

Applied Econometrics
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Lecture-41
Dynamic Panel Data Model-Part IV

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and sample length

- To avoid this trade off between lag length and sample length Holtz-Eakin et. al. suggested using second lag of y_{it} as only instrument for each period and put zero for all missing observations

Holtz-Eakin et. al. :

$$Z = \begin{pmatrix} 0 & 0 \dots 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ y_{i1} & 0 \dots 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 \dots 0 & \dots & 0 & \dots & 0 & \dots & 0 \end{pmatrix} \begin{matrix} t=2 \\ t=3 \\ t=4 \\ \dots \\ t=T \end{matrix}$$

If that is the case then the Z matrix in Holtz-Eakin et al., approach will look in this way (as given in the diagram)

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Holtz-Eakin et. al. :

$$Z = \begin{pmatrix} 0 & 0 \dots 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ y_{i1} & 0 \dots 0 & \dots & 0 & \dots & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \dots & \vdots & \dots & \vdots \\ 0 & 0 \dots 0 & \dots & 0 & \dots & 0 & \dots & 0 \end{pmatrix} \begin{matrix} t=2 \\ t=3 \\ t=4 \\ \dots \\ t=T \end{matrix}$$

- We ensured enough sample length, but lag length is only 2
- even though replacing missing values by zero seems arbitrary, each of the columns of the Z matrix is orthogonal to the transformed error

$$E(y_{i,t-2} v_{it}^*) = 0$$

where $v_{it}^* = (v_{it} - v_{i,t-1})$

So, that means in this approach I am using only one instrument for each period and that is second lag of y_{i2} . So, that is why when it is $t = 3$ i get 3 minus 2 is 1, when $t = 4$ i get y_{i2} as instrument, when it is $t = T$ i get y_{iT} minus 2. So, in this way we ensured enough sample length.

Because we have now replaced all those missing values as 0 but what is happening? But lag length is only 2. So, we have ensured enough sample length. Earlier we were losing information, the more instrument you use you are losing one on observations, to avoid that Holtz-Eakin et al., said why not using only one instrument.

That is second period lag for each time period and then replace the missing observations by 0. And in that case if we do like that then we ensured enough sample length but lag length is only at 2. Now these replacing missing observation by 0 even though it looks arbitrary one great thing that is achieved by this Holtz-Eakin et al. approach. That each of this column, so even though replacing missing observations values by 0 seems arbitrary.

Each of the columns of the Z matrix is orthogonal to the transformed error. What is the meaning of this? That means $E(y_{i,t-2} \vartheta_{it}^*) = 0$

ϑ_{it}^* equals $\vartheta_{it} - \vartheta_{it-1}$. This is the transformed error and this orthogonality condition means your explanatory variable is not correlated with your error term.

So, that is how that means in doing so what actually I am achieving, this is all coming from my Z matrix. So, when each of the columns of your Z is orthogonal, that means we are saying that your instrument is actually not correlated with your error term that is what we are trying to achieve over here. So, one fantastic thing that we have achieved here in this Holtz-Eakin et al., approach, he suggested that use only one instrument that is second period lag for each time period as instrument and then replace all these missing values by 0 to overcome enough sample length. Even though this looks replacing missing values by 0, looks like random or arbitrary in nature it is proved that each of this column of the Z matrix. That means the instrument matrix suggested by Holtz-Eakin et al., they actually satisfied orthogonality condition. That means $E(y_{i,t})$ and this transformed error term ϑ_{it}^* is zero. That means in doing so I am achieving a Z which is actually uncorrelated with my error term.

And if you recall the validity of the instrument requires it should not be correlated with my error term and that is what I am achieving here. But only problem with this Holtz-Eakin's et al.'s is that my lag length is still at 2. That means I am not able to exploit all the information available in the system because I do not know whether other orthogonality conditions are available in the

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the following

$$Z = \begin{pmatrix} y_{i,t-2} & \dots & \dots \\ y_{i,t-3} & y_{i,t-2} & \dots \\ y_{i,t-4} & y_{i,t-3} & \dots \end{pmatrix}$$

- So, there is a trade off between lag length and sample length

- To avoid this trade off for lag length and sample length Holtz-Eakin et al. suggested using second lag of $y_{i,t}$ as instrument

Then when we include third lag also as an additional instrument the instrument matrix looks like this. So, that means initially we lost only one observation for each panel inclusion of 3rd lag in the instrument matrix resulted in losing 2 observations for each panel. So, that means the moment we have enough lag we are losing too many observations. So, my length of the sample is going down, so that is why there is a 2 drop between lag length and sample length. And to avoid that Holtz-Eakin et al., they first of all suggested, we should use only second period lag as instrument that means y_{it-2} would be the instrument for each period and to avoid loss of information we will assume 0 for all these missing values. So, even though it looks arbitrary, we have achieved one great thing that expectation of this is 0.

Now Arellano and Bond's model is basically based on this Holtz-Eakin et al.'s idea where they say that if you want to have more orthogonality conditions or moment conditions we should include all available lags of the untransformed variable. So, Arellano and Bond suggested that include all available lags for the untransformed variable. So, for endogenous variable only second lag and higher is available but for predetermined variables even lag 1 is also available because that will be correlated with the error term only for $t = 2$ or earlier.

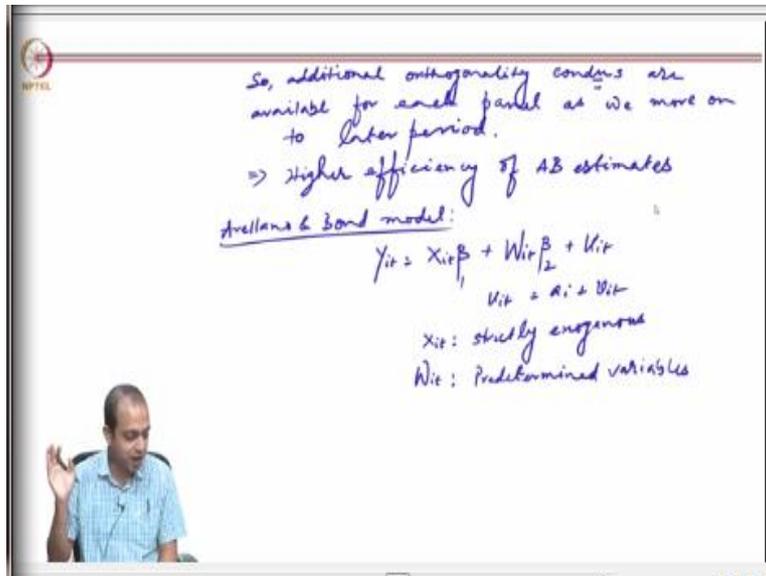
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$$Z = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \delta_{i1} & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & \delta_{i2} & \delta_{i1} & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \delta_{i3} & \delta_{i2} & \delta_{i1} & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Number of instruments
 $t=2 \rightarrow 1$
 $t=3 \rightarrow 2$
 \vdots

So, then in Arellano Bond's approach the Z matrix actually looks like this. So, the Z matrix is like this (as given in the diagram) then for the second period y_{1i} is used. So, earlier in second period there was no observation and now as suggested by Arellano Bond, you include all available lags. So that means this is following by the Arellano Bond's approach. Following the earlier style that means the y_{it-2} approaches in second period there was no instrument available. So, first y_{i1} is available and then for the third period what is happening? y_{i2} and y_{i1} . y_{i1} is followed by the standard IV approach and then y_{i2} is Arellano Bond's approach. Number of instruments in Arellano Bond's approach, so $t = 2$ implies a number of instrument is 1, when $t = 3$ the number of instrument is 2 and so on, so this is how. So, that means you can understand that how the number of instruments are increasing as we move on to the lateral period. So, additional orthogonality conditions are available as we move on to higher later period of each panel.

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So, that means, so what we can write that additional orthogonality conditions are available for each panel as we move on to later period. And this availability of additional orthogonality conditions actually results in higher efficiency of AB estimates. So, even though the original idea came from Anderson-Hsiao then followed by Holtz-Eakin et al., the contribution of Arellano and Bond is to use all available lags in the instrument matrix. That means more number of orthogonality conditions for each panel and that basically increases the efficiency of AB estimates. Now what we will do? We will formally then write the structure of Arellano Bond's dynamic panel data model because that is something what we estimate. So, this is basically the history of dynamic panel data model we have discussed how the Arellano Bond's 1991 model has been derived starting from Anderson and Hsiao and then by Holtz-Eakin et al.

So, the Arellano Bond model, model basically their model looks in this way

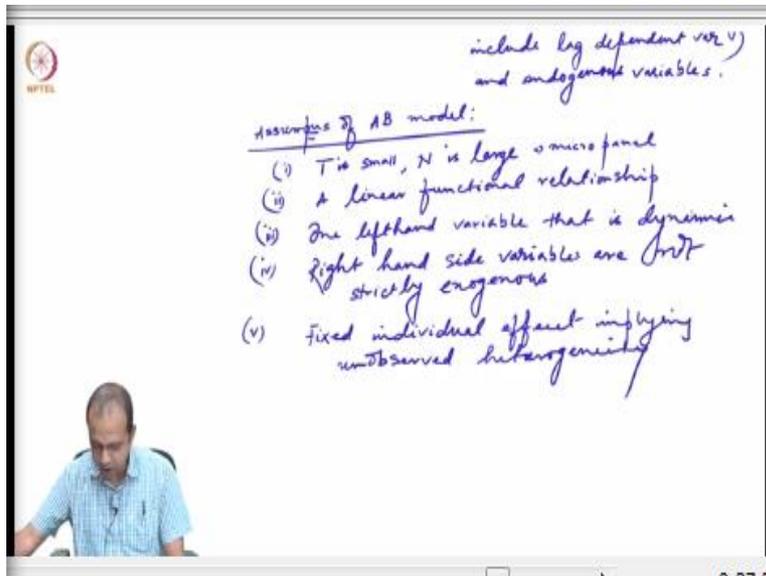
$$Y_{it} = X_{it}\beta_1 + W_{it}\beta_2 + U_{it}$$

$$U_{it} = a_i + v_{it}$$

X_{it} is strictly exogenous, that means they are not at all correlated with the error term.

W_{it} includes predetermined variables. That means it may include lag dependent variables.

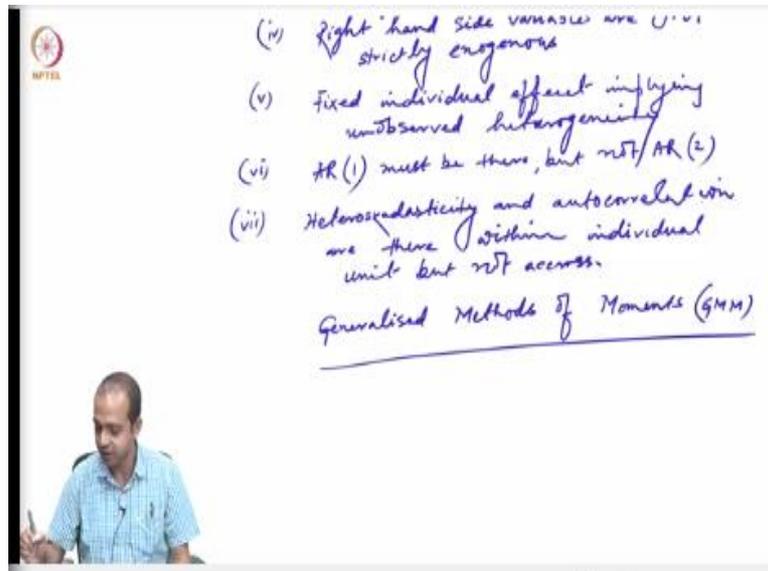
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Some important assumptions about of AB model. First and fore most assumption number 1, so this model is applicable only when T is small, N is large that is the context, we must remember this. Because as we have discussed that first differencing and fixed effect transformation both of them leads to Nickell bias and that bias is of order $1/T$. Now obviously this bias is higher when T is small if T tends to infinite, then this bias will also tend to 0. So, that means there is no point in discussing dynamic panel letter model if you already have t is sufficiently large let us say 100, 200, 250 like that. So, that is why we are discussing the Arellano Bond model in the context of when T is small number of time period is small and number of observations is large that means we are talking about a micro panel.

Second, this model is linear, a linear functional relationship between your dependent and independent variable. Third, one left hand variable that is dynamic, that means Y_{it} is dynamic, that means it includes Y_{it-1} in the right-hand side. Fourth, right-hand side variables are not strictly exogenous because it includes X_{it} as well as W_{it} . And W_{it} may include lag dependent variable as well as predetermined variable. So, that means it is a summation of predetermined variable and endogenous variable. Right hand side variables are not strictly exogenous. Fifth, then we have individual effect fixed which implying unobserved heterogeneity.

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Then assumption number 6, the way we have constructed, autocorrelation of order 1 will definitely be there. AR(1) must be there but not AR(2) because if you look at what we said that inclusion of lag dependent variable generates autocorrelation in the system. So, that is why AR(1) should be there because if AR(1) is not there that means basically dynamism is also not there significantly in the system and no need of going for all this dynamic panel data modelling and estimation at all. So, when we estimate dynamic panel data model, post-estimation we must carefully look at whether our result is showing autocorrelation of degree 1 or not. Then final assumption is presence of autocorrelation and heteroskedasticity within each individual unit but not across the individual. These are the assumptions of dynamic panel data model follow suggested by Arellano and Bond. And the estimation technique is known as Generalized Methods of Moment, in short GMM.

Now so far whatever econometrics we have learned based on ordinary least square technique where we minimize the sum of the error square. But this is the first time we are hearing a second approach of estimating our econometric model which is based on generalized method of moment. That means there is another approach which is called method of moments which is an alternative to OLS estimation. That is required because then only we will be able to understand, what is the generalized method of moments suggested by Arellano and Bond is.