

Quantitative Investment Management
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Lecture 39
American Option - 2

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AMERICAN OPTIONS: PUT CALL PARITY				
	t=0	t=τ	t=T	
PORTFOLIO A		$S_t < K$	$S_T < K$	$S_T > K$
BUY EUROPEAN CALL	-c	≥ 0	0	$S_T - K$
INVEST	-K	$Ke^{(rt)}$	$Ke^{(rT)}$	$Ke^{(rT)}$
TOTAL	$-(c+K)$	$\geq Ke^{(r\tau)}$	$Ke^{(rT)}$	$S_T + Ke^{(rT)} - K$

Welcome back. So, towards the end of the last lecture, I was talking about the equivalence of put-call parity for American options. Let us pick it up from there. What we did was we constructed two portfolios and then invoked the principle of arbitrage free pricing. In portfolio A, we had a European call, a long position in a European call and we invested an amount equal to the strike price of the options.

Please note this difference when we worked out the put call parity for European calls or European put call parity, the investment was Ke^{-rt} . That is the present value of K. In this case, we are investing an amount equal to the exercise price K. So, there is another difference and that is that because we are talking about American options, we cannot rule out ab initio.

We cannot rule out the possibility of early exercise. And therefore, we need to consider any arbitrary point in time between 0 and capital T where the options may be exercised. So, we introduced arbitrary point in time t equal to tau where tau lies between 0 and capital T, where capital T is the maturity date of the options.

So, if S_t is less than K, then the European call will either be worth 0 in the worst-case scenario, it would be worth 0. But otherwise, being an asset, being a right, it would be worth some positive value. Worst case scenario, limiting value, you must say, would be 0 and the

investment of K will grow up to the amount of $K e^{r\tau}$. And this amount will obviously be greater than or equal to $K e^{r\tau}$, the aggregate amount of the call value, the European call value and the amount of investment.

Now, at maturity, we can have two scenarios S_t less than K and S_t greater than K . If S_t is less than K , the call expires worthless and if S_t is greater than K then pay off from the call is S_t minus K , the investment will grow to an amount equal to the future value of K that is $K e^{r\tau}$ and this value will be independent of the evolution of the stock price. In other words, it would be independent of whether S_t is less than K or S_t is greater than K . So, the total payoffs and the total values of this portfolio A are given in the last row of this slide and we shall carry them over to the next slide.

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	t=0	t=τ	t=T	
PORTFOLIO A		$S_t < K$	$S_T < K$	$S_T > K$
TOTAL	$-(c+K)$	$\geq Ke^{(r\tau)}$	$Ke^{(rT)}$	$S_T + Ke^{(rT)} - K$
PORTFOLIO B				
BUY STOCK	$-S_0$	S_t	S_T	S_T
BUY AMERICAN PUT	$-P$	$K - S_t$	$K - S_T$	0
TOTAL	$-(S_0+P)$	K	K	S_T
$\checkmark S_0 + P \leq c + K$ but $c = C$, so $S_0 - K \leq C - P$ \checkmark				

So, this is what we have from the previous slide, the total pay offs from the portfolio A as well as the cost. The cost of this portfolio is c plus K . You can see that the cash outflow at $t=0$ is c plus K which represents the cost of the portfolio. In portfolio B, we have one unit of the stock, a long position in one unit of the stock that cost us S_0 at $t=0$ and that would be what? S_t at $t=\tau$ S_T at $t=T$ and all these values would be independent of the relationship between K and S_T or S_t and we take a long position in the American put.

Because it is an American put, we represent it by capital P and because it is a long position, we are buying the option and therefore there is a minus sign attached to it. It is a cash outflow, it represents a cost. And if S_t is less than K , then there would be a payoff of K minus S_t in the event of early exercise of this American put.

And if the option is not exercised early, then it would give a payoff of $K - S_t$ and 0 otherwise. The total payoffs from the portfolio B is given in the penultimate row on the slide. Now, we can clearly see that in every scenario, in every situation at t equal to τ or at t equal to capital T , whether S_t is less than K or S_t is greater than K , the aggregate payoff from portfolio A exceeds the aggregate payoff of portfolio B, except in the limiting case when r is equal to 0.

And therefore, the cost of portfolio A must be higher than the cost of portfolio B higher than or equal to in the event of r being equal to 0. That is a limiting case that is of little practical importance. But nevertheless, for mathematical completeness we also keep it into consideration and therefore, the cost of portfolio A would be greater than or equal to the cost of portfolio B which gives us this expression.

Now, as we know because the early exercise of American calls on non-dividend paying stock which are the type of stocks we are considering here, what happens is that they are not optimal. In other words, early exercise of American calls on non-dividend paying stocks is never optimal and therefore there is no additional value attached to the Americanness of such call options, that is, options on non-dividend paying stocks. And therefore, we have $c = C$ and on substitution, we arrive at this particular formula $S - K \leq C - P$. We pick it up from here in the next slide.

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From previous slide: $S_0 - K < C - P$ (A)

$c + Ke^{(-rT)} = p + S_0$ (1)

$c - p = S_0 - Ke^{(-rT)}$ (2)

$c = C; p \leq P$ and $c, p > 0$

$C - P \leq c - p$ (3)

$C - P \leq S_0 - Ke^{(-rT)}$ (4)

$S_0 - K \leq C - P \leq S_0 - Ke^{(-rT)}$ (5)

WHAT ABOUT $S_t > K$??

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AMERICAN OPTIONS: PUT CALL PARITY

	t=0	t=τ	t=T	
PORTFOLIO A		$S_t < K$	$S_T < K$	$S_T > K$
BUY EUROPEAN CALL	-c	≥ 0	0	$S_T - K$
INVEST	-K	$Ke^{(r\tau)}$	$Ke^{(rT)}$	$Ke^{(rT)}$
TOTAL	$-(c+K)$	$\geq Ke^{(r\tau)}$	$Ke^{(rT)}$	$S_T + Ke^{(rT)} - K$

	t=0	t=τ	t=T	
PORTFOLIO A		$S_t < K$	$S_T < K$	$S_T > K$
TOTAL	$-(c+K)$	$\geq Ke^{(r\tau)}$	$Ke^{(rT)}$	$S_T + Ke^{(rT)} - K$
PORTFOLIO B				
BUY STOCK	$-S_0$	S_t	S_T	S_T
BUY AMERICAN PUT	-P	$K - S_t$	$K - S_T$	0
TOTAL	$-(S_0 + P)$	K	K	S_T
$S_0 + P \leq c + K$ but $c = C$, so $S_0 - K \leq C - P$				

Now, from put call parity for European options we have what is equation number 1 here, equation number 1 can be arranged in the form of equation 2 by simple transposition. Now, we use the fact that c is small c is equal to capital C for the reason that I just alluded to. However, in the case of American puts there can be situations, there can be circumstances which mandate the early exercise of the American put with optimal considerations and as a result of which the Americanness right in the American put options will command a price and in the limiting case we must have that small p is less than equal to capital P .

Although because of the possibility of early exercise of American puts because it can be optimal in certain circumstances, capital P would usually be greater than P . But for the limiting case we also incorporate the possibility of equality when the early exercise is mostly unlikely very unlikely.

We may have small p and capital P almost coinciding. Now, c and p are obviously greater than 0 as we know they are right. So, the value of the rights is always greater than 0. So, capital C minus P must be less than equal to small c minus p because capital C is equal to small c and capital P is greater than or equal to small p .

Therefore, capital C minus capital P would be less than equal to small c minus small p because this number, this number is greater than this number. So, this is let us say equation number 3. On using equation number 3 and equation number 2 what we end up with capital C minus capital P is less than equal to S naught minus $K e$ to the power minus rT . So, combining these two relation this is equation number 4.

So, from equation number 4 and equation number let us call it equation number A here equation number A and equation number 4 we have what is the put call parity equivalence for American options which is equation number 5. Now, what about the situation when S tau is greater than please note, let me go back to the slide, to the slide here. We have assumed here that S tau is less than K . What happens if S tau is greater than K ?

Now, if S tau is greater than K then obviously the value of the European call will increase relative to the situation when S tau is less than K . So, whatever we have discussed earlier in the context of S tau is less than K would hold even more strongly in the event that S tau is greater than K because this greater than equal to 0 will become even more towards greater than 0 more inclined towards greater than 0 as S tau increases.

As you know, the higher the value of S tau higher is the pay-off from the call option and therefore higher is the value of the call option. And therefore, if S tau is greater than K , all the consideration, all these steps that we have followed will hold with more strength and the portfolio A's pay offs at t equal to tau would be higher, even higher than the portfolio B's pay offs at that point in time. So, the rest of the modus operand will hold pretty much in the same manner.

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PUT CALL PARITY WITH DIVIDENDS

- As shown in the case without dividends, when there are no dividends $C - P \leq S_0 - Ke^{-rT}$
- Dividends reduce C and increase P. **Why??**
- Hence relationship must also be true when there are dividends.

$t=0$ $t=\tau$ $t=T$

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Now, we talk about put call parity with dividends. Now, I will not devote too much time to it. The important thing here is the point that dividends reduce C and increase P. This is one point which needs explanation. As I mentioned just now, a couple of minutes back, higher is the value of the underlying, higher is the price of the underlying, higher is the value of the call and lower is the value of the put option.

Now, if a dividend is anticipated let us say t equal to 0. Let us say this is t equal to tau. Let us say this is t equal to capital T. Let us assume that setting a t equal to 0, we anticipate that there would be a dividend payment on the stock at some point in time within 0 and capital T, let us say t equal to tau. So, at t equal to tau, we assume that there can flow on account of dividend or the stock would go x dividend.

Now, when the stock goes ex dividend, all of us know that the price of the stock will go down correspondingly because the price of the stock will go down by the amount of dividend. Once the stock becomes ex dividend, the value of the call option is obviously likely to decline, likely to decrease. So, the possibility of dividend during the life of the call option reduces the call premium and similarly it increases the put premium for the same reason.

So, let me repeat quickly because the price of the underlying asset, the price of the stock will decline in the eventuality of the stock or the underlying going ex-dividend the value of the call option will decline because the payoff on the call option declines corresponding to a decline in the value of the underlying asset. So, in this situation, what do we have? C decreases, P increases and therefore because C falls, P rises. This inequality would still hold even in the situation where the dividend is payable during the life of the options.

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AMERICAN OPTIONS: PUT CALL WITH DIVIDENDS				
	t=0	t=τ	t=T	
PORTFOLIO A		$S_τ < K$	$S_τ < K$	$S_T > K$
BUY EUROPEAN CALL	-c	≥ 0	0	$S_T - K$
INVEST	$-(K + D_0)$	$Ke^{(rτ)} + D_τ$	$Ke^{(rT)} + D_T$	$Ke^{(rT)} + D_T$
TOTAL	$-(c + K + D_0)$	$\geq (Ke^{(rτ)} + D_τ)$	$Ke^{(rT)} + D_T$	$S_T + Ke^{(rT)} + D_T - K$

Now, the rest of the steps are absolutely similar to what we have here, except for the fact that in this case, instead of dividend, instead of investing K, we are now investing K plus D_0 , where what is D_0 ? D_0 is the present value of the dividend that is expected during the life of the option.

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	t=0	t=τ	t=T	
PORTFOLIO A		$S_τ < K$	$S_τ < K$	$S_T > K$
TOTAL	$-(c + K + D_0)$	$\geq (Ke^{(rτ)} + D_τ)$	$Ke^{(rT)} + D_T$	$S_T + Ke^{(rT)} + D_T - K$
PORTFOLIO B				
BUY STOCK →	$-S_0$	$S_τ + D_τ$	$S_T + D_T$	$S_T + D_T$
BUY AMERICAN PUT	-P	$K - S_τ$	$K - S_T$	0
TOTAL	$-S_0 - P$	$K + D_τ$	$K + D_T$	$S_T + D_T$
Since $c = C$, we have $S_0 + P \leq C + K + D_0$ so $S_0 - D_0 - K \leq C - P \leq S_0 - Ke^{-rT}$				

AMERICAN OPTIONS: PUT CALL WITH DIVIDENDS				
	t=0	t=τ	t=T	
PORTFOLIO A		$S_t < K$	$S_T < K$	$S_T > K$
BUY EUROPEAN CALL	-c	≥ 0	0	$S_T - K$
INVEST	$-(K + D_0)$	$Ke^{(r\tau)} + D_\tau$	$Ke^{(rT)} + D_T$	$Ke^{(rT)} + D_T$
TOTAL	$-(c + K + D_0)$	$\geq (Ke^{(r\tau)} + D_\tau)$	$Ke^{(rT)} + D_T$	$S_T + Ke^{(rT)} + D_T - K$

Why is that? Let us look at it. This is so because if you look at this particular row this is a long stock position in portfolio B. When you have a long stock position or when you create a long stock position in portfolio B at t equal to 0, you not only recover the price of the stock at t equal to capital T but you also recover the future value of dividend that is going to arise during the period from 0 to capital T.

Let me explain this a little bit. What happens is if you create a long position in the stock at t equal to 0, then between t equal to capital T, you will get that dividend because your name would be there in the register of members corresponding to your long position and that dividend's future value would be an additional cash flow plus in addition to the market price of the stock. So, the total cash flow at maturity due to the long position of the stock will not be S_T , but will be a combination of S_T plus D_T , S_T plus D_T .

To compensate for this factor of D_T coming up in portfolio B at t equal to capital T, we are now making an investment of D_0 at t equal to 0 in the portfolio A because this D_0 will now grow up to D_T here. You can see it here at t equal to capital T and this D_T neutralizes the D_T that we saw in portfolio B due to a long position in the stock at portfolio B at t equal to 0. Otherwise, the rest is absolutely parallel to what we have done so far and therefore, let us not devote too much time to it.

The result that we get is $S_0 - c$. You can see again that the S_0 factor is now replaced by $S_0 - c$, as has been the case in the context of forwards and in the case of put call parity for European options or in the case of the American options as well, as you can see here. So, this is the put call equivalence in the case of American options when you have a

dividend payment, a dividend amount, a dividend dollar payment during the life of the option.

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EARLY EXERCISE OF AMERICAN CALL ON DIVIDEND PAYING STOCK

- We have already shown that in the absence of dividends American ^{call} options should never be exercised early.
- It can be similarly argued that, when there are dividends, it can only be optimal to exercise at a time immediately before the stock goes ex-dividend.



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Early exercise of American call on dividend paying stock. Now, so far, what have I mentioned? I have mentioned emphatically that the early exercise of an American call on non-dividend paying stock is never optimal. Early exercise of American call on non-dividend paying stock is never optimal. What is the situation? How does the situation change if there is a dividend payment expected or anticipated during the life of the option that is between 0 and capital T? That is our next question that we need to address.

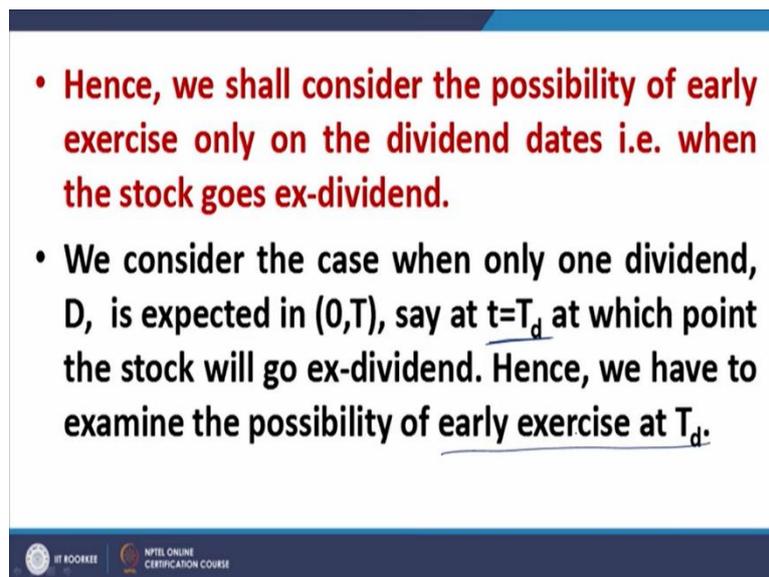
Now, we have already shown that in the absence of dividends American options, American calls rather call options should never be exercised early. So, that is what we have established in considerable detail in the previous lecture. It can similarly be argued that when there are dividends, it can only be optimal to exercise at a time immediately before the stock goes ex-dividend. Now, proceeding the same lines of argument, proceedings on the same lines of rationality that were alluded to when I established this particular part we can see that the possibility of there being an optimality in the exercise of an American call is only strongest when the exercise is closest to the point at which the stock goes ex-dividend.

The greater is the distance between today and say, the point at which the stock goes ex-dividend the lesser will be the chance of the early exercise of the American call being optimal. So, we need to investigate this particular issue at point which is infinitesimally close to the point at which the stock goes ex-dividend. Let me repeat the early exercise at a point which is distant from the point which is earlier let us say this is 0.

Let us say t equal to t_d . This is, let us say T_d and this is let us say capital T . What is T_d ? T_d is the point at which the stock was ex-dividend on account of the possibility of dividend payment during 0 and capital T . T_d is any arbitrary point earlier between 0 and T_d . Then it can be shown using the same lines of reasoning that we did when we established that early access on a non-dividend paying stock is never optimal of an American call.

Using the same line of reasoning, we can say that early exercise within this interval is not optimal and it is optimal only when we are infinite, the possibility of optimality is strongest when we are infinitesimally close to T_d . So, the relevant point for consideration, the relevant point for discussion of this particular issue that we have here is when the stock is close to or infinitesimally close to going ex-dividend. That is the point at which we need to do the analysis and analysis at an earlier point will only reinforce what the analysis at t equal to T_d and outcome gives us. So, let us do the analysis at t equal to T_d and see what we end up with.

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- Hence, we shall consider the possibility of early exercise only on the dividend dates i.e. when the stock goes ex-dividend.
- We consider the case when only one dividend, D , is expected in $(0, T)$, say at $t = T_d$ at which point the stock will go ex-dividend. Hence, we have to examine the possibility of early exercise at T_d .

Hence, we shall consider the possibility of early exercise only on the dividend date which that is the date when the stock was ex-dividend. We consider it the case when there is only one dividend D or let us call it D because there is only one dividend, so it does not matter is expected in 0 to capital T , say at t equal to t_d . So, this is the point at which the dividend is expected at which the stock is going to go ex-dividend at which point the stock will go ex-dividend. Hence, we have to examine the possibility of early exercise at t_d .

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- If the option is exercised at $t=T_d$ the payoff received by the option holder is $S(T_d)-K$.
- If the option is not exercised, the stock price will drop down to $S(T_d) - D$.
- With this stock price, the lower bound on the option price will be $[S(T_d) - D] - K \exp[-r(T-T_d)]$.
- Thus, if $[S(T_d) - D] - K \exp[-r(T-T_d)] > S(t) - K$, it is **surely** not optimal to exercise the option i.e. if $D < K\{1 - \exp[-r(T-T_d)]\}$ it cannot be optimal to exercise the call.

If the option is exercised at t equal to t_d , what will you get? If you are long in the option, you will pay the price K and you will receive the stock, you can sell the stock in the market at S_{T_d} . So, the net payoff that you will get at S_{T_d} minus K , S_{T_d} minus k . Let me repeat. If you are long in the option, what happens? You have a right to buy the stock. You buy the stock at the exercise price K and you sell it in the market at S_{T_d} . So, the pay-off that you can get by exercising the option is equal to S_{T_d} minus K .

If the option is not exercised, what will be the price of the stock immediately after it goes ex-dividend? It will fall by the amount of dividend that is undeclared on the stock. So, the stock price will go down to S_{T_d} minus D , where D is the amount of dividend. Let me repeat, S_{T_d} is the price of the stock immediately before the ex-dividend date and S_{T_d} minus D is the stock price immediately after the dividend payment date, that is the ex-dividend date, that is the date on which the stock was ex-dividend. So, with this stock price, this stock price what is the lower bound on the American option? That is the next question.

The lower bound on the American option will now become this particular quantity. Why it is so? Because what is the lower bound in the normal case? The lower bound in the normal case is S_{τ} minus $K e^{-r(T-\tau)}$ where this is the remaining life, this is the remaining life of the option. So, this is in a sense the present value of K , this expression and S_{τ} is the price as on the date of calculation of the bound.

So, now this S_{τ} is this quantity and when you substitute this quantity you get this expression. The life of the option is t and T_d is the point at which we are trying to work out the bound. Therefore, the remaining time term to maturity of the option is $T - T_d$.

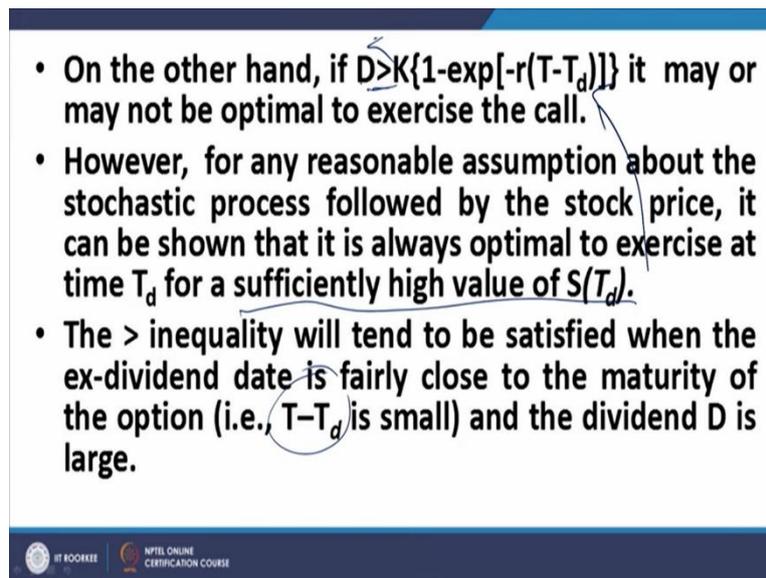
Therefore, the present value of the strike price is this expression and this is the lower bound of the option on the basis of the ex-dividend price of the stock price .

Now, if this quantity, that is the price of the option after the lower bound of the option after the stock was ex-dividend is greater than this pay off, then obviously you will not exercise the option because this is a lower bound. This quantity, this expression, this expression is the lower bound.

So, the value of the option could be this or it could be something higher and if this quantity, this number is higher than the pay-off that you expect from exercising the option, then obviously there is no rational in exercising this option, that is what this is. Therefore, if this lower bound, this quantity, this expression is higher than STd minus K which is the pay-off from the option by exercising the option and this is what you get without exercising the option and this is a lower bound. Please note that the price of the option could be more than this as well.

Therefore, if this expression the lower bound is greater than the payoff then obviously it is not optimal to exercise the option. If you realigning in this stuff that we have here, you get your D is less than K into 1 minus exponential minus rT minus Td it cannot be optimal to exercise the call option.

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The slide contains three bullet points. The first point is annotated with a blue circle around the expression $D > K\{1 - \exp[-r(T - T_d)]\}$ and a blue arrow pointing to it. The second point is annotated with a blue circle around the phrase "sufficiently high value of $S(T_d)$ ". The third point is annotated with a blue circle around the phrase "T - T_d is small".

- On the other hand, if $D > K\{1 - \exp[-r(T - T_d)]\}$ it may or may not be optimal to exercise the call.
- However, for any reasonable assumption about the stochastic process followed by the stock price, it can be shown that it is always optimal to exercise at time T_d for a sufficiently high value of $S(T_d)$.
- The $>$ inequality will tend to be satisfied when the ex-dividend date is fairly close to the maturity of the option (i.e., $T - T_d$ is small) and the dividend D is large.

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On the other hand, if D is greater than this expression, then it may or may not be optimal to exercise your call. Now, please note this point. This analysis that we have done so far is not conclusive in so far as the optimality for exercising the option is concerned. It gives you the criteria for optimality for not exercising the option, that if this less than holds, then it is not optimal to exercise the option. It is not optimal to exercise but if this greater than holds here then we are not sure whether it is optimal to exercise the option or it is not optimal to exercise the option.

But it is seen that for any reasonable assumption about the stochastic process followed by the stock price, it can be shown that it is always optimal to exercise your option at time T_d for a sufficiently high value of $S(T_d)$ if this greater than holds. Let me repeat if this greater than holds, this analysis does not give us any conclusive result insofar as the optimality about the exercise of the option or not it is concerned if D is greater than this. But for any reasonable assumption about the stochastic process followed by the stock price, it can be shown that it is always optimal to exercise at time T_d for sufficiently high value of $S(T_d)$.

So, if the current stock price value is high enough, then on the basis of a reasonable assumption of other stochastic process as a stock is going to follow, it may be optimal to exercise the option if this greater than holds. Of course, if the less than holds here, then it is not optimal to exercise the option conclusively on the basis of this analysis.

The greater than equality inequality will tend to be satisfied when the dividend rate is fairly close. As I mentioned just now, the greater than inequality will tend to be satisfied when the ex-dividend is very close to the maturity of the option that is this is small and the dividend is

large. That is D is large. So, you can see here the possibility of D being greater than K into 1 minus exponential minus rT minus Td will be more as D is larger that is quite obvious. And as T minus Td is smaller, that is the closer we are to the maturity of the option.

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EXAMPLE

- An Indian trader constituted a portfolio today ($t=0$) comprising of
- (i) a long 180-day European call covering 100 USD with a strike of $\text{INR } 72 = \text{USD } 1$ and
- (ii) a short position in a 180-day forward contract for 100 USD. *Deliver USD*
- The premium on the call was $\text{INR } 20.00$.

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Here is an example. An Indian trader constituted a portfolio today t equal to 0 , comprising of a long 180-day European call covering 100 USD with a strike of $\text{INR } 72$ equal to US dollar 1 . A short position in 180-day forward contract for US dollars 100. Short position please recall short position under a forward means what you are going to deliver US dollars delivered.

So, basically what you are going to do is you are going to buy US dollars under the call option. If the call option needs to be invoked or in the market if the call option needs not to be invoked and deliver it under the short forward position that we have undertaken. The premium on the call was $\text{INR } 20$.

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- The spot and 180-day forward INR/USD exchange rates were respectively $\text{INR } 70 = \text{USD } 1$ & $\text{INR } 75 = \text{USD } 1$.
- The trader expects the exchange rate on maturity to take either of only two values viz. $\text{INR } 71 = \text{USD } 1$ and $\text{INR } 74 = \text{USD } 1$ with probabilities of 0.75 & 0.25 respectively.
- Ignoring time value of money, calculate the expected profit from this portfolio (in INR) at maturity.

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The spot at 180-day forward INR US dollar exchange rate are respectively INR 70 equal to US dollar 1 and INR 75 equal to US dollar 1. So, the forward rate, the rate at which you are going to deliver the US dollars at T equal to 180 days is US dollar 75. This is your selling price and this is fixed by virtue of the forward contract. The trader expects the exchange rate at maturity to take either of two values INR 71 or INR 74 for US dollars 1 with probability 0.75 and 0.25 respectively.

Ignoring time value of money calculate the expected profit from the portfolio at maturity. Now, please note this point that we are having two possible values in a sense it is a binomial tree. You have got two possible values INR 71 and INR 74 for the US dollar spot rate at T equal to 180 days with respect to probabilities of 0.75 and 0.25. Now, please note this point. What was the exercise price of the option? The exercise price of the option was INR 72.

So, in the event that the price at maturity turns out to be INR 71, you will not exercise the option. You will buy the US dollars in the market and deliver it against the forward contract at INR 75 per dollar and in the event that the price at maturity in the market turns out to be 74, you will exercise the option and you will collect the dollars under the option contract at 72 and deliver it at 75 to the party who is long in the forward contract. The cost of the option that you have incurred is 20 and please note you are allowed to ignore the time value of money in the current problem.

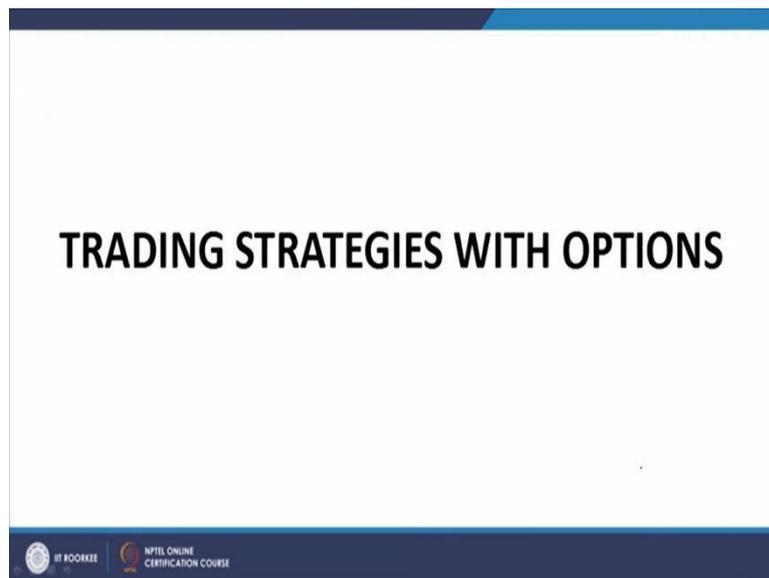
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	SCENARIO A	SCENARIO B
SPOT PRICE AT MATURITY (INR/USD)	71	74
STRIKE PRICE	72	72
CALL PAYOFF	0	200
CALL PRICE	20	20
NET PAYOFF ON CALL	-20	180
SPOT COST OF 100 USD	7100	7400
NET COST OF 100 USD	7120	7220
SELLING PRICE PER USD	75	75
SALES PROCEEDS OF 100 GBP	7500	7500
NET PROFIT	380	280
PROBABILITY	0.75	0.25
EXPECTED PROFIT		355

So, this is the Excel sheet showing the solution to the problem. These are the two possible values 71 and 74. This is the exercise price 72 of the options and therefore, in this case the payoff from the option is 0 because the strike price is higher than the market price. In this case the payoff is equal to Rupees 2 which is S_t minus K that is 74 minus 72 that is 2 and because you are having 100 units, 100 US Dollars so the pay-off from the option is 200. The cost of the option is 20. You deduct the cost of the option from the pay-off you have the profit from the option strategy of minus 20 and 180.

The spot cost of buying the US dollars is 7100 if the price is 71 and 7400 if the price is 74. Now, after accounting for the option pay off, the price turns out to be 7120 and 7220. The selling price is 75 which is fixed by the forward contract. And therefore, the selling price for the whole consignment of 100 dollars is 7500 and that leads us to a profit of 380 in situation 1 and 280 in situation 2. The respective probabilities are 0.75 and 0.25. That gives us the expected value of the profit as 355.

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We shall continue from here in the next lecture with trading strategies. Thank you.