Production and Operation Management

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Lecture 20

Examples of Safety Stock Calculation

Welcome friends. So, we are discussing about inventory management models in last few sessions. This is the last session in this particular week. So, we will be coming to end of fourth week of this course. And in our last session if you remember, we discussed the concept of safety stock. We discussed that safety stock is very necessary for meeting the fluctuations. and these fluctuations we discussed are possible because of two reasons.

One fluctuations are possible, because of variations in your lead time, that your supplier has promised you that he will supply you products in 10 days, but there may be some variations in those 10 days time, maybe some time he can supply you at the 9th day on some other time, he may be able to supply you on the 12th day. So, there are fluctuations in lead time. The second is there may be fluctuations in the daily consumption rate.

We normally assume when we developed our basic EOQ model, that our daily consumption rate is constant. But in practice, daily consumption rate may also vary. So, these are the two sources of fluctuations. But in real life, we need to do something so that we can handle these fluctuations. And for that purpose, we introduced the concept of safety stock in our previous session.

In our previous session, we also discussed about the concept of a different type of inventory management system that is the P type of inventory management system, where we used to order after a fixed interval and here each time the quantity of order was varying. So, we discussed two important things in the previous session. One is the concept of safety stock and another is P type of inventory management system or you can say Fixed Time Interval Inventory Management System.

Now, in this particular session, we will do some numerical examples to understand the use of those formulas which we discussed in our previous session. So, this whole session is basically a practice session, where we are going to discuss some numerical examples. So, we have taken certain examples to demonstrate different type of cases, which we have discussed in our previous session. So, actually, you can say the previous session was more theory session, where we discuss the concept. Now, it is the application session, where we are going to see how we need to apply those particular theory.

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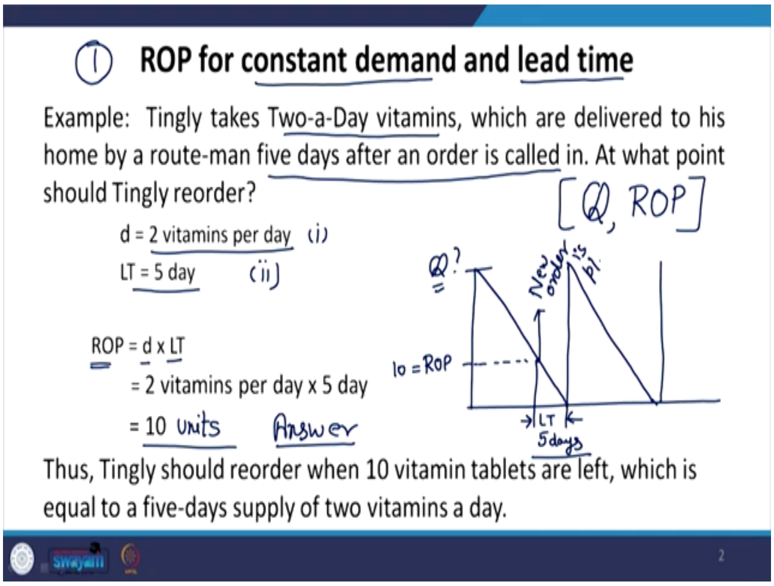

Now first example, which is to be discussed, is a very simple example. And it is you can say, we have taken from the concept of our basic EOQ model, basic EOQ model. As we have discussed many a time we need 2 answers, these are the two questions what is my order quantity and what is my reorder point? What is my order quantity and what is my reorder point? These are the 2 questions we need to answer.

Now, in this particular case, the first example which we are going to discuss here, we are not considering any kind of fluctuation, the demand is constant, daily consumption rate is constant and the lead time is also constant. So, there is no fluctuation of any type. So, in this particular case if you read the data, so, it takes a 2 vitamins per day, the daily consumption rate of this product is 2 units per day which are delivered to his home by a route-man five days after an order is called in. So, lead time is given as 5 day. So, this is information number one this is information number two available to us.

And there is no fluctuation mentioned in this example. So, we are considering that both these things are constant. So, simply the reorder point will be d into Lt. So, d is a 2 units per day into 5 days. So 10 units that is my answer. So, whenever, if you go to this particular saw teeth pattern since, I do not know the annual requirement, so, this Q I am not able to tell you at the moment, but whenever this stock is coming, this is my ROP that is equals to 10 units and this is the lead time and that is 5 days.

So, this becomes the scenario that whenever I have 10 units left in my stock, I will trigger a new order. So, you can say that from here new order is placed, new order is placed. So, you will place a new order of Q quantities, you will place a new order of Q quantities whenever 10 units are left in your stock and these 10 units are going to survive for remaining 5 days. And at the end of 5 day you will receive a new supply of, for which you have placed the order that is of Q quantities. So, that is the simplest case for this a reorder point calculation, where demand is constant and lead time is also constant.

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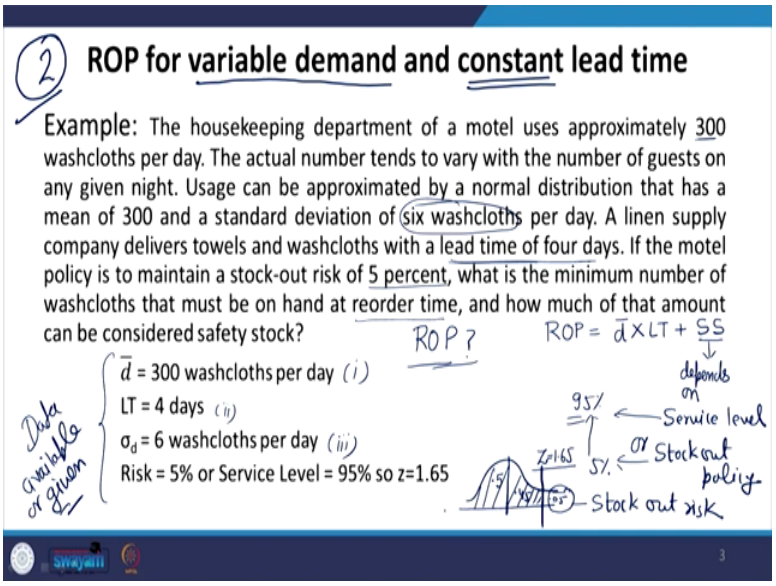

Now, let us see some variation, this is the second example. Now, what we are going to do that here we have considered variable demand and lead time is constant there are two sources of variations. So, in the previous case both these sources were constant. Now, in this example, one source is variable another source is constant. So, fluctuations are coming only because of variable daily demand.

Now in this question, if you read the data, the housekeeping department of a motel uses approximately 30 washcloths per day. The actual number tends to vary with the number of guests on any given night. Users can be approximated by a normal distribution that has a mean of 300 and a standard deviation of six units per day. A linen supply company delivers towels and washcloths with a lead time for four days.

So, lead time is constant lead time is constant that is four 4 days and the uses of this washcloth is varying with a normal distribution, where mean is 300 and the standard deviation is given as 6 per day. So, this is the additional information that the standard deviation of the demand of washcloths per day is 6 units and if the motel policy is to maintain a stock out risk of 5 percent.

Now when we have, we if you remember that now, our reorder point will be the consumption rate during the period of lead time plus safety stock, this will be the formula for calculation of our ROP. Now, this safety stock calculation depends upon service level or stock out policy. So, service level or stock out policy are related to each other. So, if my stock out risk is 5 percent, so, I want to achieve, if it is 5 percent stock out risk then the service level is 95 percent that 95 percent of the time I want to serve my customers and for getting the value of service level corresponding to 95 percent, we have already discussed that we need to go for Z table. So, we will see that how to use that.

What is the minimum number of washcloths that must be on hand at reorder point? Means my question is, what is the value of ROP? What is the value of ROP and how much of that amount can be considered safety stock? So, what is ROP and out of that rip what is the value of safety stock? So, the whole data which is mentioned in this text is given here, data available you mention it as data available or given.

So, this is the data given to us that mean demand is to 300 units per day, lead time is 4 days, standard deviation of demand 6 units per day, your stock-out risk is 5 percent or service level is 95 percent and corresponding to this 95 percent service level because the demand is normally distributed my demand is normally distributed. So, I the meaning is that you have area under this side is 0.5 and out of this side you want to fulfill 0.45.

So, this whole area you want to cover only this 0.05 is area which is not being covered by your, so, this is the area which is stock-out area. Now, for this level the value of Z if you go to normal distribution table, so, we have directly taken it is 1.65. So, for calculation of safety stock, the value of Z which we will be requiring that is 1.65 this is taking care of this 95 percent area, which is shown as hatched area.

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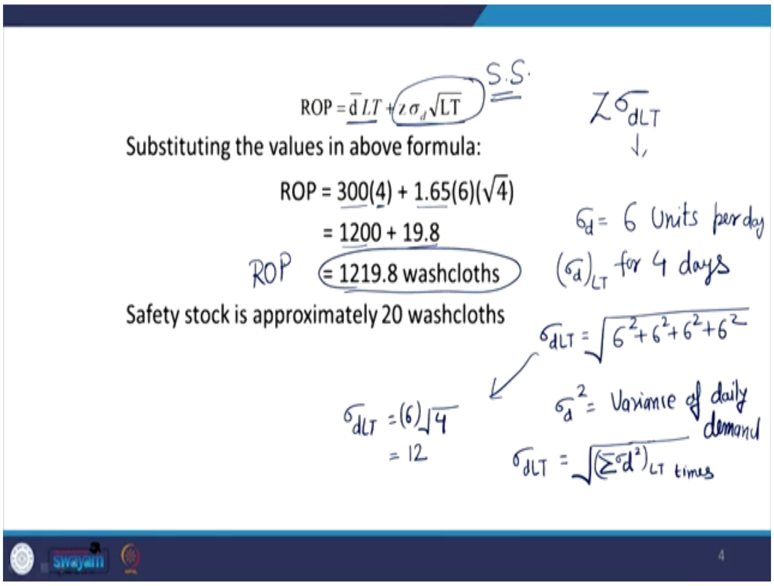

Now, let us see the calculation part. So, this ROP will be the demand during lead time plus safety stock, this component is the safety stock that is the, actually Z into sigma dLT because in this particular case, you see that your average demand is 300 units per day, the lead time is 4 days, Z value you have already taken as 1.65. And now, I need to explain you that how this sigma dLT is coming into sigma d into LT for that purpose the sigma d equals to 6 units per day.

So, my job is, to determine sigma d for entire lead time period that is for 4 days so, that sigma dLT will be actually under root of sigma of variance of each day. So variance of each day is 6 square plus 6 square plus 6 square plus 6 square, this will be the 6 squares that is, so, sigma d square is equals to variance of daily demand. And to calculate sigma dLT we have taken the under root of sigma of d Square for LT times. So, that gives me the formula here.

And if I do this calculation you can take out 6 and that it becomes under root 4, and then it is 6 into 2, 12. So 12 into 1.65 this is coming to be 19.8 plus 1200. So, total units is 1219.8 that is your reorder point. So, whenever you reach to 1220 units, whenever you reach 1220 units, you will trigger a new order and out of this 1220 your 20 units approximately under the safety stock 1200 you are going to consume, but 20 units you are keeping for the safety purpose. So, that is the case when we are having only one component as a variable component. And right now in this example that component is the variation in the daily demand.

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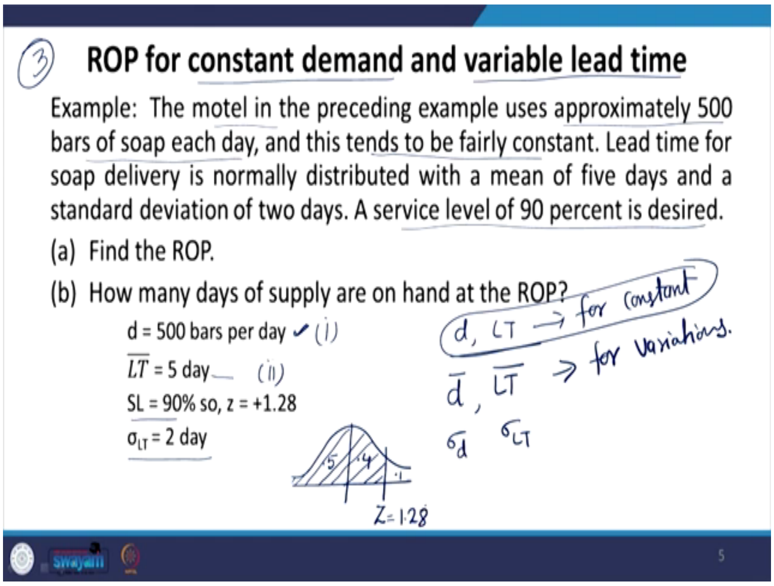

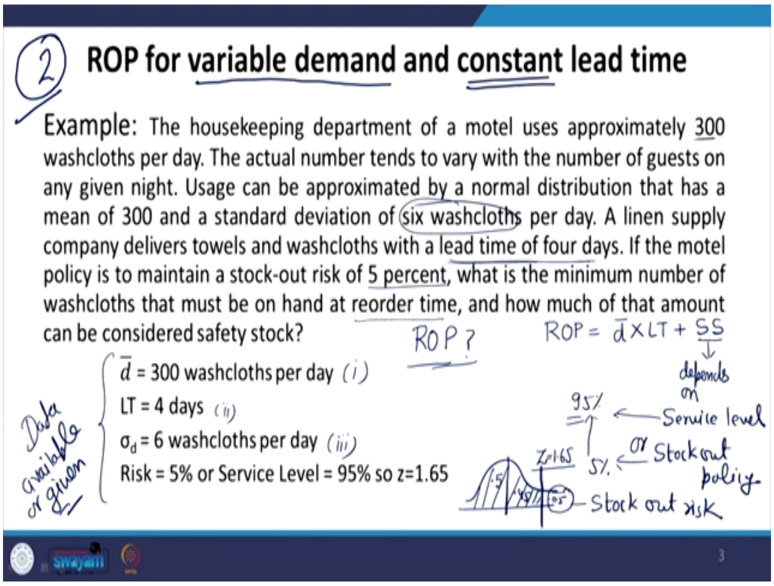

Now, let us go to third type of case where our daily demand is constant, here daily demand is constant and the lead time is variable, lead time is variable. Now, let us have an example, the same motel, which we discuss in the second example, uses approximately 500 bars of soap each day and this tends to be fairly constant. So, this d is constant, d is constant, you can also notice one important thing that things which are given constant to us, we use only this type of symbols for them.

Those things which are given in terms of variable, we use these type of symbols for them. So, for constant, for variations because these d bar LT bar these things represent mean values and then with these mean values you require sigma d sigma LT etcetera. also because these are the mean values and these are the standard deviations of these fluctuating parameters. So, you can understand that how do we write. Like in the previous cases, our daily demand was variable. So, we wrote it d bar you remember if you go back to the previous slide, so, you can see that here we wrote d bar and LT we wrote without any bar because it was given as a constant value.

Now, in this particular case, I am writing d as a constant value without any bar sign, but I am writing LT as a bar value because this is the mean value for lead time. So, you see the lead time for soap delivery is normally distributed with a mean of 5 days. So, here this is coming with a bar sign, because it is representing the mean value then with a mean of 5 days and a standard deviation of 2 days sigma LT is given as 2 days.

A service level of 90 percent is desirable. So, you can take care of 10 percent stock-out risk. So, service level is 90 percent again, it is like this way, you have this normal distribution curve, this area represents 0.5 and now you are taking only 0.4 of this side. So, this is the point where you are having 0.1 on this side. So, you are taking this much area under your consideration and here the value of Z is coming 1.28, because 90 percent you want to fulfill with this kind of situation.

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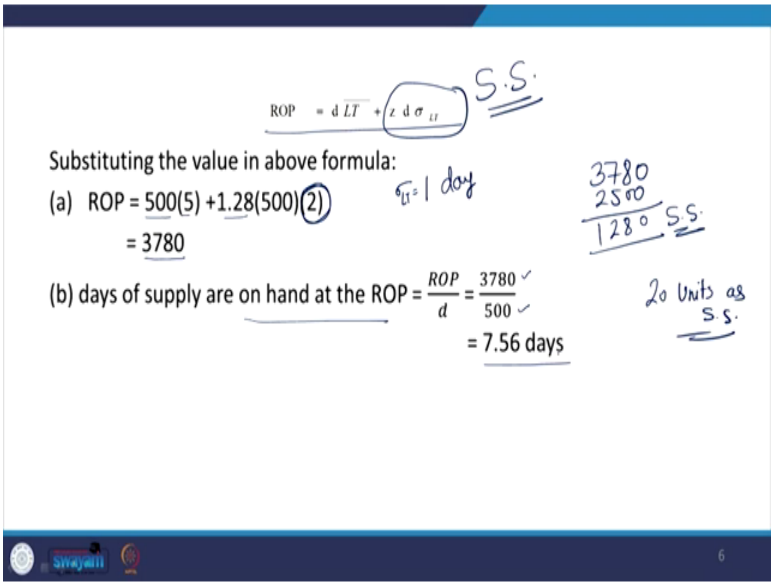

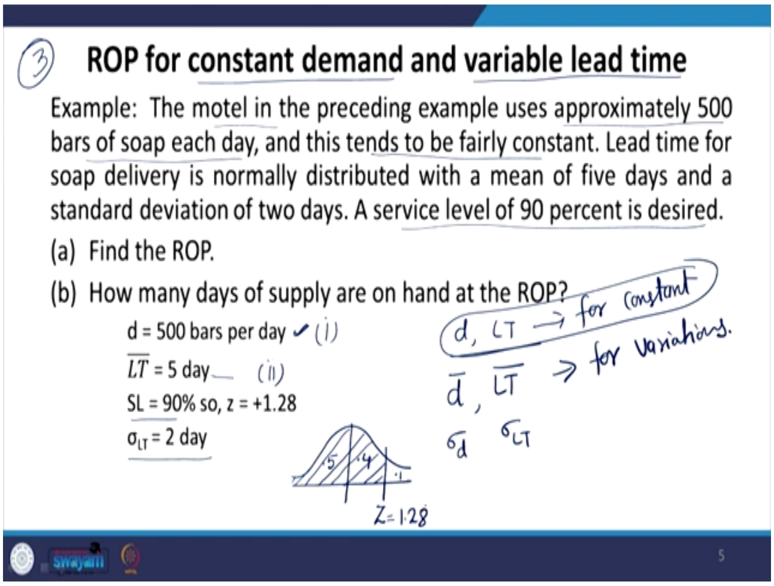

Now, let us see how do we calculate. Now, in this particular case, this is the formula, where this term represents Safety Stock. So, this is the third formula, if you remember from our previous session for calculation of reorder point. So, this first term is representing the consumption during the lead time period. So, our daily demands which is fixed is 500 units, the mean value of lead time is 5. So, this is the first term.

Now the second term is Z d sigma LT. So, Z value is 1.28 because service level is 90 percent, d is 500. And variation, standard deviation of lead time is 2 units or 2 days. So, that is 2. So, when we are doing this calculation, it is coming to be 3780. So, out of 3780, you see your 2500 is the quantity you are going to have for fixed consumption and remaining this is the safety stock. So, 1280 units you are keeping for the safety stock.

Now, it is important to understand with these two examples, the previous example and this example. In the previous example, we had only 20 units as safety stock. In this example, we have 1280 units as safety stock. The point is that if there are variations in your lead time, if there are variations in your lead time, all of a sudden the requirement of safety stock increases too much, there is a fluctuation in daily demand, but that fluctuation in daily demand will not increase the value of safety stock too much, but fluctuation of lead time will increase the daily (dem), increase the value of safety stock to a very high limit.

Therefore, we want a very reliable supplier, so, that we can minimize, so, that we can minimize the, if you see, if the fluctuation here it is 2, here it is 2 that is the fluctuation of your lead time if we have a better supplier who can supply with a fluctuation of 1 day. If my Sigma LT comes to one day, automatically this entire term will reduce by 50 percent it will become half of this. So, in place of 1280 units as your safety stock, you will have only 640 units.

So, by improving the performance of your supplier with respect to reducing the variability in the lead time, you can reduce your safety stock drastically. So, we should focus more on reliable suppliers, so, that you have minimum lead time fluctuations, because lead time fluctuations can cause very high value of safety stock. So, with the help of these two examples, now, we are able to understand very clearly that safety stock increases with a very fast rate, if there are more fluctuations in the lead time.

But here the fluctuation of lead time is 2 days, if you remember in the previous case, the fluctuation of daily demand was 6 unit, standard deviation of demand per day was 6 unit, but still the safety stock calculation gave us just 20 units here with two days of lead time your safety stock calculation has increased so much. So, that is a very important thing, we must remember that safety stock calculation is more sensitive to the fluctuations in the lead time.

Now, another part of this question is that days of supply are on hand at the ROP. So, whenever you are going to place new order what is the supply on hand available with you? So, ROP divided by d, so, 3780 divided by 500 is your daily demand. So, you are having around 7.5 days supply available with you. So, you are having that much of supply, your orders are normally expected in 5 days and there is a situation of up to 7 days also. So, you can easily maintain, you can easily maintain the stocks even up to 7 days. So, that is the answer for this particular question.

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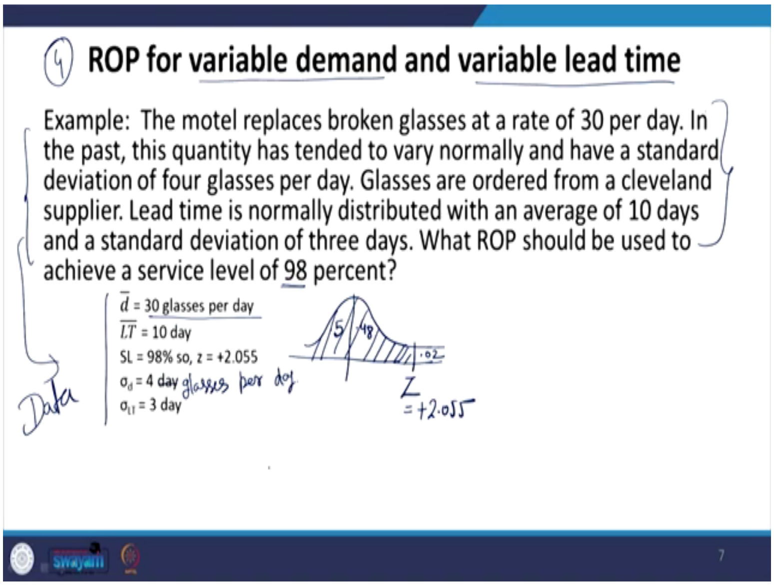

Then we go to next example, where we have more complex situation, here the demand is also variable and lead time is also variable, both demand and lead time both are variable and in the same way we go with this example, here the average demand, if you read this text, so, we have simplified the data, whatever data is available in the text, we have tabulated it separately. So, the average demand is 30 glasses per day therefore, there is a bar sign over d average lead time is 10 days.

So, therefore, a bar is on LT also. The service label which we have chosen here, that is 9 percent. And if I give you again that normal distribution curve, so, this is 0.5 and this is 0.48. So, here you see a very small tail is available of just 2 percent and this much area you want to support and here the value of Z is plus 2.055. So, that is the service level we have expected. Now, the standard deviation in the demand is given as 4 units, 4 glasses per day and lead time standard deviation is 3 days. So, these are the data available to us.

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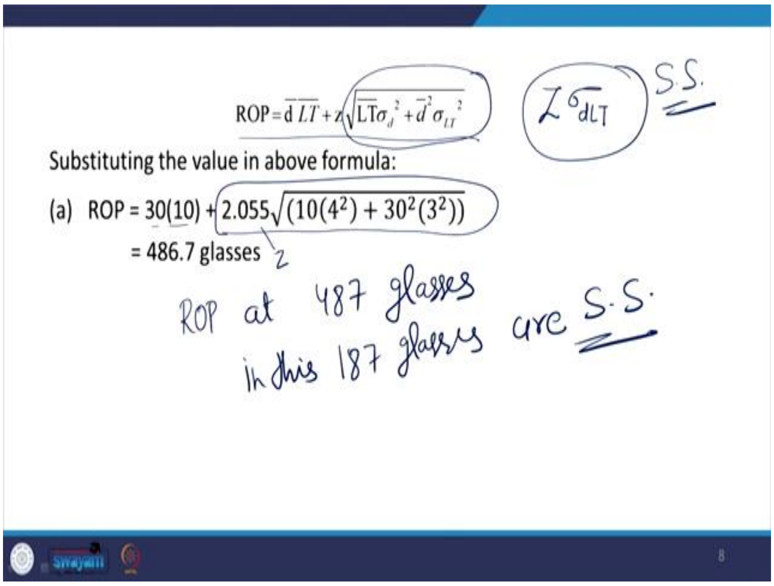

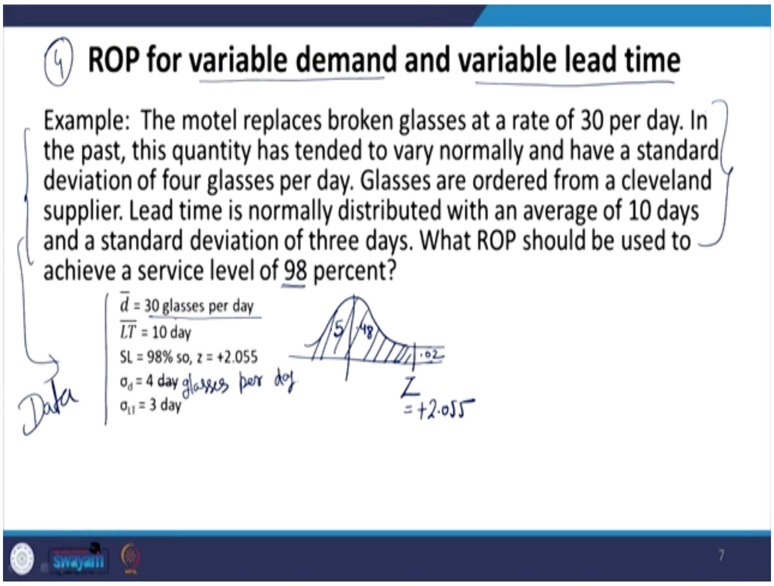

For this purpose, since, it is a complicated case, you have both d bar LT bar that is the demand during lead time and this is the dZ into sigma dLT. This is the sigma of demand during lead time and here both demand and LT both are fluctuating. So, you need to take the variance of both these things and then this particular calculation gives you safety stock. So, this was the fourth formula, if you remember we discuss in our previous session. So, now substituting all these values in this formula. 30 units is our average demand and lead time is 10 days.

So, 30 into 10 that is our normal consumption and this is giving you the safety stock 2.055 is the value of Z this is my LT, average LT and this is the demand fluctuation that is the standard deviation of the demand. Then this is the average demand and this is the fluctuation in the lead time.

So, that is the complete calculation of my formula and on the basis of that, we will have ROP at around 487 glasses and in this 187 glasses are my safety stock, because 300 you will be consuming within 10 days. So, 187, remaining 187 are part of your safety stock, because you have fluctuations in lead time and in demand. Therefore, again the value of safety stock is sufficiently high, value of safety stock is sufficiently high. So, that is the usefulness of the fourth formula where we have variations in both the quantities.

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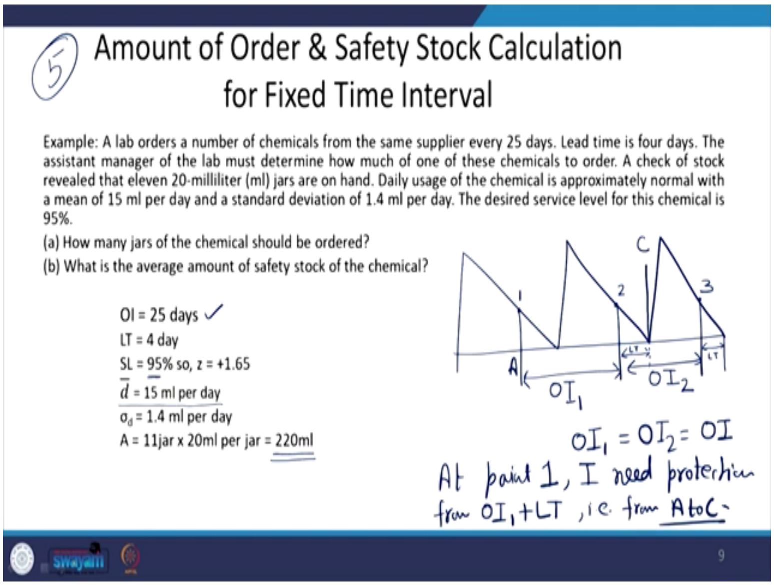

Now, the another type of situation which we discussed in our previous session, that is about the Fixed Time Interval cases. When we are going to order after a fixed time interval and in this particular example, we have discussed that particular type of example where you are going to have so, from here to here, from here to here, so, these three are my point of order. So, this period of order interval, this is OI1 and this is OI2. So, OI1 is equal to OI2 equals to OI and that is given as 25 days in this particular case. So, I have these OIs and here this is my lead time, this is the lead time, this is the lead time.

Now there may be fluctuations in the lead time there may be fluctuations in the daily consumption rate and on the basis of that we are going to discuss this particular example. So, here the order interval is 25 days, lead time is 4 days, the service level which is desirable is 95 percent, the average consumption rate is 15 milliliter per day and the standard deviation in the demand is 1.4 milliliter per day. And on a particular day, when I am checking my stock the available quantity in my stock in my inventory is to 220 milliliter. So, how much I should order? How many jars of chemical should we order?

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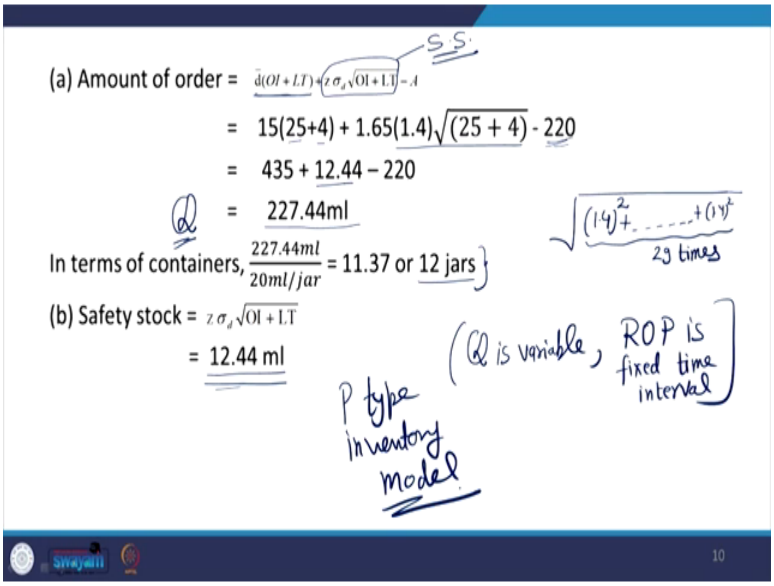

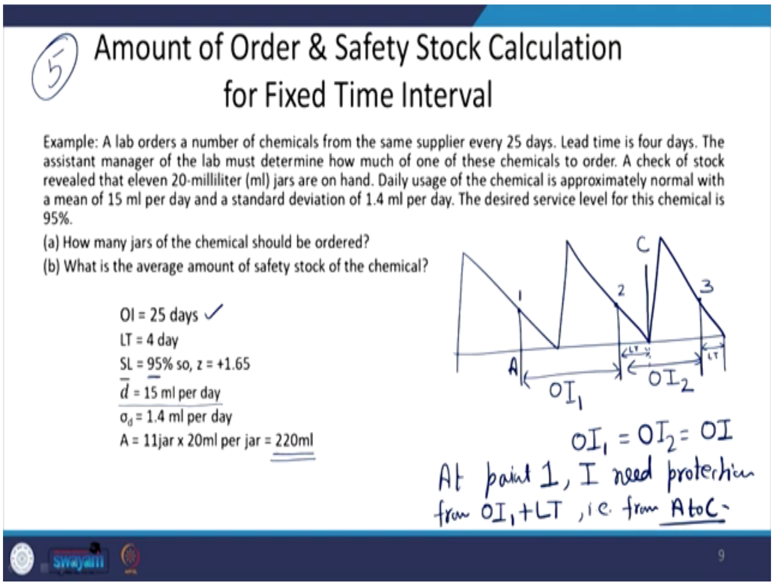

So, let us see the calculation of this particular case. So here this is the inventory which I am going to consume during the period of protection that is my order interval plus lead time. Because when I am ordering at point number one, I need to have a protection from at point 1 I need protection from OI1 plus next LT. So because from here to here, this is point A that means, from A to C for this much long period, I want protection.

And that is coming 15 is the average consumption rate, 25 is my order interval, 4 is the lead time. So, that is my actual consumption. Now, the second component of this formula, this is going to give me the safety stock and since, again you see I have the variation in daily demand. So, I am going to use if you remember the second case where I am going to get the variance of daily demand. Variance will be the square of standard deviation and then I sum up for the entire protection period and then I take the under root of that. So, this is this calculation.

So, actually what I am doing here I am putting 1.4 square these are 29 times and that is what I have written in the simplified manner and the inventory on stock is 220 units. So, when I have done the calculation to 227.44 milliliter, I need to order, that is my quantity to order, this is my Q and in terms of container because each container contains 20 milliliter so I have to order somewhere around 12 jars, I have to order somewhere around 12 jars and this 12.44 is my safety stock, this is my safety stock. So, around less than means 60 percent of one jar is my safety stock. So, this is the calculation when we are going to order at the fixed interval.

So, here you see your order quantity Q is variable and ROP is fixed time interval. So, the meaning is whatever is available at that particular time in my earlier case, this a the 220 unit used to serve ROP point. But now, in this particular case, ROP will come on the basis of fixed time interval and that is 25 days in this particular example, so, that is this P type inventory case. So, with this, we discuss the examples of both our safety stock calculations and P type of inventory management models. We will do more such problems in our forum discussions and in our assignments also with this thank you very much.