Operations and Supply Chain Management
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Lecture - 6
Aggregate Planning, Transportation Model

In this lecture we will see the Transportation model for Aggregate Planning. In the previous lecture, we saw the linear programming formulation for aggregate planning.

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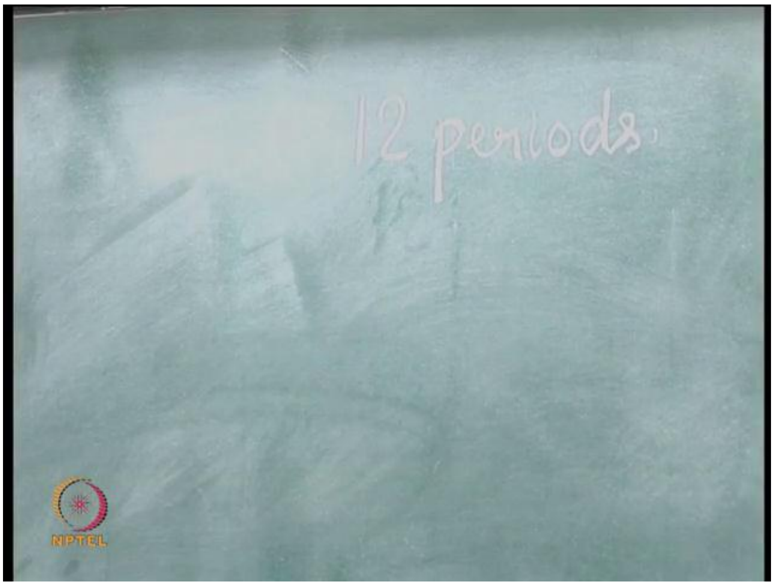

We considered demand data for 12 months or 12 periods, the linear programming formulation had several sets of constraints. We had one set of constraints where, we added the production to the beginning inventory, and then we subtracted the demand to get the ending inventory. We also had another set of constraints, relating the regular time production under utilization and capacity; and we had a third set of constraints involving the workforce.

Now, we are going to describe a transportation model, which does almost everything that the linear programming model does, there are a few differences which also we will see, as we move along. We take a much smaller example, to illustrate the use of the transportation model, and towards the end we also try and show how the transportation model would behave, if we considered the same example involving 12 periods.

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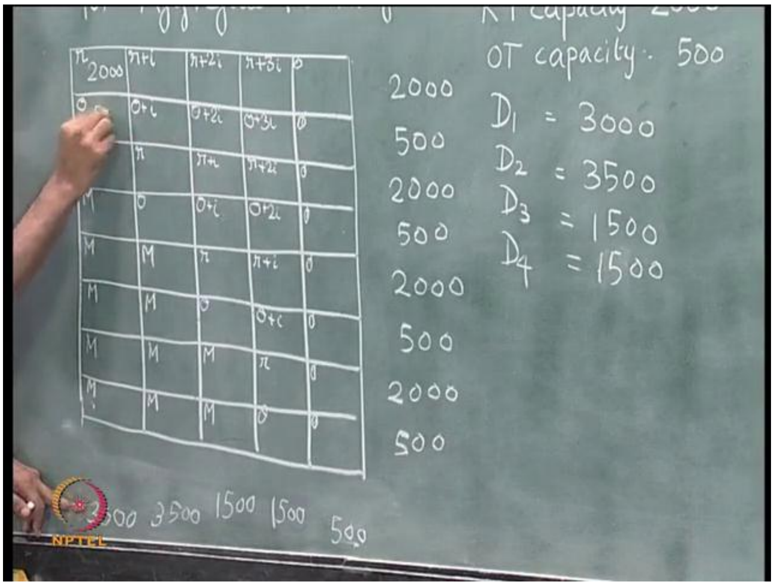

Now, for the sake of illustration, let us assume that we are considering 4 periods as against 12 periods, each period say is a month. And for the sake of illustration let us assume that, a regular time capacity and overtime capacity are the same and are known. So, we will have regular time capacity, let us say is 2000 and overtime capacity is equal to 500; in the last lecture we saw that it is customary to denote all these as man hours instead of units.

So, each month we assume has a RT capacity of 2000 and OT capacity of 500, so totally a capacity of 10000 man hours are available in the entire production period. We have 4 periods, so let us have the 4 demand values, demand is also represented in terms of man hours. So, let us say the demand values are, 3000, 3500, 1500 and let us say it is 1500, so let us now try and draw the transportation matrix for this.

So, the transportation matrix will look like this ((Refer Time: 03:27)), now each month there is a supply of 2000, which comes from regular time capacity and there is a supply of 500 that comes from overtime capacity. So, there are 4 periods, so each period has a regular time capacity and an overtime capacity, which are the supplies. So, we will say that 1st period 2000 comes from RT, 500 comes from overtime capacity, again 2000, 500, again 2000,500, again 2000 and 500.

As I mentioned in this particular example we are assuming that, the regular time capacity is 2000 in all the months and overtime capacity is 500 in all the months. In practice it can be different for each period, as we saw in the tabular method; the overtime capacity and the regular time capacity depend upon the number of days that are available. And therefore, they can be different for different periods, now we have to meet the demand of these 4 months with these capacities.

So, we start drawing the 4 demand columns, so the 1st period demand is 3000, 2nd period demand is 3500, 3rd period demand is 1500, and the 4th period demand is 1500. So, the 4 period demands are written as 3000, 3500, 1500 and 1500 respectively, now we have created the basic structure for a transportation graph. Now, we all know that we always solve balanced transportation problems, therefore we have to find out the total supply and the total demand.

Total supply is 2500 into 4 which is 10000, total demand is 6500 plus 3000 is 9500, so it is customary in transportation problems to balance it, and in the event where the total supply exceeds the total demand, we create a dummy demand column, so that the total supply becomes equal to the total demand. In situations where the total demand is higher, we would create a dummy supplier, so in this case the total supply is higher than the total demand, so we create another dummy which has a dummy demand of 500, so that we have a balanced transportation problem.

Now, we have assumed here that there is no initial inventory, so if there were an initial inventory, then we would have one more row and the initial inventory would act as a supply. Right now, we are assuming that at the end of the 4th period, we are not going to have a final inventory, so if there were a final inventory, then that final inventory would appear as another column in the transportation table. So, right now we are not assuming initial inventory and final inventory, otherwise we will have one more row and one more column in the transportation table, the dummy comes; so that, we balance the total supply and the total demand.

Now, we start writing the various costs, now if we are going to use the 1st month regular time capacity to meet the demand of the 1st month, then we assume that there is a regular time production cost which is given by R. If the 1st month overtime capacity is used to meet the 1st months demand, we are going to assume that it involves a overtime production cost of O, now I call this as O and to distinguish it from 0, let us say we call this as O.

If I meet the 1st months regular time production to meet the 2nd months demand, it means I am producing it in the 1st month, I am holding it in inventory for 1 month, so that I can meet the 2nd months demand. So, the cost becomes r plus i where I add 1 period inventory cost into this, similarly if I use this capacity to meet the 2nd months demand, it becomes O plus i, where O is the overtime production cost per man hour, because both demands and supplies are represented as man hours.

So, let us assume the cost are suitably represented as, so much per man hour, similarly if I use the 1st month's regular time to meet the 3rd month's demand it will become r plus 2 i and this will become r plus 3 i. Similarly, this will become O plus 2 i become O plus 3 i, now if I use this supply of 2000 which is regular time that is available in the 2nd period to meet the demand of the 1st period, which means I am producing in the 2nd period and I am going to meet the demand of the 1st period.

Now, that I can do only if I allow back order, it means if I have to use this to meet this, then I am not meeting the demand entirely in the 1st period, I am back ordering. And from the 2d month's regular time I am trying to meet the demand of the 1st month. Now, for the present, let us assume that we are not going to allow back ordering and therefore, it is a minimisation transportation problem. So, it is customary to put a big M, where M is large and positive, so that we do not have this allocation and this is not allowed.

Similarly, overtime capacity of the 2nd month to meet the 1st month's demand is not allowed, so automatically all these will become big M, because we are not allowing back ordering in our 1st assumption. Regular time capacity of 2nd month to meet 2nd month's demand will be r, this will be r plus i, this will be r plus 2 i, the r plus 2 i comes, because I am using the 2nd month's regular time capacity to meet the 4th month's demand, which means I am producing in the 2nd month. And I am keeping it in inventory for 2 periods, therefore this becomes r plus 2 i.

Similarly this will become O, this will become O plus i, this will become O plus 2 i, now from the 3rd month's capacity, I cannot meet the 2nd month's demand, because I am assuming right now that back ordering is not allowed, therefore all these will become big M. Now, I am using the 3rd month's capacity to meet the 3rd month's demand, so the cost will be r here, r plus i here, O here, O plus i here and I cannot use the 3rd month's capacity I cannot use the 4th month's capacity to meet the 3rd month's demand, so this will be big M, this will also be big M.

So, 4th month I will have r here, I will have an O here please note that these are O, so that I can distinguish them from 0. Now, I will need to fill the cost associated with this, in any transportation problem it is customary to fill the dummy row with cost equal to 0. So, we put cost equal to 0 for all of these, so I am going to write this as 0 and please note that this is o, while this is 0 and they are different. So, just to distinguish that this is 0, I am just going to add a like a phi, so that we know that this cost is 0 and this is not O.

So, now we have completed the cost for this transportation problem, and we now need to solve this transportation problem. Now, in earlier lectures in different courses, you would have been introduced to how to solve the transportation problem, there are many ways of solving it. Usually transportation problems are solved in two stages, in the first stage we try and get a good basic feasible solution; either through the North West corner rule, or through the minimum cost method, or through the Vogel's approximation method.

We have also seen in a different course earlier, that the Vogel's approximation method is used usually, because it gives a better objective function value compared to the North West corner rule. And then the second stage, we use the basic feasible solution that is obtained to try and get to the optimal solution. Now, let us try and apply the North West corner rule to this, North West corner rule is the simplest of the methods to get a basic feasible solution to the transportation problem. So, in the North West corner rule, we start with this 2000 ((Refer Time: 15:57)) and this 3000, so we allocate 2000 here, the balance 1000 is here, so there is a 500; so we would allocate a 500 here, then there is a remaining 500 which is there, which has to be met.

So, in this particular example, ordinarily we would put a 500 here for the North West corner, so ordinarily we would have met this through this, but since we have a big M and we cannot do any of these. So, we will not be able to proceed with the North West corner rule, but now for the sake of illustration let me assume, that I am going to start with a 500 here and then, I would possibly try and improve it or make it better. In this particular example I would not be able to, because I am not allowing back ordering, the 1st month's demand itself is 3000.

So, the 1st month's total production available is 2500, so in this case I will not have a solution, because I am not allowing for back ordering, so for the sake of illustration and for the sake of getting a very good solution.

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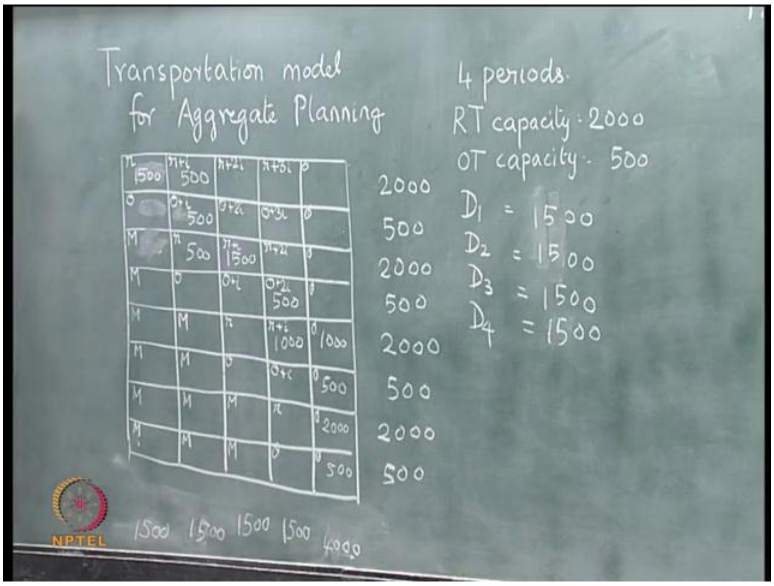

So, for the moment let me say that this demand is 2000, and let us say this demand is 3000, so that this demand is 2000 and this demand is 3000, so now what happens is your total supply is 10000, total demand is 3 plus 2 5 plus 3 8000, so the dummy becomes 2000, so now we have balanced it. So, now if we apply the North West corner rule, we have 2000 here, we have 2000 here, so we will use up all the 2000 here, so this will go and this will also.

In fact, let me also make a very minor change in the data, so that we can show few more things as we move along, so I am going to make this as 1500 instead of, this also I am going to make 1500, so that the demands are 6000, so this will become 4000. So, I am going to make the demands as all the 4 demands as 1500, now what happens is when I start solving using the North West corner rule. Now, before I start solving from the beginning, let me explain to you that the four capacities are 2500 in each month, with 2000 as regular time capacity and 500 as overtime capacity.

The 4 demands are now 1500 for me, so what I do is I apply the North West corner rule, so I first get 1500 out of it, so this column is completed another 500 remains here, there is a 1500 here, so I will put 500 out of this, so this is completely met. So, this will become 1000, 1000 is demanded 500 is available, so I put a 500 here this is also completely met, so I put another 500 here, so that this is met completely. Then I put a 1500 here, so this is met completely this is also met completely.

Then I go back and do a 500 and then I have a balance of 1000, so this is met, so there is a 1000 that comes out of this ((Refer Time: 19:15)), there is a 500 that comes out of this, there is a 2000 that comes out of this and there is a 500 that comes out of this, so it is balanced. So, now we have got the first solution using the North West corner rule, now let us try and see what kind of solution it is. Now, in transportation problem, if we have M supply points and N demand points, every basic feasible solution should have M plus N minus 1 allocations or less, but these allocations have to be independent.

If we have less than M plus N minus 1 allocations, then it represents a degenerate basic feasible solution, so in this particular example, we have 1 to 8 supply points and 5 demand points. So, we have 13, 8 plus 5 is 13, M plus N minus 1 is 12, so the basic feasible solution should have 12 allocations, now let us see 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 allocations we have. It is still a basic feasible solution with 11 allocations which is less than M plus N minus 1, so it is a degenerate basic feasible solution.

And the degeneracy comes because somewhere when we made this particular allocation, I think when we made this particular allocation, the balance of 1500 was met here and the demand of 1500 was met here. This particular allocation did not leave behind anything either in this row or in this column, therefore we started here with this 500 for example, when we made this allocation of 1500, we had a balance of 500 here which came here. So, only one row or one column was entirely filled or met by the allocation, but at this point both the row and column were met entirely.

And therefore, both these were met and we started from here, so we had 1 allocation less, and we also know that if we have a degenerate basic feasible solution. In the next method which is to optimise it, in the UV method or MODI method, Modified Distribution method, it is customary to put an epsilon to denote the place where there could be a degenerate allocation, and then we proceed.

Now, let us assume that this is the basic feasible solution, that we have using the North West corner rule, we have already learnt elsewhere that, if we had gone for a minimum cost method or a Vogel's approximation method, we may have got a better basic feasible solution. Right now I have not defined what the values of r and i etcetera are, so that we will not be able to do the minimum cost method or the Vogel's method here, unless we know the values of r, i, O and so on.

Whereas, we can do the North West corner, because North West corner depends only on the positions and the values that we actually know, so we are fine doing a North West corner. Now, we have to improve the solution to try and get to the optimal solution, so in a transportation problem the improvement is done using two methods, one is called the stepping stone method and the other is called the modified distribution method or UV method or MODI method and so on.

Now, let us try and look at the stepping stone method, now what is the stepping stone method do, stepping stone method believes that if this is not the optimal solution, then there is at least 1 allocation which is currently blank, which means which is currently non basic, which will appear in the optimal solution. So, stepping stone method will take each one of these blank positions, and it will try and enter a plus 1 or a theta to see whether there is a gain in the cost.

For example, ((Refer Time: 23:40)) this is a position which does not have any allocation right now, so if this were in the optimal solution, the belief in the optimal basis, the motivation or the belief in stepping stone is that; if I put a plus 1 here, then it will be advantageous, then it can enter the basis. So, if I try and put a plus 1 here, now to balance it I will have to put a minus 1 here, I have to put a plus 1 here and I have to put a minus 1 here. Now, the net gain will be plus O minus O plus i plus r plus i minus r.

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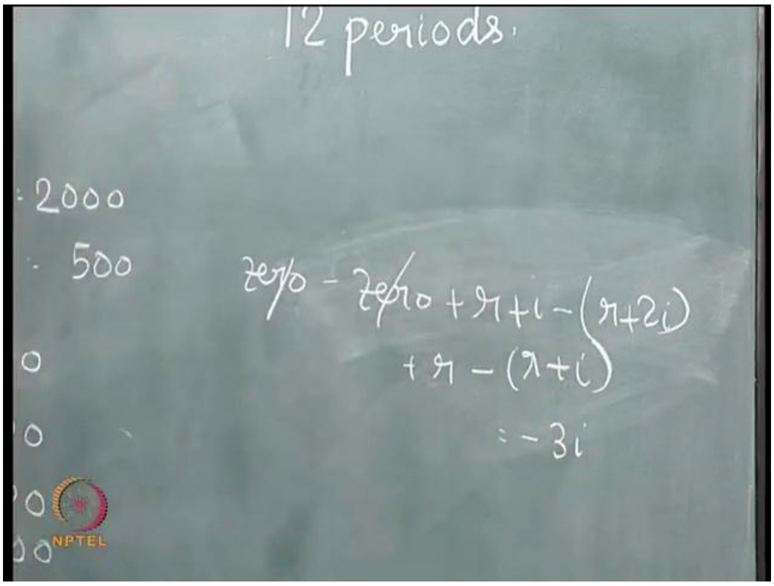

So, let me write the net gain, so plus O that comes out of this by putting a plus 1 I am incurring a cost of O, by taking away 1 I gain a O plus i, so minus O plus i by adding a 1 here, I am adding an r plus i; and then by taking away 1 I am taking away an r, so minus r. So, the net cost this is O ,this is not zero, so O minus O minus i plus i plus r minus r is 0, so we observe here that if we try and put a plus 1 here, now we realise that there is not there is not a gain the cost remains the same, so we try and do this.

So, stepping stone will try and see all the places where we do not have a variable we will try and put a plus 1, now let us try putting a plus 1 here for example, so if we put a plus 1 here then we have to balance it with the minus 1 with the plus 1 and a minus 1. So, the net will be r plus 2 i minus of r plus i plus r minus of r plus i, which will also be 0, so it is not going to be an advantage doing this. But, let us try and put a plus 1 here, now what happens I add a 0, I subtract a 0, so this will become a minus 1 here, become a plus 1 here, it will become a minus 1 here.

Now, because of degeneracy I would not be able to proceed, so I have to put an epsilon here, so this will become plus 1 here, this will become minus 1 here and then it will become ((Refer Time: 26:24)) this is not the thing, so plus 1 here minus 1 here plus 1 here minus 1 here, so that it gets balanced. Let me repeat it, if I put a 1 here, so to balance it I have to put a minus 1 here, I have to put plus 1 to balance here, then I am not able to move, because of degeneracy, so I add an epsilon here.

So, to balance it I would put a minus 1 in this epsilon, then I would put a plus 1 and then I would put a minus 1, now the net will be plus 0, here I have to be very careful, because some values involves 0 some values involves O, so this is plus 0. So, I am going to write it as plus 0 minus 0 plus r plus i minus r plus 2 i plus r minus r plus i, so this is r plus i r minus r plus r minus r is 0 i minus 2 i minus i is minus 3 i. So, this gives us a non zero value.

Let me repeat it 0 and 0 gets cancelled, ((Refer Time: 27:57)) this r will cancel with this r, this r will cancel with this r, so plus i minus 2 i is minus i minus i minus i is minus 2 i. So, there is a gain when we do this, so what we try to do is we try to put the maximum possible here, so it will become plus theta minus theta, plus theta minus theta, plus theta minus theta, again we realise that theta will become epsilon, because of degeneracy. So, this becomes theta minus epsilon, so the epsilon will shift from here to here and so on.

We know how to proceed in a transportation problem by doing this, the basic idea is we try and solve the transportation problem. Now, what have we learnt by this, if we apply the North West corner rule to this, we do not know the values of r i yet and we can proceed by applying a North West corner rule, because North West corner rule does not depend on the values of r i and so on. So, if we apply the North West corner rule and then, we apply the stepping stone method to solve it, we realise that under the assumptions that we have in this transportation table.

The important assumptions in this table are, we are considering only regular time production cost overtime production cost; we are considering inventory cost, we are not considering back ordering cost. So, we are considering only 3 costs regular time, overtime and inventory, right now do not have a beginning inventory, we have only capacities based on this. And if we apply North West corner rule here and then, we want to apply the stepping stone we realise that, all the positions remember that when we put 1 here the net was 0, when we put a 1 here the net was 0, but when we put a 1 here the net was minus i.

So, only when we put a plus 1 and compare the net gain or loss, and if there is a gain we can proceed towards the optimality, now when the net effect is 0 it is not going to help us in anyway. So, when we apply the stepping stone method after applying the North West corner rule, we understand that in all the possible non dummy positions, example ((Refer

Time: 30:39)) this, this, this, this, this, this, this, this, this and this and this. I am not including these, because these are not possible positions, because they have a big M, these are not possible positions.

So, for all the possible non dummy positions, one can easily show that the net by putting a plus 1 is only 0, so when we apply the stepping stone method under all these assumptions, it is not necessary to evaluate it at all non dummy non basic possible positions. Example this is a non basic non dummy position, but it is not possible, because there is a big M, this is a non basic non dummy position it is possible, because it has a value of O, but if we put a plus 1 here, then the net is only going to be 0.

So, the simplification is that, if we apply North West corner rule and we apply the stepping stone method, it is enough only to consider the vacant positions in the dummy, and try to use them as candidates for improvement. Because, every non basic non dummy allowable position is going to have a net equal to 0, so when we apply the North West corner rule and the stepping stone, it is enough to consider only ((Refer Time: 32:20)) this, this, this and this and proceed with the transportation.

Now, this simplification is called the Bishop's simplification and that is useful when we apply the North West corner rule and the stepping stone method, so that the computations that are involved in the transportation is minimised and are simplified. But, it is not always or absolutely necessary to use the North West corner rule followed by stepping stone, today with more and more packages available and spreadsheets solutions through transportation problems available.

One does not take the trouble of actually solving it like I am doing it on the board, one could simply put it into a solver and get the solution or use a spread sheet model to try and get the solution. But, in the earlier days when people were actually solving it by hand, the simplification was helpful, because it reduced the number of computations that were there. Now, let us come back and do a few more things, now one of the assumptions that we had of course, before I proceed one can now apply the stepping stone method or the MODI method, to try and get to the optimal solution for this problem.

Now, let us look at some more things, now let us relax one important assumption that instead of back ordering not allowed, let us allow back order and see what happens. So, when we allow back order, we want to remove all these, so if we allow back order which means. I can use regular time capacity of 2nd period to meet the demand of the 1st period. which means I back order by 1 period.

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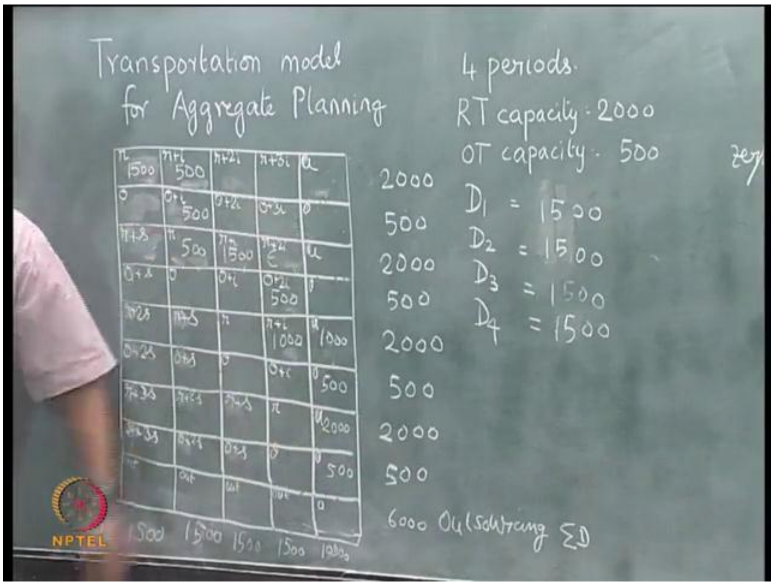

So, there is a regular time production cost of r plus a back order cost of s, so the cost becomes r plus s, 2nd periods overtime to meet this the cost becomes O plus s. Now, the 3rd periods r t capacity I am using to meet the 1st periods demand, which means I am going to produce here and back order for 2 periods. So, this will be r plus 2 s and O plus 2 s, and this will be r plus 3 s and O plus 3 s, now this is 3rd period capacity to meet the 2nd periods demand.

So, this will be r plus s, this will be O plus s, this will be r plus 2 s, this is O plus 2 s comes, because this is the 4th periods capacity is used to meet the 2nd period's demand, so there is a back order for 2 periods, so r plus 2 s O plus 2 s, this is 4th period capacity to meet the 3rd periods demand, so it will be r plus s and it will be O plus s. Now, the transportation model is complete, please note that your North West corner solution is going to be the same, it is not going to change, because North West corner is only based on positions.

So, now after from this ((Refer Time: 35:58)), under this assumption where shortages or back orders are allowed, we can now use this starting solution to try and get to the optimal solution of the problem. So, the next question that we have to answer is, is the

Bishop's simplification still valid for example, if I put a plus 1 here what is going to happen to me. Now, when I put a plus 1 here, it will be a plus 1, it will be a minus 1, it will be a plus 1, it will be a minus 1, so the net change will be r plus s minus r which is s, s plus r plus i minus r which is s plus i.

So, what happens is when I put a plus 1 here, there is a net change which is s plus i, it is not 0, we can still proceed by considering that the net change here is s plus i, and keep proceeding towards the optimality by applying the stepping stone method or by applying the MODI method. The only difference is that, if we apply the stepping stone method now, now we have to in this case there are 5 columns 5 demand points and 8, so it is 8 into 5 40.

The transportation should have 12 allocations M plus N minus 1, but in this case it has 11, but we have added an epsilon to get 12, so there are 12 basic variables, 28 non basic variables. So, if we apply the stepping stone method, then we have to evaluate the net increase or decrease for all the remaining 28 positions, because the simplification rule does not work when we consider back order. If we considered back ordering then it was enough to do it only in 4 positions, now we have to do it for all the 28 positions.

But, as I mentioned earlier, now most of the times we do not use North West corner rule or we do not solve by hand, if we are going to use a solver to solve this problem, it actually does not matter, whether we are going to gain advantage of the simplification rule or we are not going to gain any advantage, because of the simplification. But, then this is also possible now the transportation problem has 4 costs, cost of regular time, cost of overtime, cost of inventory and cost of shortage or back order. Now, let us try and do one more thing to it, now earlier when we did the linear programming and we did the tabular, we said there are 8 costs were considered.

Regular time overtime as one pair, inventory and shortage was as a pair, outsourcing and under utilization was considered as a pair, hiring and laying off was considered as a pair, out of these four pairs which add up to 8 cost, we have now considered four cost. Now, can we also try and add some more costs into the transportation table, now let us try and do that. Now, if we look at these capacities, we have now only two types of capacities which are regular time capacity and overtime capacity.

So, the total capacity is 10000, 2000 into 4 months plus 500 into 4 months, but when we did the linear programming formulation, we also assumed that it is possible to outsource and to try and get. So, outsourcing can be another capacity, so we add one more capacity here which is the outsourcing, so let us add outsourcing as another capacity. In the tabular method or the linear programming method, we did not have any limit on the extent to which we can outsource.

So, in principle we can assume that, the entire demand of 6000 can be outsourced, so we have the demand of 6000 as outsourced. So, in some sense the outsourcing capacity becomes sigma D, which is sum of the demands, now these capacities are already 10000. So, there is a demand 6000, so now your total capacity is 16000, the total demand is only 6000, therefore the dummy will become 10000. So, this part is sigma D which is sum of the demands, this part is sigma S which is the sum of the regular time and overtime capacities.

((Refer Time: 41:28)) This part is sigma S sum of the regular time and overtime capacities, this part is sigma D, the nice thing about this modelling is that, whether supply is greater than demand or demand is greater than supply. This total is always sigma S plus sigma D, this is always sigma S plus sigma D, and the moment we allow outsourcing even in the event where we have this capacity is less than the total demands, the balance can be got out of outsourcing.

Or going back to the example that we had, where I actually made a change here, initially I had put a higher value here and then when we were doing it, if we had put a 3000 here, and then we realises when we were doing it 2000 could come, 500 could come, because these had big M's back ordering was not allowed, when I had to actually change this. Now, in order to have a situation where, if my 1st month's capacity is 2500 and my 1st month's demand is 3000, if there is no beginning inventory, then there is definitely going to be a shortage of 500.

And now in order to meet that we have to do one of these two things, which is either allow back ordering or allow outsource. So, if we had allowed back ordering, then we would have something like r plus s here and I would have put some allocation here, if I had not allowed back ordering this would have been a big M, and it will come here the balance would be outsourced. So, at the optimum the balance would be outsourced, if I am not allowing for backorder, so outsourcing helps in multiple ways and prevents infeasibility to the transportation problem.

So, we build an outsourcing, now the nice thing as I mentioned is that, if we call this thing as sigma S, so this part is sigma S sigma D, this is sigma D sigma S, now it does not matter whether sigma D is bigger or sigma S is bigger. We will still be able to meet the entire demand, because outsourcing is available as an option, now the cost of outsourcing, so let me simply call it as o u t, o u t, o u t, o u t, so there is a cost of outsourcing.

So, if I want to outsource and buy something from the market to meet each months demand, then I go and buy only in that market, so I am not going to say plus i and so on. Every month, if there is a shortfall, if there is a way by which I can go and buy it from outside, I will assume that this is available all the time and therefore, I will buy only when it is required, so I have outsourced as a cost that comes. Now, the most important thing is to fill this, now ((Refer Time: 44:25)) this has to be 0, because if for some reason I do not have to outsource and I can meet all my demand using these capacities.

Then all these capacities and excess will have to go and sit here, only then the transportation problem is balanced, therefore I need to put cost equal to 0 here, this is very important, so we need to put cost equal to 0. The next question is, if I have this regular time capacity and for some reason I am not utilising this regular time capacity fully which means, if I have 2000 and in this row, I have not allocated 2000 here, there is some allocation that comes into the dummy.

For example, if you look at this, this is a regular time of 2000 available 1000 has been utilised, 1000 goes to dummy, which means this 1000 is not utilised, which means it results in a under utilisation cost, so there is an under utilisation cost here. So, there will be a under utilisation cost here ((Refer Time: 45:37)), there will be under utilisation cost here, there will be under utilisation cost here. And as I mentioned we are not going to put under utilization cost for overtime capacity, consistently we did not do that in the L P formulation also as well as in the tabular method.

So, there is no under utilisation cost for the overtime capacity, but there could be under utilisation cost for regular time capacity, this is a example where the 2000 regular time capacity is not utilised fully, it is used partly only. So, now this table uses 6 costs instead of eight inventory, shortage, regular time, overtime, outsourcing, under utilising, so it is able to use six costs. Now, we can solve this problem optimally, if we want to consider 6 costs, it does not consider and it is not possible to easily model hiring and lay off.

So, transportation model can be used up to six costs, we do not model hiring and lay off, if we also want to put hiring and lay off into the problem, then we have to go to the linear programming model, which is capable of solving it, transportation does not do that. Transportation also does not explicitly consider what is called the set up cost associated with the production, because every time we produce, there is a set up cost and a set up time that has to be used. And that cost cannot be brought in to the transportation model, the linear programming model that we saw in the earlier class, also does not include the set up cost.

But, there is a provision or there is a way by which it can be brought into the linear programming formulation. Now, before I wind up I can tell you that, if we go back to the examples that we considered, both in the tabular method and in the linear programming method. And if we modelled it as 12 periods which means, we will have 24 rows plus 1 initial inventory 25 rows plus 12 periods 12 columns, if we did not consider the outsourcing and under utilization, we considered only this, and solved a transportation problem that involved 25 rows, 12 columns and a dummy.

Then the solution that we would have got would be exactly the same solution that the L P optimum we get, if we added this outsourcing as one extra row and then, we solved it this way. And if we had solved a corresponding linear programming problem by considering 6 costs, including outsourcing and under utilization and not considering hiring and laying off. Then such a linear programming problem and the transportation problem will give the same solution, but linear programming has the additional advantage of being able to model hiring and lay off, which we would not be able to do in the transportation problem.

So, under the same set of assumptions, wherever if the problem it is possible to solve using transportation, then the same solution we will obtain, if we solve by linear programming under the same assumptions. But, L P model is little more versatile than the transportation model, if one can solve it by modelling it as transportation, it is always preferable to do that, because from operations research we would know that the time taken to solve transportation problems can be quicker than the corresponding L P's. So, transportation is preferred, wherever it can be modelled as transportation and transportation can handle a maximum of 6 out of the 8 costs. Now, I mentioned the transportation cannot handle set of costs explicitly. So, the next question is how do we model set up costs into production planning, and we will try and look at models to do that in the next lecture.