

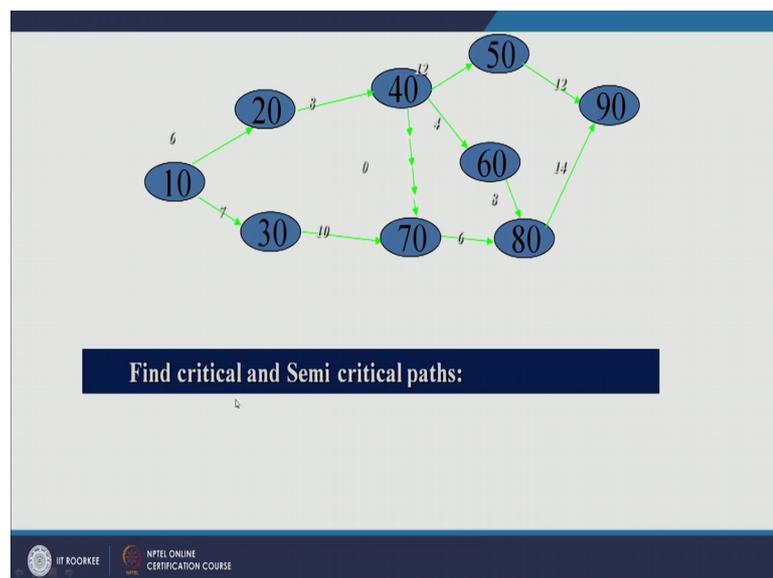
Project Management for Managers
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Lecture – 37
Project Time Management - CPM

Good morning friends, I welcome you all in this session. In this session, we are going to talk about how to find critical path, what are different semi critical paths, how to find out variance of an activity, how to find out standard deviation of an activity; and at the end of the day, we will talk about how to find out variance along critical path of a network. So, let us gets started.

In previous class, we discussed something about what is critical path. And critical path as I said is the longest path connecting first node to last node in a network. In other words, we define critical path as a path along those nodes where slack is zero. Let us look at some more refined version of critical path and we will discuss something about semi critical path also. Let me take up an example and we will find out what is critical path and semi critical path.

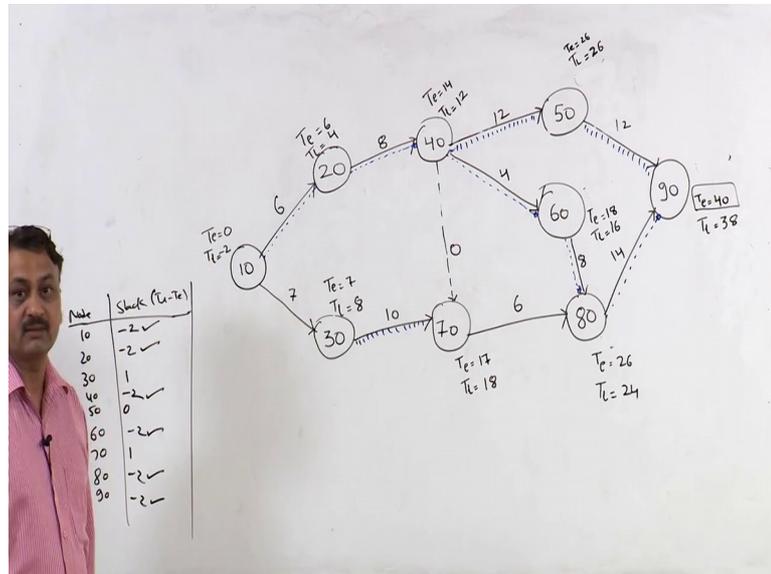
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So, this is the network where in you have got 9 nodes and different activities. So, we will try to find out what is critical path and semi critical path as far as this particular network is concerned. So, I have drawn this network on board. If you look at this, you have been

given durations 6, 7, 8, 10, since this is a dummy activity. So, duration is 0. This is 12, this is 4, this is 12, this is 14, 8 and 6.

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So, this how you can write all the durations. Now, you need to find out T_e and T_l set all these nodes. So, T_e here is earliest start time. So, you can start this particular these two activities as early as zero unit of time. . So, T_e here is 6, T_e here is 7, T_e here is 6 plus 8 – 14; T_e here is 7 plus 10 is 17. Now from here if you calculate T_e then it would be 14 plus 9 which would be 23, but from this path this is 17. So, 14 and 17 you have to write the higher value. So, 17 is T_e at this particular node. T_e here this node is 26, how 26, it is 20, it is 14 plus 12. So, let me write T_e here, T_e is 26. So, in fact what we have done we have written T_e and below it T_l . So, everywhere we have kept this uniformity.

So, T_e at this point is what 18, how come 18. From this route 14 plus 4, this is 18; and at this point T_e is 26, why 26, it is 17 plus 6 is 23, but 18 plus 8 is 26. So, you have to take the higher value, so this is 26. At this point T_e is 40, it is 26 plus 12 is 38. And from here 26 plus 14 is it is 40. So, this is the time in which this project would be completed, and the probability of completion is 50 percent. But you have been given scheduled completion time for this project as 38, you have to complete this project in 38 days. So, we will take T_l at this node is equal to T_s .

Since we have been given T_s we are taking T_l is equal to T_s otherwise we would we would have taken T_l equal to 40. So, when we take T_l is equal to 38, we will calculate

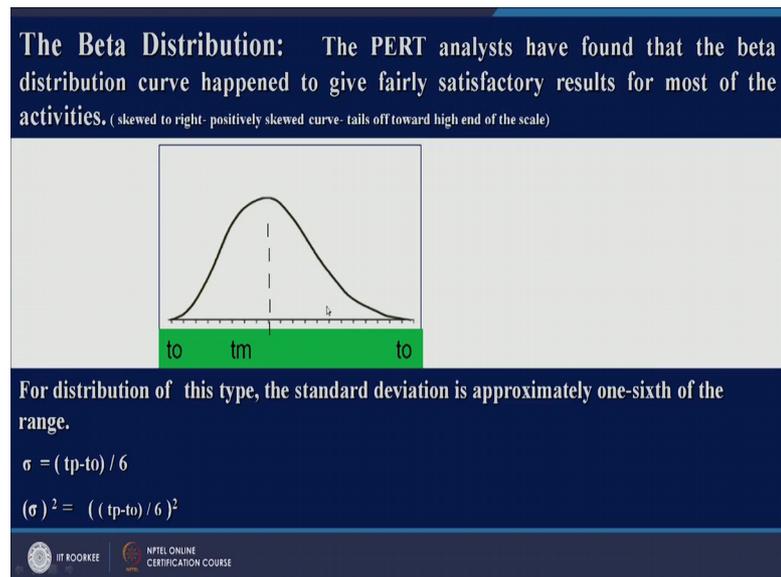
now Tl in backward paths. So, this is $38 - 12 = 26$, $38 - 14 = 24$, $24 - 8$ is 16, $16 - 4$, you will get 12. And from here also, if you get let look at what is Tl from this route. So, $26 - 12$ is 14, but from this route it is 12. So, we will take the lower one. So, Tl here is 12. Now, Tl here is 18. So, this is $24 - 6$ is 18. In fact, you should have taken Tl from this route also. So, if you Tl from this route at this node, so $18 - 0$ is 18, so 18 from here, 12 from here, and 14 from here. So, the least the lowest one is 12. So, we have taken 12 here. Then $12 - 8$ is four at this node at this node its $18 - 10$ is 8, $8 - 7$ is 1, $4 - 6$ is minus 2. So, we will the lower one value. So, Tl at this point is minus 2.

Now, let us find out let us write down slake at all these nodes. So, node number 10, 20, 30, 40, 50, 60, 70, 80, 90 all these are nodes. And what are the slack values here. So, first slack values write this it is minus 2 similarly for at node 20, 30, 40 and 50 you can calculate. So, just look at this these are different slacks right slake 20, it is minus 2; at 30 also it is it plus 1, so this is plus value. So, $8 - 7$ is 1. At 40, 40 it is minus 2; and at 50, it is 0. So, it is Tl minus T e right, Tl minus T e, this how you are calculating slacks. So, for 60, node number 60, it is minus 2; node 70, it is 1; node 80, it is minus 2; node 90, it is minus 2.

So, if you look at all these slacks then this is minus 2, minus 2, minus 2, minus 2, minus 2, so this is your critical path. So, 10, 20, 20 to 40 then 40 to 60, 60 to 80, and then 80 to 90, this is your critical path. Now, which path is semi critical path? Now these are lowest slack values, which has given us critical path now after minus 2 which is the higher value, higher value is 0, which is at node 50. So, node 50 is here, 50 is connected to which two nodes 40 and 90. So, this is your semi critical path, let me write like this, this is your semi critical path. So, this is your first semi critical path. Similarly, the next higher value is 30 and 70, so 30, 70 and 80. So, this will also be your second semi critical path. So, 30 to 70, this is also your semi critical path.

So, let us look at what is there in next slide. So far we talked a lot about CPM network, where in the duration of the activity was known right or it was deterministic. Let us look at PERT network I have already told you that there are three types of time estimates in PERT networks. So, you have got optimistic time, pessimistic time and most likely time. Now, here is a network wherein you have been given three time estimates for each of these activities. So, for activity 1, 2 these are time estimate 6, 7, 8.

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So, this is optimistic, this pessimistic time, most likely time and optimistic time. As I said optimistic time would be the least one. So, for all other activities you have been given these three time estimates. So, from 3 to 4, you have got a dummy activity and for this all these three time estimates would be 0. So, find critical and semi critical paths.

So, let us move on to next question. Now, once you convert this diagram into CPM network, you can easily calculate critical and semi critical paths. Let us move on to next point, which is beta distribution. So, as I said in PERT you have got three time estimates and these time estimates generally take a beta distribution ship. So, PERT analysis have done research and they found that the beta distribution is the correct representation of an activity. So, an activity in a network will always take a beta distribution which is a right skewed distribution like this. So, it is not a activity, it is not a normal shape, but in a project you will have got you will have thousands of activities, hundreds of activities and project as a whole it is a normal distribution. So, keep this point in mind.

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The image shows handwritten mathematical work on a whiteboard. At the top left, the formula for standard deviation is written as $\sigma = \left[\frac{(t_p - t_o)^2}{6} \right]^{1/2}$. Below this, two data sets are listed: for person X, $t_o = 6$, $t_m = 8$, $t_p = 10$; and for person Y, $t_o = 5$, $t_m = 7$, $t_p = 11$. To the right, the variance calculation for X is shown as $\sigma_x^2 = \left[\frac{(10-6)^2}{6} \right] = \checkmark$. Below that, the variance for Y is circled and given as $\sigma_y^2 = 11$.

So, let us find out how to find out standard deviation and variance of beta distribution. So, it is very simple the beta distribution is standard deviation is it is T_p minus T_o divided by 6 this is the standard deviation and variance is of course, you just take square of this. So, variance is this. So, this is nothing but variance. So, let us take two activities and try to find out which activity has got more uncertainty. So, let us take this example consider the time estimates for two persons x and y for execution of a particular job. So, x has given three time estimates, and y has also given three time estimates.

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The Beta Distribution:

Consider the time estimates for two persons, x and y, for the execution of a particular job.

	t_o	t_m	t_p
Estimate by x	6	8	10
Estimate by y	5	7	11

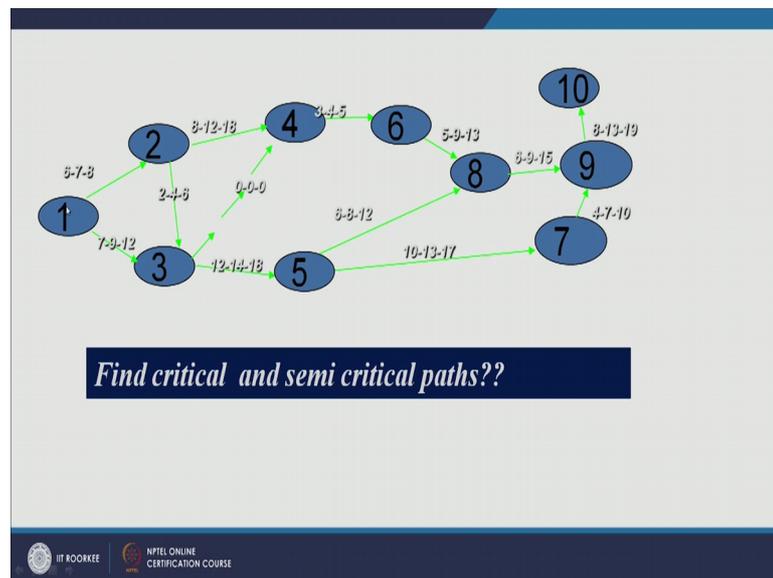
Who is more uncertain????????????

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Now, we want to know who is more uncertain about these time estimates. So, let us look at x has given 6, 8 and 10, so this is t_p , t_m and t_o . Y has given 5, 7 and 11, who is more uncertain. So, this dispersion is to be measured by variance. So, let us look at this standard deviation is $10 - 6$ divided by 6 and whole square would give you square. This 4 by 6^2 by 3 , 0.6 whole square, whatever is the value here. Similarly for this of course, that value would be less than 1 . So, this is for x this for y. So, this is $5 - 11$ is 6 divided by 6 is 1 , 1 square is equal to 1 . So, this was less than 1 this one. So, who is more uncertain, this fellow is more uncertain because variance is higher in this case. So, 0.44 for first person and 1 for second person.

Now, since you have got a PERT network and there are three time estimates first of all you should find out expected time for that activity. In fact, I gave you one assignment also, yeah when I said when I asked here to find out critical and semi critical path, first of all you should find out expected value of this of all these activities right.

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And how to find out expected value or mean value, this formula for that. It is optimistic time plus 4 times pessimistic time 4 times most likely time plus pessimistic time divided by 6. In PERT average, in PERT networks average time is called as expected time.

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EXPECTED TIME OR AVERAGE TIME.

After finding SD and variance, let us find average time taken for completion of a job.

In PERT, average time is called as expected time. There is 50-50 chance of getting the job done within that time.

$$t_e = \frac{1}{6}(t_o) + \frac{2}{3}(t_m) + \frac{1}{6}(t_p)$$
$$= \frac{(t_o + 4t_m + t_p)}{6}$$

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So, either you call it average time or expected time one and the same thing. And there is a 50-50 chance of getting job done within that time. So, it is either it is true for activity as well as for project. So, you can find out critical and semi critical path if you are given a PERT network.

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Lags in Precedence Relationships

The logical relationship between the start and finish of one activity and the start and finish of another activity.

Four logical relationships between tasks

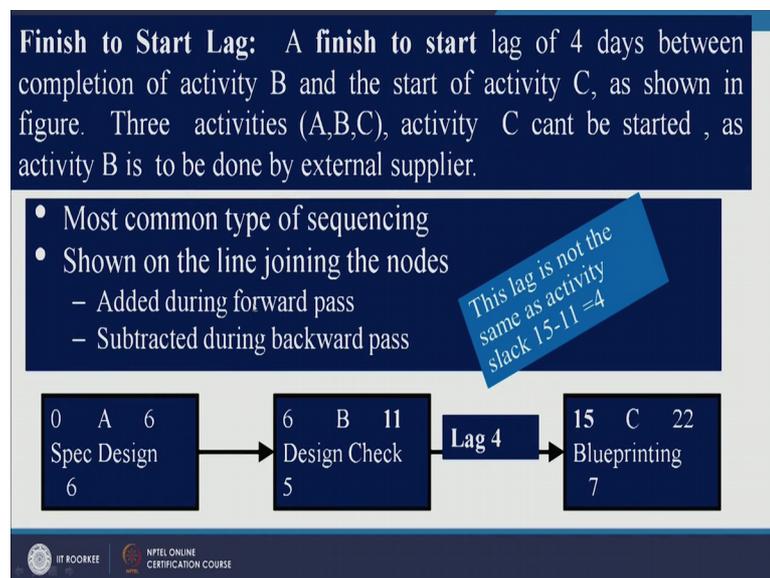
1. Finish to Start
2. Finish to Finish
3. Start to Start
4. Start to Finish

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So, now, let us move on to some other point it is called lag in precedence relationship. You have already seen what is slack, slack is the surplus time available for you in which you can shift your activities without affecting the total duration of the project. So, that is

slack. Now, let us look at something called lag, a lags in precedence relationship very important topic. What is the meaning of lag? So, the logical relationship between start and finish of first activity and start and finish of other activity. So, we will see what is that relationship. So, there are four types of relationships possible, the first one is finish to finish sorry finish to start, second is finish to finish, third is start to start, and fourth is start to finish. So, it is very simple in fact, you can number this as 1, 2, 3 and 4 no problem right. So, start to start, start to finish, finish to start and finish-to-finish right. So, this how you can remember.

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So, let us look at first activity what is the meaning of finish to start lag. So, finish to start lag means if you look at in this network there are three activities. Specification design activity A, which is taking 6 minutes or 6 days time. Once specification of design is over the next activity is design check. Now this activity will take 5 days here. The third activity is also there, it is called blueprinting, it is taking 7 days right. So, finish to start lag is there is a lag of four days between activity B and C. So, you have done specification design, but the next activity which is design check, this is being performed by this activity is being performed by a third party. And third party will take 5 days to complete this activity this one.

So, after completing this activity he has to supply let us say that particular part in your plant and that time is 4 days. So, before getting this part from third party you cannot start

activity C. So, there is a lag of 4 days. So, this is known as finish to start, you cannot start activity C, you cannot start activity C unless and until you complete activity B. Though B completed at let us say on 11th day, but you did not receive that designed check the material which you have to receive from supplier or the third party you will be receiving it on 15th day, and after receiving that on 15th day you are doing blueprinting and you are taking 7 days. So, total duration is 22.

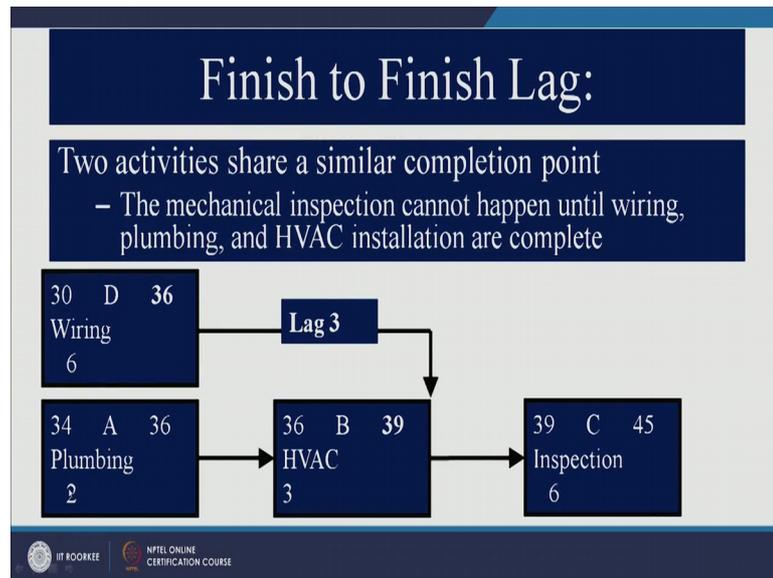
So, a finish to start lag of 4 days between completion of activity B and the start of activity is shown in figure it is there in this figure. So, there are three activities A, B, C, C cannot be started as activity B is to be done by external supplier right. So, keep this point in my mind right. So, this lag is not same as slack right this not slack, slack is something different slack means you have got freedom to move activities without affecting duration of the project, but lag is something given to an activity before which you cannot start your next activity. So, if lag is in between finish to start then you need to add that lag time in forward pass. So, this is most common type of sequencing shown on line joining the nodes yes, it is there added during forward paths.

So, if there is any lag it is to be added during forward pass for calculation of critical path and subtracted during backward pass. So, you are starting specification of design on 0th day completing it on 6th day. So, on 6th day you are starting design check, it is taking 5 days, so 11th day. This activity is being as I said is outsource to third party right or external supplier. So, this fellow did it in five days, but taking 4 days in supplying to you. Let us say this is transportation time. So, you cannot start activity C on 11th day because you do not have this particular activity finished on 11th day, it is getting finished on fifteenth day. So, total duration of this project is 22 days. So, this known as finish to start lag, you cannot start an activity unless and until you finish previous activity.

Let us look at finish-to-finish lag. When you say finish to finish means the time between finishing of preceding activity and succeeding activity is same or they will both of them will finish at the same time right not at the same time exactly. But if you do not finish both of them then you cannot start the next activity. So, finish to finish lag is so two activities share a completion point not completion time right completion point. So, unless and until you complete both of them you cannot start next activity.

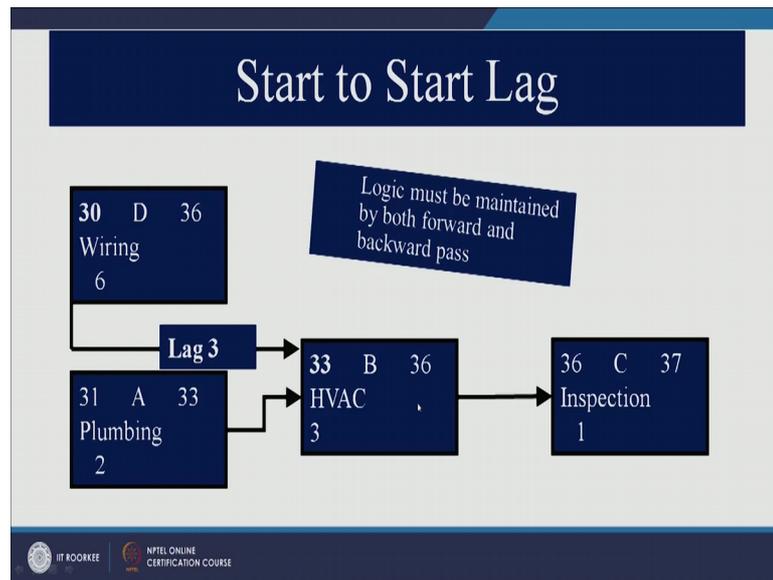
Let us take this network. In this network, there are four activities, the first one is wiring. So, let us say it is a construction of a building project and you are doing let say wiring activity right wiring. So, starting wiring on 30th day completing it on 36th day. Plumbing starting on 34th day, since it is taking 2 days time, so you are completing it on 36th day.

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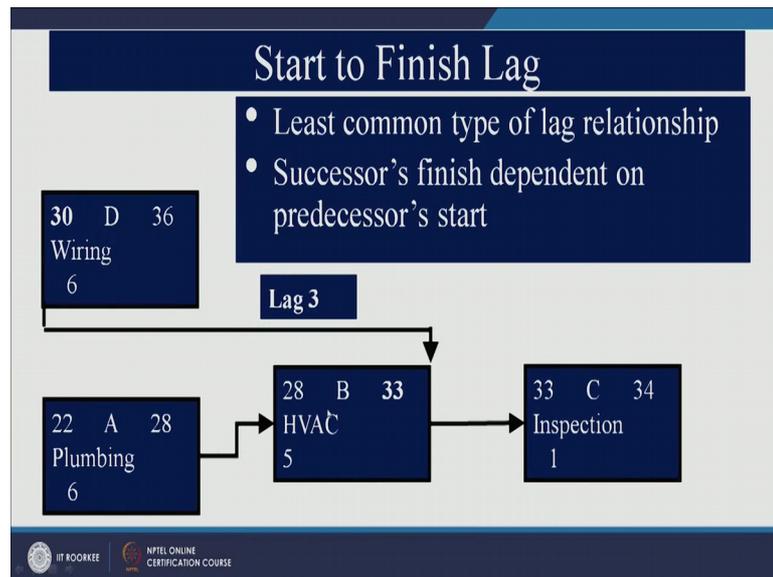
Now, there is one more activity, it is called B it is called heat ventilation and air conditioning - HVAC. So, this activity will start on 36th day, it will take 3 days time and it will complete on 39th day. Now there is a lag between activity D and activity B. So, this lag is of a three days why there is a lag of three days and what is the meaning of this. Before starting inspection you have to complete wiring, plumbing and HVAC. Though you completed wiring on 36th day, but since you did not complete HVAC, you need to wait for three days after completion of wiring. So, this is finish to finish lag of 3 days. So, inspection can be started only after you complete wiring HVAC and of course, HVAC you cannot start until you complete plumbing. So, inspection cannot be completed until you complete wiring and HVAC right. So, this is finish to finish lag.

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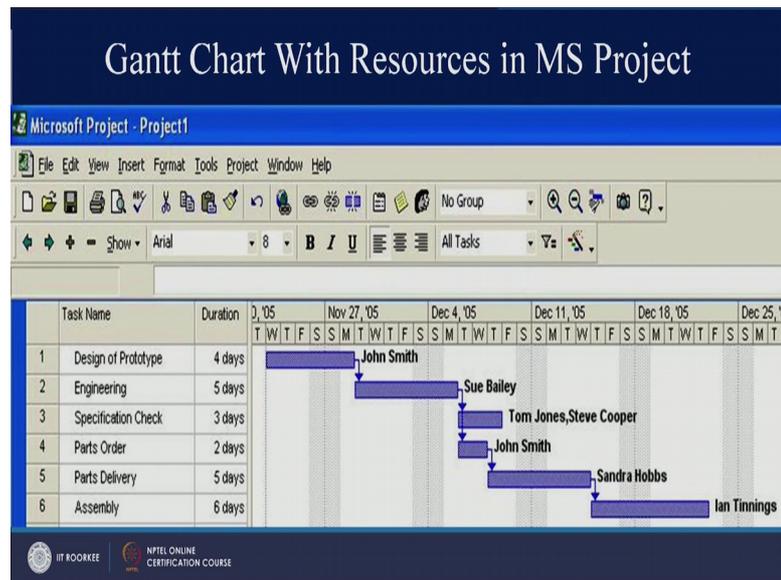
The next is start-to-start lag. So, there is a lag between a start of you know preceding activity and start of succeeding activity. So, let us look at again this same example wiring, plumbing, HVAC, inspection. So, there is a lag of three days between start of wiring and activity HVAC. So, this is taking 30 to 36, this is 31 to 33 because duration is 2. Though in fact, if you look at these two activities, so there is a lag of three days over here and you can start inspection once HVAC is finished. So, this is start to start. And keep in mind that you have to maintain this logic must be maintained by both forward and backward pass. And what is the logic? Logic is that in forward pass you need to add lag and in backward pass you need to subtract lag. So, this is start to start.

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Now, the final is what, what is the final lag relationship, start to finish. So, the least common type of lag relationship and the successors finish. So, this is the succeeding activity, this successors finish depend on predecessor's start. So, rarely this situation occurs, but this is start to finish. So, it is start of activity and activity which is the preceding one and finish of succeeding activity, which is activity B. So, 30 to 36, it is 22 to 28 right because it is taking 6 days time; and this is 28 to 33, because this is 5. So, what we are saying that the successors finish depends on predecessor's start right and inspection of course, you can start after this or on 33rd day, and it is taking 1 day. So, it will be over on 34th day. So, with this we have completed this lead and lag relationship.

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Let us look at some more points related to Gantt chart. And why we are discussing it again is because I wanted to show you how Gantt chart looks in MS project software. So, let us say these are different activities, design of prototype, engineering, specification check, parts order, parts delivery and assembly. So, design of prototype is taking 4 days assembly is taking 6 days right. So, how Gantt chart would look like in MS project. So, design of prototype its taking 4 day. So, you are starting on let us say Wednesday. So, Wednesday, Thursday, Friday, but these two are holidays right Saturdays and Sundays are holidays right. So, on Monday this activity would be finished. So, the duration is 4 days, but it is taking how many days 6 days, because you are not working on Saturday and Sunday. And this is a fellow who is responsible for completion of designer design of prototype.

If you look at engineering is taking 5 days, so again first day, second, third, fourth and Saturday, Sunday, so again Monday you are finishing it actually it is taking 5 days, you are not working on two day. So, while calculating critical path of a project you need to look at whether there is a holiday or not in a particular week, and you need to also look at resource availability. So in fact, theoretically when say that this activity will take 5 days; but in real life, it may take 10 days right because of non availability of resources, and because of non availability of let say one of key members or non availability of or let say due to holiday and some other reasons. Now, assembly, assembly is taking 6 days.

So, this is Thursday, Friday, two days here and Monday, Tuesday, Wednesday, Thursday. So, total 6 days and Ian is responsible for this particular activity.

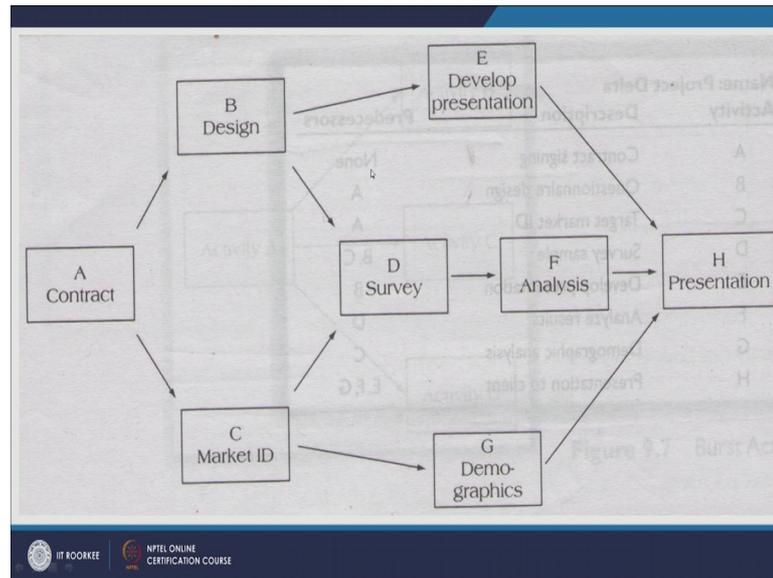
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Activity	Description	Predecessors
A	Contract signing	None
B	Questionnaire design	A
C	Target market	A
D	Survey sample	B,C
E	Develop presentation	B
F	Analyze results	D
G	Demographic analysis	C
H	Presentation to client	E,F,G



So, let me summarize what we have done in today's session. We have seen how to find out critical path and semi critical path. We have also seen lag relationship and there are four types of lag relationship; you have got start to start, start to finish, finish to start and finish to finish. And we have seen Gantt chart within in MSP. So, this is the Microsoft project window you will have different options in it. Let us look at what is there in this slide. So, in fact, this is a network and the network is having different activities A, B, C, D, E, F, G and H. So, you can always draw a network for this particular problem.

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So, this is actually the network. So, contract, design and all other activities are there.

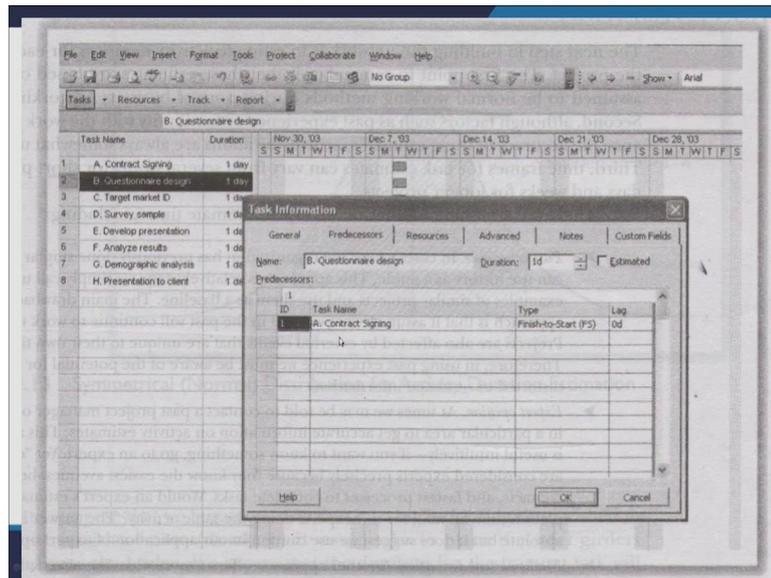
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Task ID	Task Name	Duration	Nov 30, '03							Dec 7, '03							Dec 14, '03						
			F	S	M	T	W	T	F	S	M	T	W	T	F	S	M	T	W	T	F	S	
1	A. Contract Signing	1 day																					
2	B. Questionnaire design	1 day																					
3	C. Target market ID	1 day																					
4	D. Survey sample	1 day																					
5	E. Develop presentation	1 day																					
6	F. Analyze results	1 day																					
7	G. Demographic analysis	1 day																					
8	H. Presentation to client	1 day																					

Duration is one day by default

Now if you want how to enter these activities in MSP, then it looks like this. So, all these activities are here, contract signing, questionnaire design, target market, survey sample, developed presentation, analyze results, demographic analyses and presentation to client. Now, if you look at the duration here is one day. Actually this comes by default. Now, you need to set duration in each of these cells for this particular project. As we have seen that the first activities is a contract signing, second is questionnaire design.

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Now, there is a relationship between just see this, this activity based questionnaire design, which is here. So, you can give ID number; ID number is 1 for contract signing, ID number would be 2 for questionnaire design. And you need to also mention relationship between these two. So, relationship is finish to start you cannot start contract signing you cannot start questionnaire design unless and until you complete contract signing. So, you need to give finish to the type of relationship, in this case finish to start. So, let me stop here, we will have some more topics in next session.

Thank you very much.