

Applied Econometrics
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Lecture - 10
Instrumental Variable Estimation – Part X

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Hausman - Wu - Hausman Test

$$y_1 = \beta_0 + \beta_1 y_2 + \beta_2 z_1 + u_1$$

z_2 : instrument

$$y_2 = \pi_0 + \pi_1 z_1 + \pi_2 z_2 + v_2$$

H_0 : OLS actually a valid estimation method

H_1 : IV is valid estimation method

$\hat{\beta}^c$: consistent under both H_0 and H_1

$\hat{\beta}^c$: efficient under H_0 but not consistent when the H_0 is not true

$$J = (\hat{\beta}^c - \hat{\beta})' [\text{var}(\hat{\beta}) - \text{var}(\hat{\beta}^c)]^{-1} [\hat{\beta}^c - \hat{\beta}]$$

$\sim \chi^2_{df}$, no. of endogenous var.

Welcome to our discussion on instrumental variable estimation technique. And yesterday we were talking about we were discussing how to detect endogeneity in a model before we actually implement instrumental variable estimation. And the test what we are discussing was the Hausman test which is also known as Durbin Wu Hausman test because the test was developed by these three econometrician Durbin, Wu and Hausman.

So, quickly we will recap what we are doing in this test basically let us $Y_1 = \beta_0 + \beta_1 Y_2 + \beta_2 Z_1 + U_1$, Y_2 is the endogenous variable and this is z_2 which is instrument that we are using. So, basically what we are doing we were running a reduced form equation $y_2 = \pi_0 + \pi_1 Z_1 + \pi_2 Z_2 + \pi_3 Z_3 + v_2$. So, we are trying to get the this is \hat{Y}_2 and this is \hat{v}_2 and we were, what we are doing?

We are estimating the predicted value of the error term from this reduced form equation and we are putting the predicted value of the term in the equation as an additional explanatory variable. And then we are checking whether the coefficient of that predicted error term is significant or not.

If significant then we say that there is endogeneity if not then we will say that there is no endogeneity. So, that is the mechanism we discussed.

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So, once again we will use the same data set, we will see. So, this is our data set and what we will do? We will first regress education on experience let us say experience and for father's education or an mother's education. So, this is the model this is the reduced form equation for integration we are regressing on the excluded exogenous variables father's education and mother's education and included exogenous variable is experience.

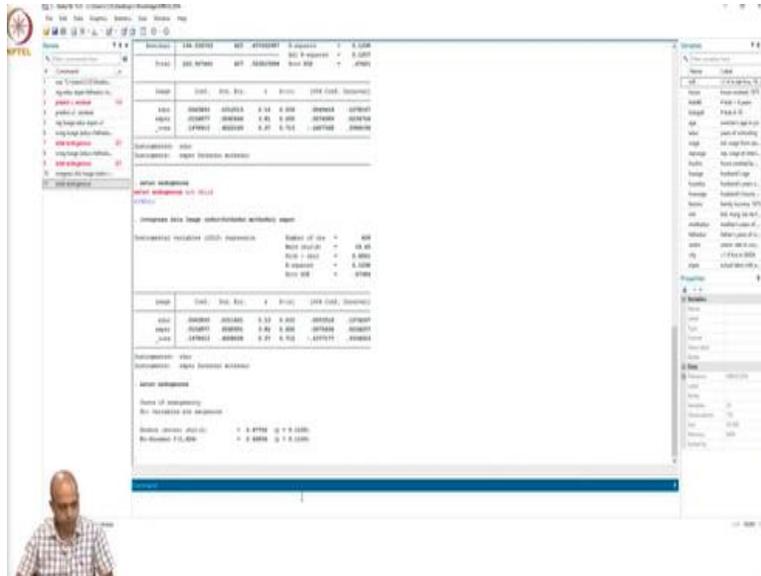
Then predict let us say V residual and then variable we already defined then what we will do? We will take predict some different name predict u_1 residual. So, then we will regress what we will do? Regress log wage on education, experience and this predicted error term. And if we look at the coefficient of this predicted error term the T value is 1.60 and the P value is 0.10 so that means this is not significant.

So, that means what would be our conclusion that education is actually not an endogenous variable in this particular context. Now the manual detection of instrumental variable estimation technique we can actually do by these using the statistical using the status command. In that case what we have to do? We have to first estimate the equation using IV reg then we will put education equals to father's education, mother's education and then we have experience so this is the model.

After this what you have to do estat endogenous that is what we discussed. So, this is our model IV reg then the command what we are using estat endogenous. Yesterday it was working, what is the problem of this command estat endogenous we will use instead of this IV regress 2 SLS.

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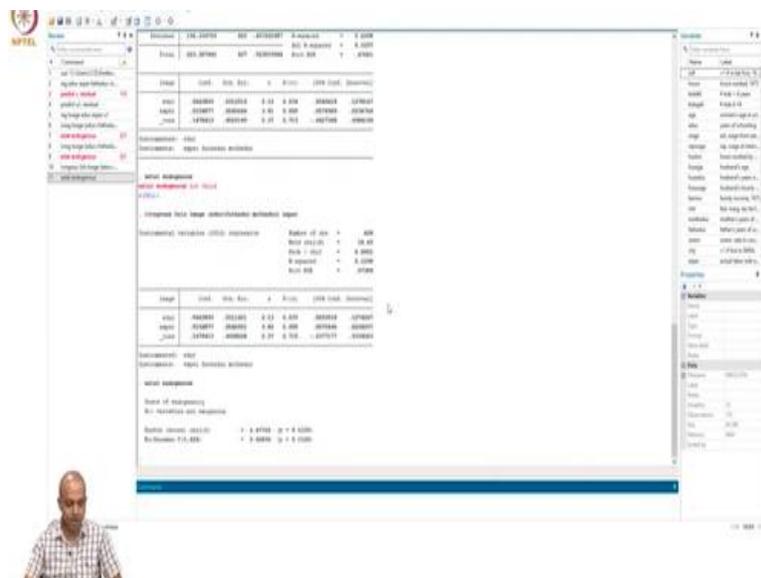
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So, instead of IV reg we have used IV regress 2 SLS and then this estat endogenous command is working. Now what we have to take this Durbin Wu Hausman statistic which is the F value and corresponding T statistic is 2.46. Now this F value this Durbin this Wu Hausman statistic is basically the F value is nothing but if we go back then it is actually the square root of this T value.

So, that means when we are doing it manually, we are using T stage T statistic to test the individual significance of the predicted value of the error term.

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And when we are using this Durbin Wu Hausman test that is the F statistic we are using where the value of this F is nothing but this square root of the previous T value. Now what exactly we are

doing here in this Durbin Wu Hausman test that is something we need to discuss. Now in Durbin Wu Hausman test what they suggest basically if you recall what is the philosophy of Hausman test.

We need to estimate the model using OLS, we need to estimate the model using IV and then we need to check whether the coefficients estimated value of the coefficients derived from the OLS and that from the IV they are statistically different or not. If there is a significant difference in between OLS and IV estimates then we will say that OLS is actually not an appropriate method of estimation.

So, let us say that the same model this model we are using fast OLS or null hypothesis is OLS is actually a valid estimation method that is the null hypothesis. And what is the alternative? Alternative hypothesis is that IV is valid estimation. Let us assume that β_1 that means consistent estimate of this β_1 that what we are doing let us say this is the endogenous variable. So, β_1 that which is consistent under both H_0 and H_A .

So, what we know that if OLS is actually a valid estimation technique that means if there is no endogeneity in the model then whether we apply OLS or IV both estimates would be consistent. So, there will not be any problem in consistency. So, β_1 that means is consistent under both H_0 and H_A and β_1 the efficiency property is efficient under H_0 but not consistent when the null is not true.

So, that means if OLS is actually not valid if there is endogeneity and if we still apply OLS then the resultant estimates will not be consistent, that is a problem of applying OLS. When there is endogeneity, we will lose consistency property. So, once again what I repeat that in presence of OLS if there is no endogeneity whatever we apply whether it is IV or OLS both the estimates would be consistent, consistency could be preserved.

But if there is no endogeneity and if we still apply IV then the efficiency property to some extent we have to sacrifice because IV estimates standard error are larger than the OLS standard error. And β_1 which is actually efficient estimator that means it is fully efficient under H_0 . That

means when there is OLS if we apply OLS fully efficient when there is no endogeneity. But it is not consistent when H_0 is true that means when there is endogeneity this beta hat e is actually not consistent.

So, that is why the test statistic is constructed in Wu Hausman test this is Durbin Wu Hausman test we are talking about. So, it is $\beta \hat{c} - \beta \hat{e} \text{ transpose}$, these are all in vector form. Variance of $\beta \hat{c} - \text{variance of } \beta \hat{e} \text{ inverse } \beta \hat{c} - \beta \hat{e}$ that is Durbin Wu Hausman test or in short DWH. And these follows χ^2 distribution with degrees of freedom equals to number of endogenous variable.

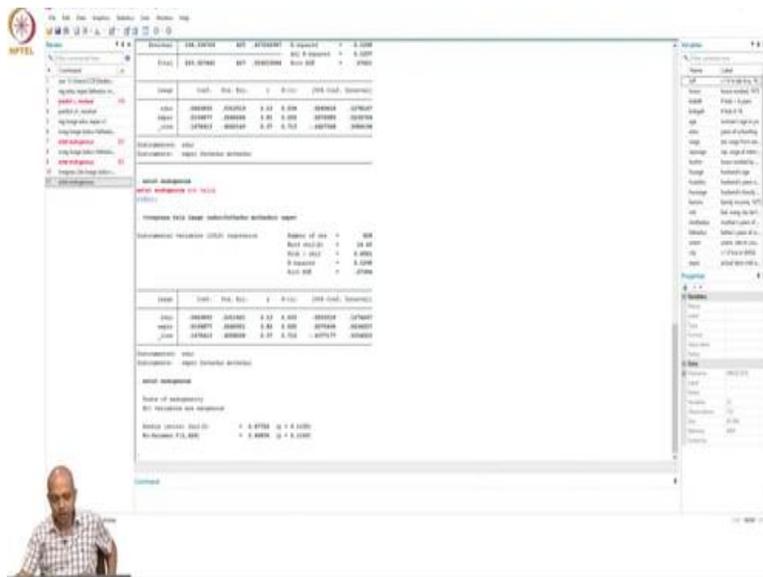
In this case we have only one endogenous variable that is why it is 1 here so this will follow a chi square distribution. So, these tests this Durbin Wu Hausman test is basically chi square test.

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So, that means here in this output we have one Durbin score and that Durbin score is actually coming from here. Because this is a chi square distribution and if you look at this is also a chi square distribution. But other Hausman test what we manually did that is basically F test which is basically the square root of this T. So, stata is reporting Durbin chi score as the Durbin Wu Hausman test.

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So, this test can also be implemented additionally what they say so that means we can say that this test is made of a test for whether OLS is valid or not rather than testing endogeneity. So, the entire philosophy of Hausman test is based on two alternative methods of estimating the same equation, 1 by OLS, another by IV and then we test whether there is significant difference between the beta hat OLS and IV because here we are writing beta hat c and beta hat e, c is consistent e is efficient.

Consistency is preserved under IV and this is under OLS. So, this is IV - OLS again this is variance of beta hat IV - OLS. So, the entire DWH test statistic is based on how far the IV estimates are different from the OLS estimates that is the philosophy that is the logic of Hausman test. So, this logic also we can implement using stata command and we will now see how to do it.

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So, we will say that let us say reg then education equals to father's education, mother's education then experience so we are estimating the model using IV then we will store this result EST store IV. Then we will use the same equation estimate the same equation using OLS. So, using command take education, experience. Then what we will do we will put Hausman IV and the command is constant, sigma this is the command.

So, we will first estimate the model using so this is what is working here. So, what I am saying we will first estimate the model using IV that means two SLS method then we will use the same equation by OLS. And then by putting this command Hausman IV constant sigma more we are actually asking stata to report the DWH test statistic. How is it doing? Look at here B that means these IV estimates are consistent under H_0 .

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The screenshot displays the EViews software interface. The main window shows the results of a regression analysis. The dependent variable is 'long' and the independent variable is 'short'. The regression equation is:

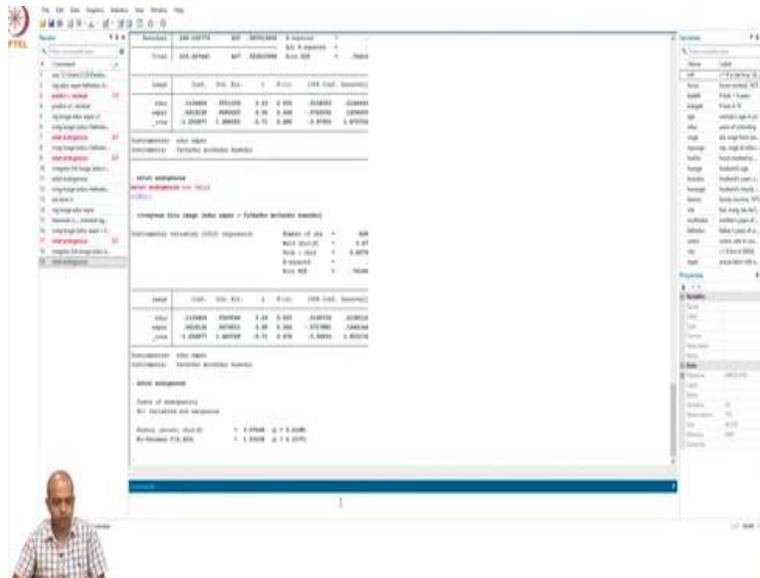
$$\hat{long} = 0.000000 + 0.000000 \cdot short + 0.000000 \cdot R^2 = 0.000000$$
 The F-statistic is 2.46. The p-value is 0.120000. The t-statistic is 0.490000. The adjusted R-squared is 0.000000. The Durbin-Watson statistic is 1.990000. The Breusch-Pagan test is 0.000000. The White test is 0.000000. The Ramsey RESET test is 0.000000. The Jarque-Bera test is 0.000000. The Skewness is 0.000000. The Kurtosis is 0.000000. The Jarque-Bera test is 0.000000. The Skewness is 0.000000. The Kurtosis is 0.000000.

So, what is H_0 OLS is appropriate. When is OLS appropriate? There is no endogeneity. So, when there is no endogeneity? IV is consistent OLS is also consistent. But this B is actually inconsistent under H_A . So, that means when there is endogeneity and if we apply OLS then that would be inconsistent. So, in our result this beta hat c and beta hat estat is denoting as b and capital B and then they are taking this and ultimately, they are arriving at the chi square value which is 2.46. (Refer Slide Time: 21:17)

The screenshot displays the EViews software interface. The main window shows the results of a regression analysis. The dependent variable is 'short' and the independent variable is 'long'. The regression equation is:

$$\hat{short} = 0.000000 - 0.000000 \cdot long + 0.000000 \cdot R^2 = 0.000000$$
 The F-statistic is 2.47. The p-value is 0.120000. The t-statistic is -0.490000. The adjusted R-squared is 0.000000. The Durbin-Watson statistic is 1.990000. The Breusch-Pagan test is 0.000000. The White test is 0.000000. The Ramsey RESET test is 0.000000. The Jarque-Bera test is 0.000000. The Skewness is 0.000000. The Kurtosis is 0.000000. The Jarque-Bera test is 0.000000. The Skewness is 0.000000. The Kurtosis is 0.000000.

And if we look at the start endogenous look at the same thing 2.46 so chi square is 2.46. So, almost this chi square sorry this is the F test this is 2.47. So, almost same value we are getting at. So, this is a chi square test statistic that means whatever we are getting here and the same things data is



Now if you look at what is the null hypothesis here, variables are exogenous and we cannot reject the null also so there is no endogeneity problem. But the difficulty is here they are saying all variables are exogenous but which particular variable is exogenous whether one variable is exogenous or all the variables are exogenous that we cannot say because it is based on an F statistic which is saying that whether the variables are significant jointly or not.

When there are two endogenous variable, we will get two reduced form equation from the two reduced form equation we will get two predicted value of the error term let us say \hat{v}_1 and \hat{v}_2 and these two predicted value of there are terms we will put it in the structural equation. And then we will Implement one F test to test whether \hat{v}_1 and \hat{v}_2 hat they are jointly significant. And what is the alternative of this F test at least one among them is significant.

So, if it is at least one then I do not know whether that education is endogenous what experience is endogenous or both of them are endogenous that is the limitation of this test. To overcome these what we have to do we have to use another command we have to keep in mind, here again we have to use IV reg.

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So, instead of this IV reg and then Iwendog we have to install this command. This is not connected I think we cannot install also. Anyway, after estimating this model if you put Iwendog and you have to put which particular variable to use for endogeneity. We have two endogenous variable so

after IV n log if you put education then that should work. Here I am not able to use this Iwendog because this command I am not able to download here since it is not connected.

So, here I am not able to connect this SSC install Iwendog. So, if it does not work nothing to bear it.

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So, what I am saying using this Iwendog command you can specify which particular variable you want to use for endogeneity. So, you can first use Iwendog education and see whether education is endogenous or not then after that you can use Iwendog experience to test whether experience is endogenous or not. So, this way you can actually test the endogeneity of a specific variable which was the limitation of the Hausman test. Since Hausman test uses the F test that is the problem.