constituting the primary investment, S_0 is the price per stock at t equal to 0 and Q_S is the number of units of the stock. So, the total investment at t equal to 0 in the primary position in the cash position in the portfolio is equal to $Q_S I_{S0}$.

So, this can be simplified as ΔS upon S_0 which is equal to what, which is equal to R_S where R_S is the return on the portfolio, unhedged portfolio, R_S is the return on the unhedged portfolio minus Q_F upon $Q_S I_{S0}$ upon S_0 into ΔF upon F_0 . This is what we have done here is simply some algebraic manipulation, multiplying by F_0 and dividing by F_0 . And we can write this expression in the form of the equation that I now put within the square box, enclosed within the square box.

And let us call it equation number 1 here. R_S minus V_{F0} , where V_{F0} is the value of the position, value of the total futures position, value of the position that we have taken in the futures market divided by I_{S0} , I_{S0} is the value of the stock investment, both of them are at t equal to 0. And what is R_F ? R_F is the return on the futures contract. Now, we move to the next slide.

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Now, $E(\tilde{R}_S) = R_f + \beta_S (E(\tilde{R}_m) - R_f)$. — (2)

Similarly, $E(\tilde{R}_F) = \beta_F (E(\tilde{R}_m) - R_f)$ — (3)

The riskfree rate does not appear because futures do not involve any upfront investment so there is no opportunity cost for funds tied up in futures.

Also $E(\tilde{R}_I) = R_f + \beta_H (E(\tilde{R}_m) - R_f)$ — (4)

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$\Delta \tilde{I} = Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}$ (1) **BETA OF HEDGED PORTFOLIO**

$\tilde{R}_I = \frac{\Delta \tilde{I}}{I_0} = \frac{Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}}{I_0} = \frac{Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}}{I_{S,0}} = \frac{Q_S \Delta \tilde{S} - Q_F \Delta \tilde{F}}{Q_S \times S_0}$

(since initial investment in futures is zero i.e. $I_{F,0} = 0$)

$= \frac{\Delta \tilde{S}}{S_0} - \frac{Q_F F_0 \Delta \tilde{F}}{Q_S S_0 F_0} = \tilde{R}_S - \frac{Q_F F_0}{I_{S,0}} \tilde{R}_F = \tilde{R}_S - \frac{V_{F,0}}{I_{S,0}} \tilde{R}_F$ (2)

unhedged portfolio

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Now, using the CAPM model, we can write the expected return on the spot investment on the unhedged investment in the form of equation number 2, R_f plus beta S into E of R_m minus R_f , this is standard CAPM formula. And similarly, we can write the expected return on a futures that we have taken that we have taken a short position in the form of equation number 3. Now, please note that this factor R_f here is missing in this equation, there is no R_f here.

Why is this R_f not here? Let us understand that. The risk-free rate does not appear, because futures do not entail any upfront investment so there is no opportunity cost for funds tied up in the futures. So, that is the reason that we do not have any R_f here, there is no R_f here. Because futures do not entail any initial investment, as I mentioned, we are not considering margins. So, futures do not entail any initial investment.

And because they do not entail any investment, no funds are blocked, when we make an investment in future, there is no opportunity cost for block funds, and therefore R_f does not appear here. And as far as the total investment is concerned, for the total investment, that is the hedged investment, we have the same expression in the form of equation number 4. Substitute all these expressions into the formula that we have that is here, R_I , R_S and R_F and we make the substitutions of these three expressions in terms of the CAPM expressions.

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$$0 = \beta_H = \beta_S - \frac{V_{F,0}}{I_{S,0}} \beta_F \quad \text{For complete hedging}$$

$$\beta_H = 0; V_{F,0} = \beta_S \frac{I_{S,0}}{\beta_F}; N = \frac{V_{F,0}}{V_{F,0}^*} = \beta_S \frac{I_{S,0}}{\beta_F V_{F,0}^*}$$

For index futures $\beta_F = 1$ so that $N = \beta_S \frac{I_{S,0}}{V_{F,0}^*}$

What we end up with the results that we get is beta H is equal to beta S minus V_{F0} upon I_{S0} into beta F. And please note the terms here, beta H is the beta of the hedged portfolio, beta S is the beta of the unhedged investment, beta F is the beta of the futures contracts, V_{F0} is your position in the futures market at t equal to 0 that you have taken for hedging your portfolio and I_{S0} is the initial investment value of your exposure.

So, if you want complete hedging, if you want perfect hedging, you equate this to 0. And when you equate this to 0, what you get is V_{F0} that is the amount of futures the value of futures that will need to take a position in the futures market is given by beta S into I_{S0} upon beta F. And the number of contracts you have to take is equal to V_{F0} divided by V^* $F0$, where V^* $F0$ is the price per contract, V_{F0} is the total value of the position that you need to take, V^* $F0$ is value per contract.

So, you divide the two you get the number of contracts, then that is equal to the expression that I have in this square box. For index futures beta F is approximated by one and therefore

we have N is equal to βS into the investment value of the unhedged investment at t equal to 0 divided by the futures value per futures at t equal to 0.

(Refer Slide Time: 18:45)

EXAMPLE 2

- A portfolio manager has a portfolio of long positions in stocks valued at USD 1.5m with a beta of 1.50 at $t=0$ when the S & P 500 index is 1500. He desires to hedge this portfolio for a period of 3 months using S& P 500 index futures with a maturity of four months. The index has a continuously compounded yield of 4% and the riskfree rate is 10.00% cont. compd.. The contract multiple is USD 250. Calculate the number of contracts required. If the index actually falls by 10% at $t=3$ months from its original value assess the performance of the hedged portfolio. **Use no arbitrage pricing wherever required.**

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Let us do an example. Portfolio Manager has a portfolio of long positions in stocks valued at US dollars 1.5 million, with a beta of 1.50 at t equal to 0 when the S&P 500 index is 1500. He desires to hedged this portfolio for a period of 3 months using S&P 500 index futures with a maturity of 4 months. Here again, we encounter that issue. The maturity of the futures contract is not coinciding with the hedged period.

The hedged period is 3 months, the maturity of the futures is 4 months, there is no necessity there is no rule in the rulebook that the hedged period must coincide with the, futures period must coincide with the hedged period that is not at all necessary. Maybe you can say that it is desirable, but that may not always be the case that it is available. The index has a continuously compounded yield of 4 percent and the risk-free rate is 10 percent per annum continuously compounded. The contract multiple is US dollars 250.

Calculate the number of contracts required if the index actually falls by 10 percent at t equal to 3 months. That is when you are lifting the hedged from its original value, original value was 1500, this is the original value of the index, it falls by 10 percent. So, its value at equal to 3 months is 1350. Assess the performance of the hedged portfolio. Use arbitrage free pricing wherever required. In the absence of data we use arbitrage free pricing.

(Refer Slide Time: 20:19)

CALCULATION OF NO OF CONTRACTS		
SPOT VALUE OF INDEX	1500	
RISK FREE RATE	10%	r
YIELD ON INDEX	4%	q
MATURITY	4	MONTHS
NO ARBITRAGE FUTURE VALUE OF INDEX	1530.302	$EXP[(r-q)T]$
CONTRACT MULTIPLE	250	z
NO ARBITRAGE PRICE OF FUTURES CONTRACT	382575.5	$V(f,0)$
EXPOSURE	1500000	$V(s)$
BETA OF EXPOSURE	1.5	β
NO OF CONTRACTS	5.881192	$\beta V(s)/V(f,0)$
ROUNDED OFF	6	

This is the solution. Let us go through step by step. Spot value of the index is 1500, we know that that is given. Risk-free rate 10 percent that is also given. Yield on index that is q, q is equal to 4 percent per annum. Maturity of the futures is 4 months. So, no arbitrary future value of the index, that is no arbitrage futures price is equal to 1530.320.

How we have got it is shown here, it is given by F_0 is equal to $S_0 \exp[(r-q)T]$, S_0 is equal to 1500, r is equal to 10 percent and that is 0.10, q is equal to 0.04 and t is equal to 4 months, that is equal to 4 upon 12th of any year. So, when you substitute these values, you get F_0 is equal to 1530.302. The contract multiple is 250, that is given to us. So, the arbitrage free price of futures at entry is equal to 382525 per futures.

And (21:35) the number of futures exposure is equal to 1.5 million, the beta of the exposure is equal to 1.5. All this is given data. Exposure is 1.5 million is given, beta of 1.5 million is given, beta of 1.5 is given, and therefore, the number of futures contract with 5.88, which is rounded off to 6. How do we calculate the number of futures contracts?

That is given by this expression beta of the spot portfolio into V_s , V_s is the value of the portfolio, beta is equal to 1.5, V_s is equal to 1.5 million divided by $V(f,0)$ that is the value of per futures contract at t equal to 0, we have worked that out, it is 382575.50. When you substitute all these values, we get n is equal to 5.88, which is rounded off to 6.00. So, this is the number of contracts, this is one part of the problem which is solved here.

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EVALUATION OF HEDGE		
HEDGE PERIOD	3 MONTHS	
DECLINE IN INDEX AT END OF HEDGE PERIOD	10%	
VALUE OF INDEX AT END OF HEDGE PERIOD	1350	
RISK FREE RATE	10%	r
YIELD ON INDEX	4%	q
MATURITY	1 MONTHS	
NO ARBITRAGE FUTURE VALUE OF INDEX	1356.767	$EXP[(r-q)T]$
CONTRACT MULTIPLE	250	z
NO ARBITRAGE PRICE OF FUTURES CONTRACT	339191.7	$V(f,0)$
PROFIT ON SHORT FUTURES	260302.7	

CALCULATION OF NO OF CONTRACTS		
SPOT VALUE OF INDEX	1500	
RISK FREE RATE	10%	r
YIELD ON INDEX	4%	q
MATURITY	4 MONTHS	
NO ARBITRAGE FUTURE VALUE OF INDEX	1530.302	$EXP[(r-q)T]$
CONTRACT MULTIPLE	250	z
NO ARBITRAGE PRICE OF FUTURES CONTRACT	382575.5	$V(f,0)$
EXPOSURE	1500000	$V(s)$
BETA OF EXPOSURE	1.5	β
NO OF CONTRACTS	5.881192	$\beta V(s)/V(f,0)$
ROUNDED OFF	6	

Now, we evaluate the hedged, the hedged period is equal to 3 months, the decline in the index at the end of the hedged period is given as 10 percent so the value of the index at t equal to 3 months, that is when you are lifting the hedged is 1350, the risk-free rate is 10 percent, the yield on the index is 4 percent.

Now is the important part of, you see, you took a position in the futures when the futures has a maturity of 4 months, this was 4 months, this was futures maturity, and this is at t equal to 0 point. Now, you have progressed 3 months down the line, 1 month, 2 months, 3 months. Therefore, you are here now, you are here. So, what is the time remaining to maturity of the futures it is equal to 1 month.

So, when we do the pricing using the arbitrary free pricing strategy, we shall be working out the arbitrary free price using a remaining term to maturity of 1 month. Please note this point very carefully. The remaining term to maturity is what is relevant, when we are doing arbitrage free pricing. In this case, the remaining time to maturity is 1 month.

So, using that we work out the arbitrage free value of the index that is 1356.767. The contract multiple is 250. Therefore, the value per contract futures contract at t equal to 3 months when you are lifting the hedged is equal to 339191. Therefore, the profit on the short futures contract, remember you got 6 futures contract and you are taking a short position.

So, the profit of the short position and futures contract is equal to 6 multiplied by, let me see that figure from the previous slide, it is 382575, 382575 minus 339192, 339192 you make take it. This is your profit on the futures position. So, that is equal to 260,302. This is the total profit on your futures position.

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LOSS ON SPOT		
	$R_p = R_f + \beta_p (R_m - R_f)$	
FALL IN INDEX	$R_p = -14.75\%$	10%
YIELD ON INDEX FOR 3 MONTHS		0.01
MARKET RETURN	$\beta_p = 1.5$	-9%
RISK FREE RATE FOR 3 MONTHS		0.025
BETA		1.5
RETURN ON PORTFOLIO USING CAPM		-0.1475
LOSS IN VALUE OF PORTFOLIO		-221250
NET GAIN ON HEDGED PORTFOLIO		39052.66
RETURN ON HEDGED PORTFOLIO		0.026035

EVALUATION OF HEDGE		
HEDGE PERIOD	3 MONTHS	
DECLINE IN INDEX AT END OF HEDGE PERIOD	10%	
VALUE OF INDEX AT END OF HEDGE PERIOD	1350	
RISK FREE RATE	10%	r
YIELD ON INDEX	4%	q
MATURITY	1 MONTHS	
NO ARBITRAGE FUTURE VALUE OF INDEX	1356.767	$EXP[(r-q)T]$
CONTRACT MULTIPLE	250	z
NO ARBITRAGE PRICE OF FUTURES CONTRACT	339191.7	$V(f,0)$
PROFIT ON SHORT FUTURES	260302.7	

Now let us look at the spot position. What does it say? It says that this the fall index is equal to 10 percent that is 0.1. Now, the yield on the index is equal to 4 percent per annum, we are talking about a 3-month period, so the yield on the index must have generated a return of 1 percent. The yield on the indexes, that means, you are generating a return of 4 percent per annum continuously compounded over the year.

So, because your investment horizon was 3 months, over the investment horizon of 3 months, you would have generated a return of 1 percent. Therefore, the market return, the market return has declined by 9 percent. That is minus 9 percent. Although the index has fallen by 10 percent, this is a catch here, although the index has fallen by 10 percent, the market has generated a return of 1 percent through the yield and as a result of it the net fall in the value or in the effective return that is generated by the market is minus 9 percent.

Notwithstanding the fact that there is a decline in the index by 10 percent. So, if you work out the return on the basis of the index alone, you will find that the market return is minus 10 percent, which is not correct, because this 10 percent is very much there. But notwithstanding this fall in 10 percent in the prices, as indicated by the index, the market has generated a return which could possibly have been siphoned out from the system of 1 percent.

So, the net return on the market in the relevant period is minus 9 percent, not minus 10 percent. So, risk-free rate for 3 months is 0.025. Please note it was 10 percent for the whole year. So, for 3 months, it is 0.025. Beta is equal to 1.50. So, if I use the CAPM model, the return on my portfolio must be equal to minus 14.75 percent. How do I get it? I use the, for standard formula, R_p is equal to R_f plus beta p into R_m minus R_f .

Everything is known here, R_f is equal to 0.025 percent, beta ρ is equal to 1.5, R_m is equal to minus 0.09. So, keep putting all these values, we get R_p is equal to minus 14.75 percent. The portfolio value V_p was equal to 1.5 million. So, the net loss on the spot position is equal to 221250. This is the loss in value of the spot position, 14.75 percent of 1.5 million. What was the profit on the futures?

The profit on the futures was equal to 360302. The loss on the spot is equal to, I am sorry, the profit on the futures is 260302, the loss on the spot is equal to 221250. So, there is a net profit net return on the hedged portfolio equal to 39052, which translates to 2.60 percent over the 3-month period.

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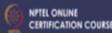
**INDEX ARBITRAGE:
NO ARBITRAGE FUTURES PRICES**

$F_0 = S_0 e^{(r-q)T}$

$F = S[1 + (r - \delta)T]$ (using simple rates)

$F = S[1 + (r - \delta)/q]^T$ (using discrete compounded rate)

$F = S e^{(r - \delta)T}$ (using continuously compounded rates)

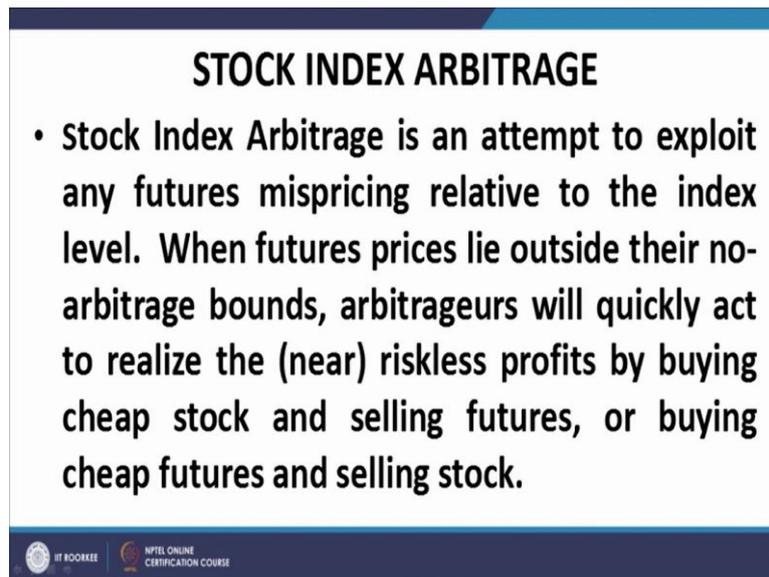
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Now, we will talk about index arbitrage. Now, as I mentioned, arbitrage free considerations mandate a certain rule or set of certain pricing insofar as the futures or forwards are concerned. What basically it says is that the future price of an asset or the forward price for that matter must be equal to, equal to what, equal to the future value of the spot price. That is F_0 must be equal to $S_0 e^{rT}$.

And of course, if there is a continuously compounded yield being generated, then it will be r minus q into T . You use this formula just know a few minutes back. Now, if this equality is violated, then there arises an opportunity for marketplace to extract risk-free money out of the system. That in fact pushes the prices again back to makes the prices converge back to this arbitrary formulation.

But nevertheless, there can be situations there can be circumstances where there is some disequilibrium and this equality is violated. But as soon as this equality is violated, arbitrage comes into the frame and their actions of extracting profits out of the system result in the prices move moving in such a direction that they tend to, that this equality tends to be satisfied and equilibrium is sustained.

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STOCK INDEX ARBITRAGE

- **Stock Index Arbitrage is an attempt to exploit any futures mispricing relative to the index level. When futures prices lie outside their no-arbitrage bounds, arbitrageurs will quickly act to realize the (near) riskless profits by buying cheap stock and selling futures, or buying cheap futures and selling stock.**

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The stock index arbitrage quickly. Stock Index arbitrage is an attempt to exploit any futures mispricing relative to the index level. When futures prices lie outside there no arbitrage bounds, arbitrageurs will quickly act to realize the near riskless profits by buying cheap stock and selling futures or buying cheap futures and selling stock.

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ARBITRAGE BOUNDS

- If $F > S + CC - CR$, then arbitrageurs' achieve arbitrage profits by purchases of stock. This will increase stock prices. Their sales of futures will depress futures prices, until equilibrium is again reached ($F \approx S + CC - CR$), and no arbitrage opportunities exist.
- **This is CASH & CARRY arbitrage.**

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So, arbitrage bounds we have already worked that out in the context of futures, so I will not spend too much time again. But however, the basic thing is, if the price in the futures markets is more and the equivalent arbitrage free price is less, arbitrage free forward price is less. Then what will you do? You will sell in the futures market or the forward markets and you will buy the underlying asset and hold it during the life of the futures contract that is called cash and carry arbitrage.

On the other hand, if the futures price is lower, and the arbitrage free pricing is more than what you do is, if the futures prices are lower, if the spot prices are higher, you borrow the asset, you sought the asset, sell it in the spot market, invest the proceeds and at the same time against the long futures position rather you will receive the asset and maturity and deliver the asset to the party from whom you have borrowed the asset in the first place. So, that is how reverse cash and carry operates. Thank you. I will continue from herein the next lecturer.