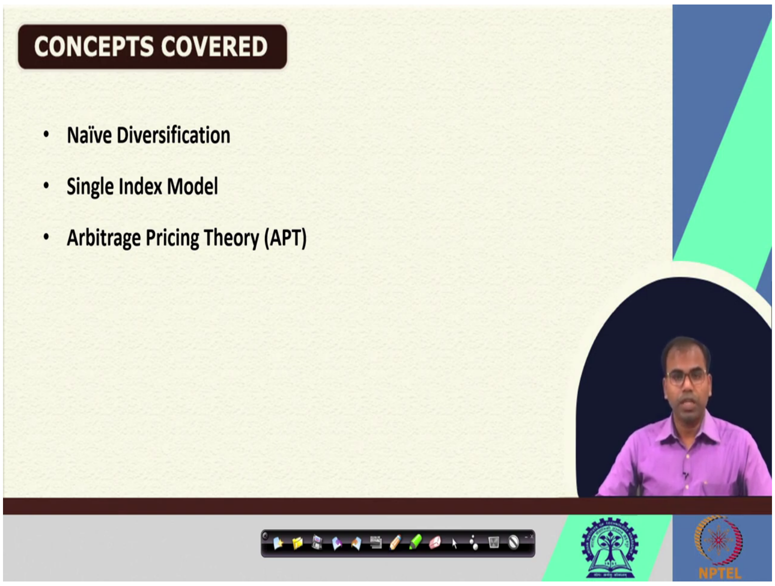
Investment Management
Prof. Abhijeet Chandra
Vinod Gupta School of Management
Indian Institute of Technology, Kharagpur

Lecture - 19
The CAPM and Index Models (Contd.)

Hello there, so to continue our discussion on portfolio theory and capital asset pricing model. In this session we will talk about Capital Asset Pricing Model and Index Model and we will continue our understanding of index model as a tool for investment management.

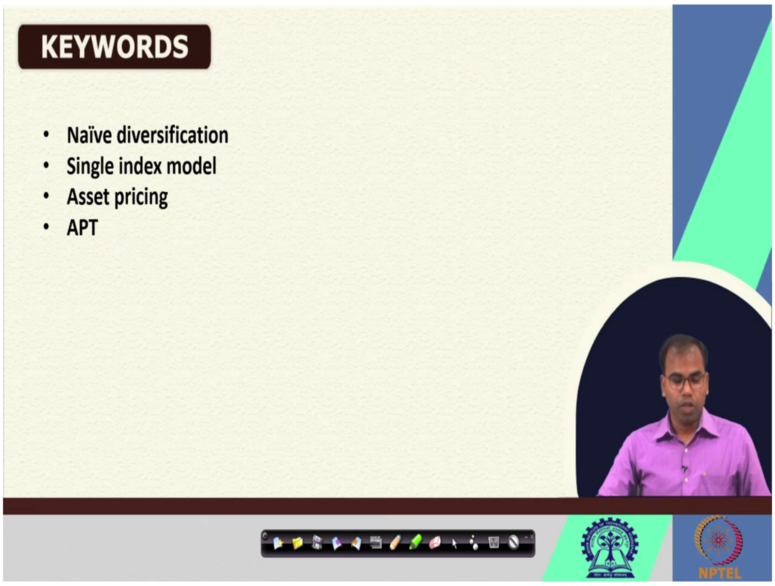
(Refer Slide Time: 00:45)



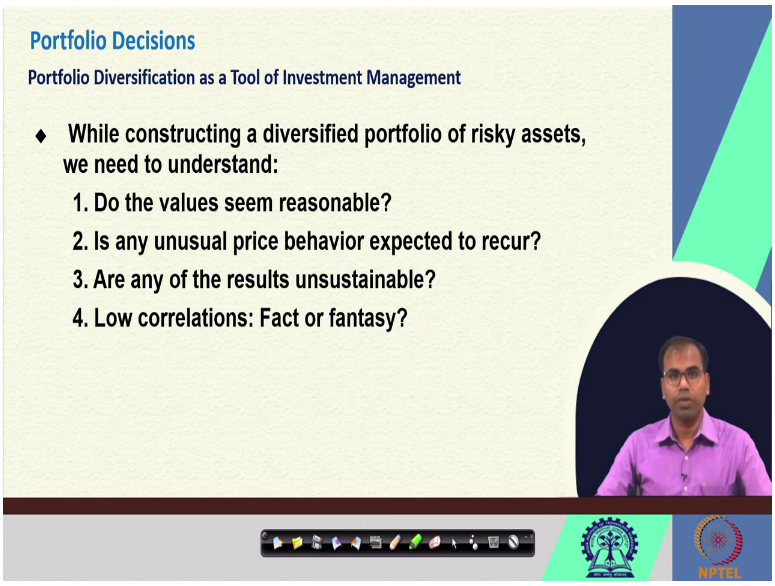
Now, we know that diversification is one of the tools through which investors try to reduce the risk for a given level of expected return or they try to increase the expected return for a given level of risk.

Now, there are different strategies through which investor try to calculate the expected return for a certain level of risk or try to gaze risk for a given level of desired return. Now, one of the strategies or concepts that we are going to discuss in this module is naive diversification and another is single index model through which investors try to understand the implication of risk on return.

(Refer Slide Time: 01:32)



(Refer Slide Time: 01:32)



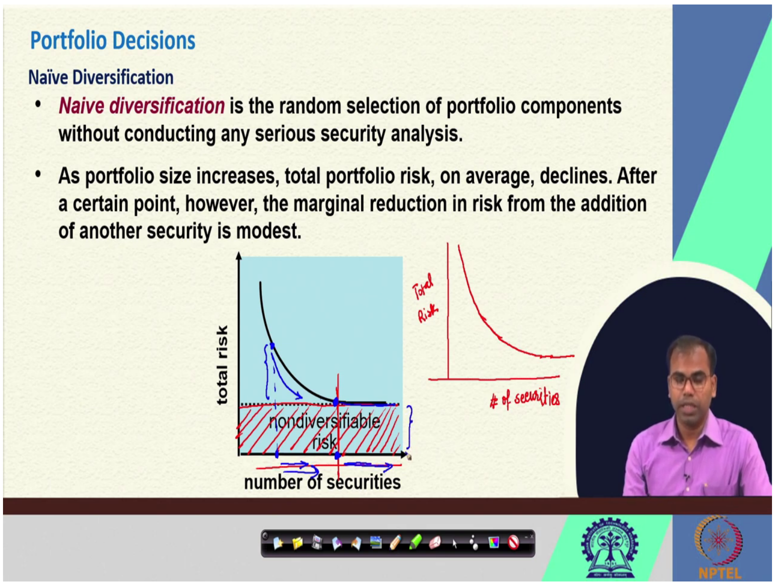
To begin with, if we try to understand the naive diversification, before that we need to understand what are the issues or questions that investors face when they try to diversify their investment portfolio of risky assets. We need to understand when we try to calculate the value of an investment to incorporate in portfolio diversification strategy; do the values seem reasonable for an investor?

As we understand, different investors can have different expectations, different lifespan, different investment horizon, different risk pairing capacity. So, do we have to consider their individual differences or heterogeneity into the diversification exercise? We also need to understand whether there is any unusual price behavior that can be expected to recur, particularly in markets with more volatile nature or more volatile behavior.

We can see that certain assets behave differently at different points of time because of so many different reasons. And when we bring in those results, bring in those events in the market, are those results unreasonable or unsustainable for any investor or any set of investors.

Of course, we need to understand the implication of correlations, particularly the cases of low correlations. Many a times we realize that the correlation coefficient between two or more assets could be very low and this does this this implication have serious inferences for portfolio diversification strategy. We should not be ignoring the fact that low correlation does not necessarily mean that those stocks are not good for including in the portfolio.

(Refer Slide Time: 03:40)



So, to understand the naive portfolio in the light of all these concerns let us try to understand to implement what naive portfolio diversification strategy is and how it can be related to the broad single index model or index model for that matter. So, as we understand naive diversification is basically the selection of portfolio components or selection of assets for constructing a portfolio in a very random haphazard manner where we do not carry out any serious security analysis.

In a typical diversification strategy, we need to bring in assets based on their risk and return characteristics. And when we try to do this exercise, we do we carry out serious security analysis before we include any assets in our portfolio. Naive diversification is contrary to this practice. Here we do not consider any serious security analysis and we just bring in portfolio components or assets to construct the portfolio.

The underlying assumption is as we increase the number of assets in the portfolio, the total portfolio risk on an average will decline. As we know that if we randomly bring more and more assets in the portfolio, the total variance of the portfolio will decline because of their correlation and other factors.

After however, we need not to forget that after a certain point, the marginal reduction in risk from addition of another security is modest and after certain point of time, this may be negligible or nil as well. The idea of naive diversification is to bring in as many securities as possible for the investor to reduce the risk.

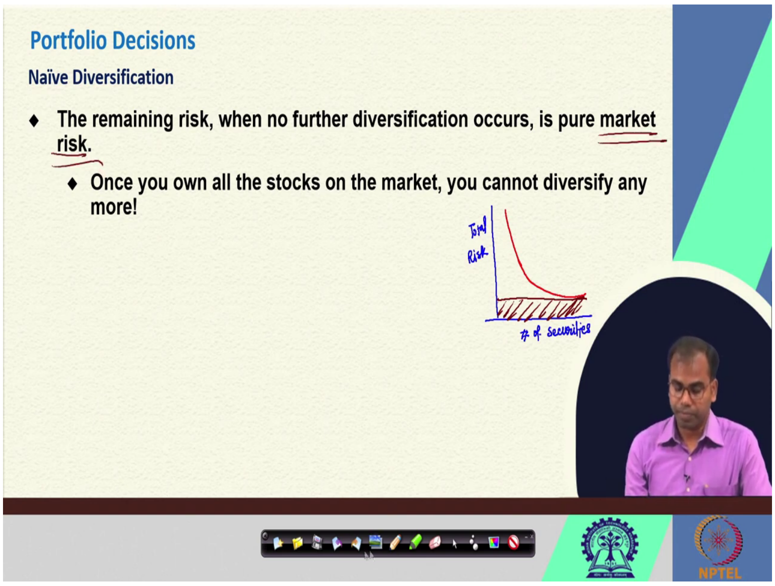
So, when we try to have this this concept called diversification where we have number of securities, as we can see in this graph and the portfolio risk on other axis, as we increase the number of stocks in the portfolio, our portfolio risk starts declining. And it so happens that after certain point, the portfolio risk the marginal portfolio risk decline will be stable and it may not change at all.

Now, this has implication for understanding of a return determination or return generation process. Now, we know that this is the level of risk that we cannot diversify or that we cannot avoid. So, if we are investing in this market irrespective of the number of security after certain point, we know that the risk cannot be reduced.

Suppose, I hold a portfolio here with this many number of securities, I know that I can I still have certain risk that I can reduce. And how do I reduce? I typically go to include more and more assets in the portfolio. So, as we move towards increasing more and more asset to the portfolio, I reduce the risk and I have the scope to reduce the risk to this extent by including this this this these many securities.

So, effectively this is the number of ideal securities or this is the optimal number of securities that I should include in my portfolio and I should be able to minimize the risk. After this, if I try to include even more securities in my portfolio, that will not have any effect on the total risk of the portfolio. Which means this component of risk is non-diversifiable risk. What implication does this have for understanding of risk return relationship?

(Refer Slide Time: 08:09)

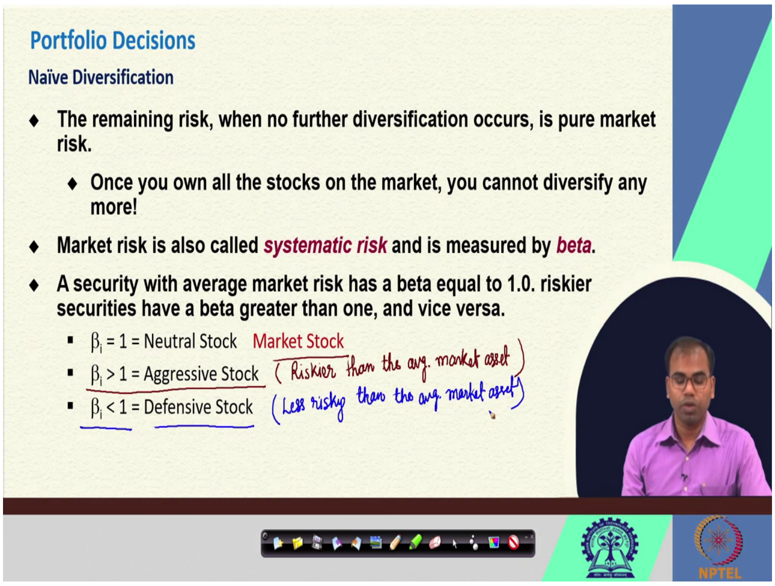


We know that when we have this remaining risk, when no further diversification occurs, is this risk is essentially driven by market forces or market factors. Once we own all the stocks in the market, we cannot diversify risk anymore.

As I was trying to show in the previous graph, where we have on one axis number of securities which means number of stocks or number of number of stocks in the portfolio or number of assets in the portfolio and on the other hand, I have total risk of the portfolio or total portfolio risk.

And we know that if we try to reduce the portfolio risk by including more and more stocks in the portfolio, after certain point of time, after certain number of securities, there will be no incremental marginal decline in total risk. And this particular risk is known as non-diversifiable or market risk in general.

(Refer Slide Time: 09:33)



So, when we have this market risk, this is also called systematic risk or beta in functional form. So, we have this systematic risk or beta measured in terms of measured by beta and it is basically that component of risk that we cannot diversify.

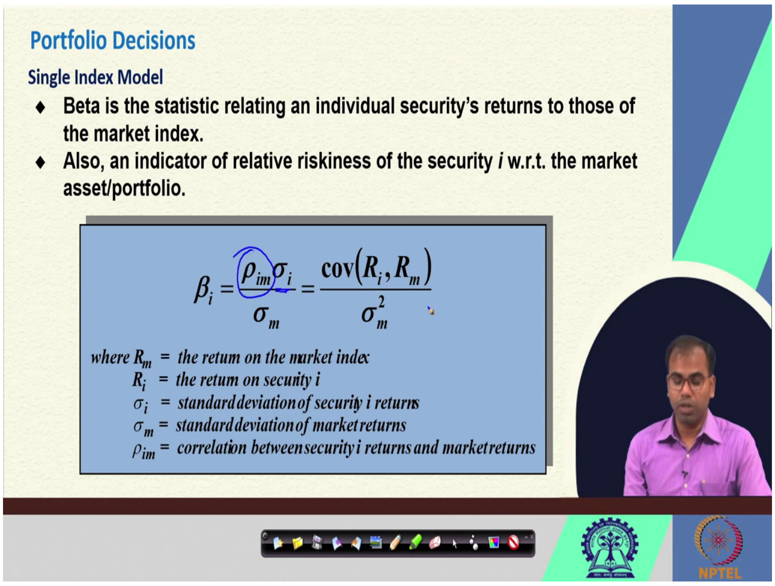
So, once we have this systematic risk identified or calculated using beta, we know what is the component of total risk, the component of total risk that we can diversify by increasing the number of securities in our portfolio and what is the component of total risk that we cannot diversify that is effectively the market risk, the systematic risk that we can measure through beta.

So, when we try to understand this concept of beta, we know that a security with average market risk has a beta of 1 and if any asset has more than 1 beta, any asset has a beta value of more than 1, then that security is relatively riskier than the market asset or average market risk. If the beta is less than 1 for any asset, we know that this asset is less riskier than the average market risk or average market asset.

So, the these three explanation for beta shows if an asset has more than 1 beta, then we know that these are aggressive stocks. Basically, these are stocks riskier than the average market asset or the average market stock. If the beta is less than 1, then it is a defensive stock and we can say that it is less risky than the average market asset.

So, if I have a security with more than 1 beta, then I can say that this asset, this security is more risky than the average market asset and if I have a security which with less than 1 beta, then we can conclude that this asset is less risky than the average market asset. And if it is having beta value of 1, it is the market asset or close to the market asset.

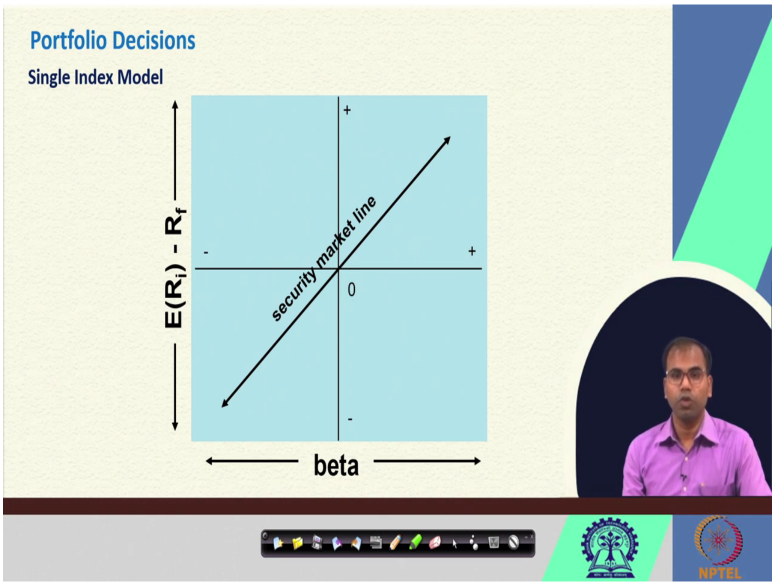
(Refer Slide Time: 12:41)



Now, as we understand the beta value is basically an indicator of relative riskiness of the security with respect to the market asset or portfolio and it is basically one statistic that is that is related to an individual securities returns along with those of the market index. So, if an asset or a security has a beta value of more than 1, it is more riskier than the market in relative terms and an asset with beta value of less than 1 is less risky than the market asset or market portfolio.

Now, the calculation for beta can be interpreted or can be done in this manner where we have the correlation between the asset and the market and the sigma of the individual security divided by the market. We can also write is like covariance of asset along with the market and the market variance and in this way, we can calculate the beta. Earlier in capital asset pricing model, we have seen how we calculate beta.

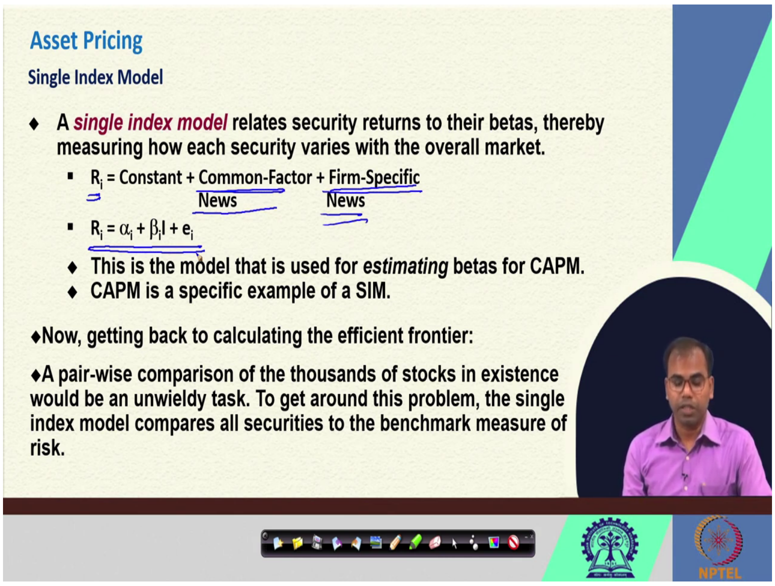
(Refer Slide Time: 14:03)



If we just go to the single asset, a single index model, we know that beta is the one factor; beta is probably the one most important factor that determines the riskiness of the asset and subsequently the expected return of the asset.

So, this particular single index model is helping us to understand the security market line which is basically the relationship between beta and the return that asset is generating over and above the risk free rate of return.

(Refer Slide Time: 14:41)

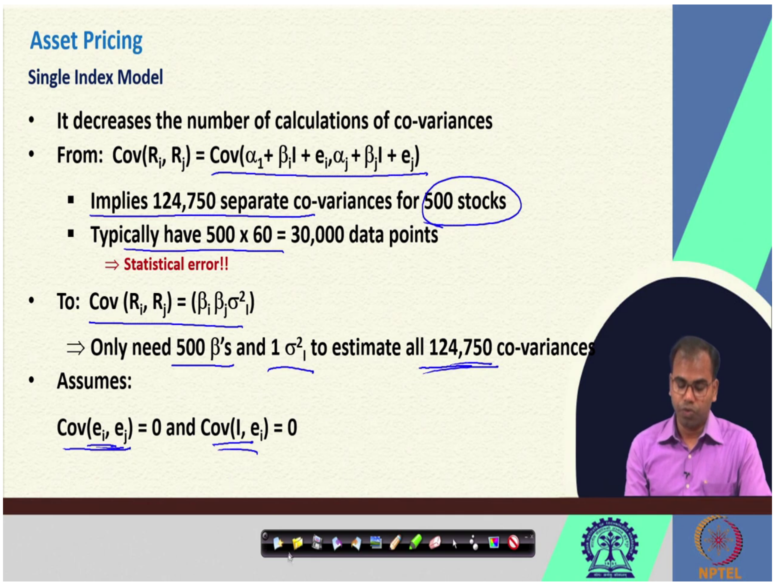


So, effectively when we try to understand the single index model, we know that a single index model relates security returns to their beta. Basically, the relationship between risk and return and thereby they measure how each security varies with the overall market.

For example, if I have a security i and I need to calculate the return of asset i and we try to understand what will be the impact of common factor let say news that is common factor for all asset. And then we have firm specific news which is related to the related to the firm i or stock i or asset security i.

We know that we can generate we can calculate the expected rate of return following this functional relationship between return on individual security i, the index model and expected rate of return can be calculated. And this is the basic or baseline model that we use for estimating beta for capital asset pricing model.

(Refer Slide Time: 15:57)



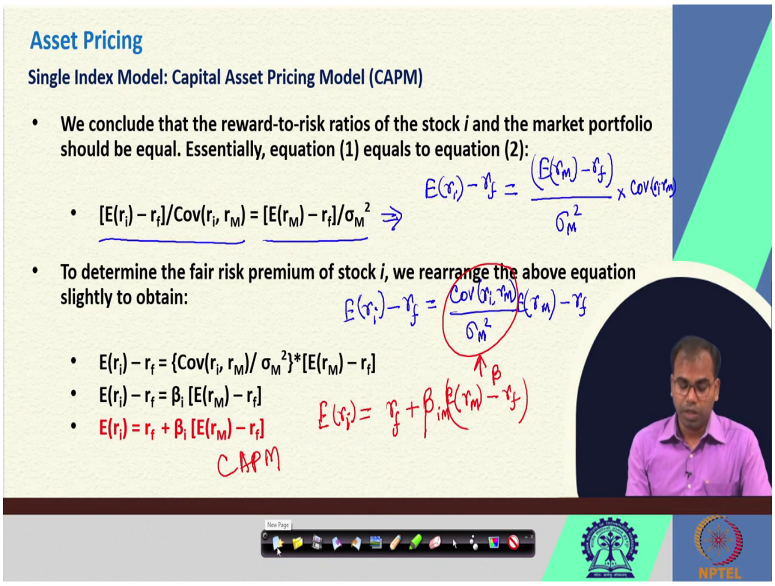
In fact, if you could recall couple of sessions earlier, we have seen that this kind of model can be used to understand the capital asset pricing model or can be explained with the help of capital asset pricing model.

Now, before we go to that capital asset pricing model once, we know that when we have single index model there are certain issues because it decreases the number of calculations of co-variances. Let say for example, if we try to calculate the covariance between asset i and j which is basically two assets i and j. We can calculate it in such a way that covariance of two assets are available. And if we have a portfolio of 500 stocks we need to calculate 124,750 separate co-variances.

And then when we use this kind of calculation typically, we have 30,000 data points and this might lead to a statistical error while calculating the expected rate of return based on such model. But if we have this beta based model then co-variance between R and i and j can be calculated using beta based model where only 500 betas are required 1 sigma is to estimate 1,24,750 co-variances.

And here it is assumed that covariance between expectation of i and j returns is 0 and the epsilon i and epsilon j and covariance of the index and epsilon i is also 0. And with this assumption we can use this beta based model or single index model for calculating the co-variances as well.

(Refer Slide Time: 17:53)



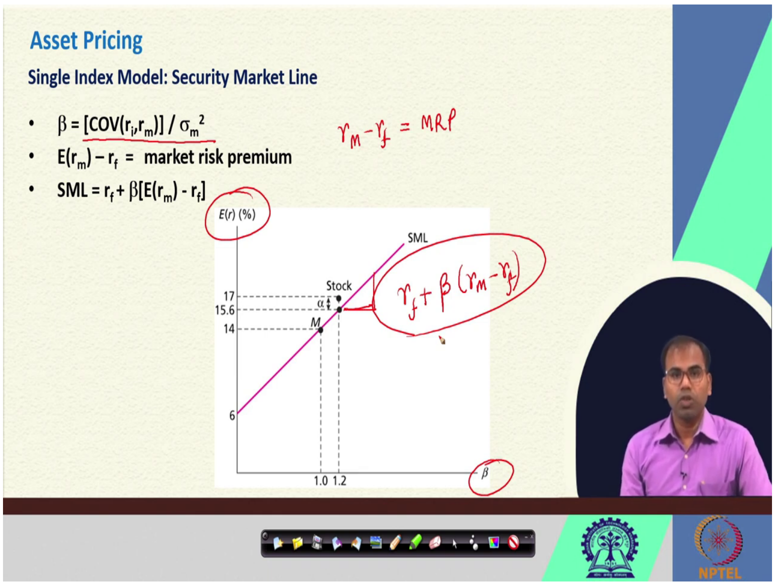
Now, just let just go back and recall how we calculate the expected rate of return using capital asset pricing model in a typical modern portfolio theory. We know that the reward to risk ratio for stock i and the market portfolio is considered to be equal. So, we have reward to risk ratio for individual assets and reward to risk ratio for the market given.

And when we try to rewrite this reward to risk ratio of individual asset and reward to risk ratio of the market asset, we can rewrite this function in such a way that expected rate of return minus risk free rate of return is equal to expected rate of return on the market minus risk free rate of return divided by market variance into covariance of r i r m.

Now, if we rewrite this once more we have this function that can be expressed as expected rate of return on asset i minus r f covariance between r i which is return on asset i and r m by variance of the market and expected rate of return on market asset minus r f. And this particular function, this particular component is indicated by beta in a single index model.

And we have this form where we can express this expected rate of return on asset i is equal to r f plus beta which is nothing but the covariance variance of the asset and the market into r m or expected rate of return on the r m and r f. And this is basically the capital asset pricing model that we work with. Now, once we have this kind of capital asset pricing model.

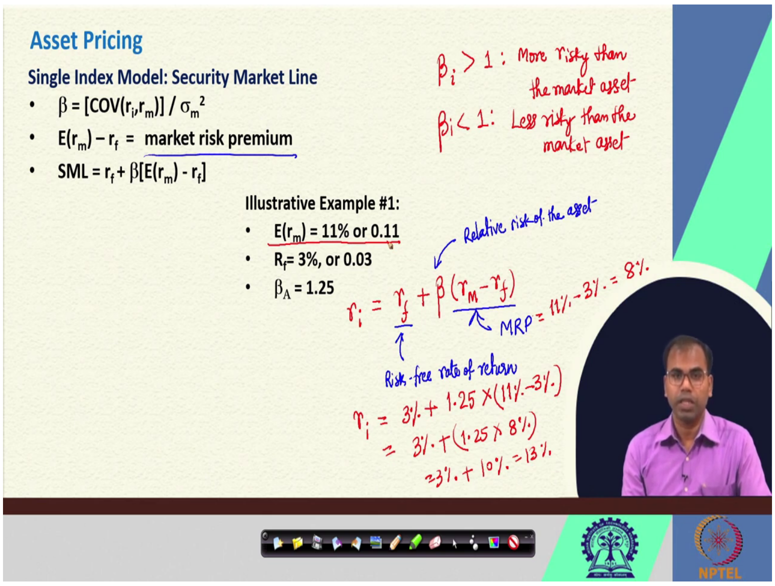
(Refer Slide Time: 20:24)



We can use this for single asset single index model and derive the security market line. Now, we know that beta can be interpreted or expressed in terms of covariance of the asset with the market divided by variance of the market and expected rate of return on market asset minus r f which is r m minus r f is market risk premium.

So, security market line or the slope of this curve relating expected rate of return on asset and beta can be expressed in terms of r f plus beta into r m minus r f in general. And this can be used to draw the security market line. We will just take a look at one example where we have some hypothetical numbers related to the security related to the beta expected rate of return and the market return market return and thereby, we can try to find the security market line and accordingly we can calculate the risk free rate of return on the expected asset.

(Refer Slide Time: 21:29)



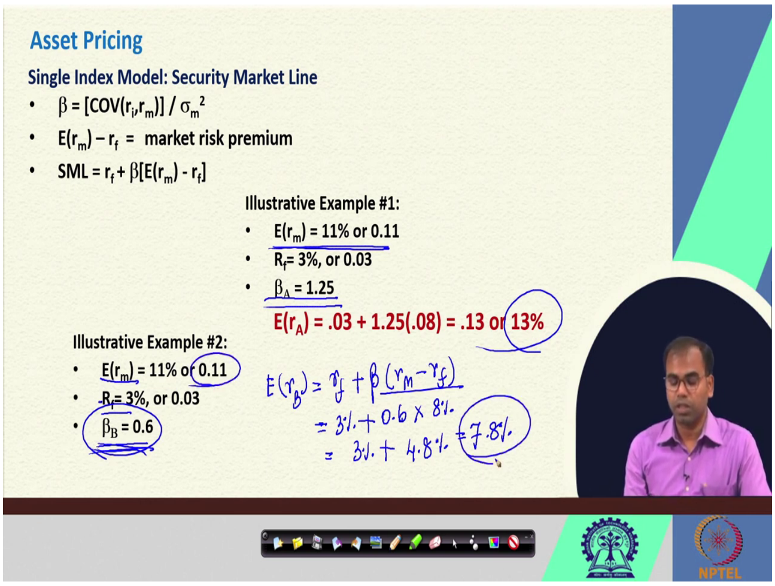
Now, suppose that this is the basic notations and we have an example where we have expected rate of return on the market to be 11 percent, risk free rate of return to be 3 percent and beta of the asset A for example, is 1.25. As we understand when we have the beta of the asset A any asset greater than 1 let us say beta of i greater than 1 this is considered to be more risky than the market asset. And if beta is less than 1 then we can say that it is less risky than the market asset.

With this inference if we can try to calculate what will be the r i the approach will be r i should be equal to r f plus beta into r m minus r f which is nothing but expected rate of return on asset i is equal to risk free rate of return that is r f relative risk of the asset which is beta. So, relative means it is relative to the market asset and market risk premium.

So, once we have this kind of formula, we all we do have all we need to do is to plug in the numbers and find the value. So, r i will be 3 percent plus 1.25 into 11 percent minus 3 percent. So, here it is important to understand market risk premium will be 11 percent minus 3 percent is equal to 8 percent.

So, this will be 3 percent plus 1.25 into 8 percent that gives us 3 percent plus 10 percent is equal to 13 percent. So, this should be the rate of return expected rate of return on asset where we have a situation where r m is equal to 11 percent, r f is equal to 3 percent and beta of the asset is given to be 1.25.

(Refer Slide Time: 24:47)



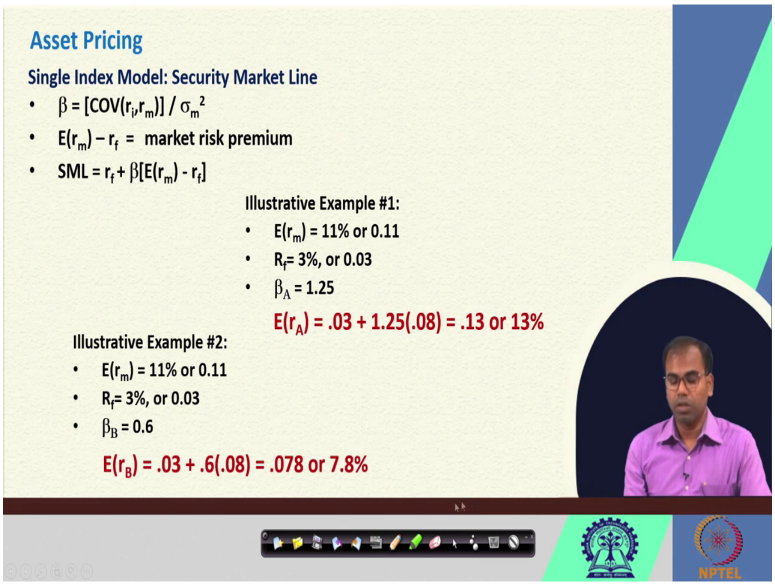
We can understand the value to be 13 percent. In a similar fashion if we have a scenario where there is another asset and this asset has a beta of 0.6 which is effectively less than 1 which means it is less riskier than the market asset.

Then we can follow the same approach where expected rate of return on asset B can be calculated using r f plus beta into r m minus r f where we have r f to be 3 percent plus 0.6 into 8 percent which is coming from r m minus r f, we have we are given r m and r f. So, we have 3 percent plus 4.8 percent. So, we have 7.8 percent of return if the beta is given to be 0.6.

Now, we can simply relate we know that higher the risk higher should be the return, lower the risk lower should be the return. Here beta shows that it is more risky than the market asset because its value is 1.25. So, it should generate more return than the market asset. Market asset is generating 11 percent return.

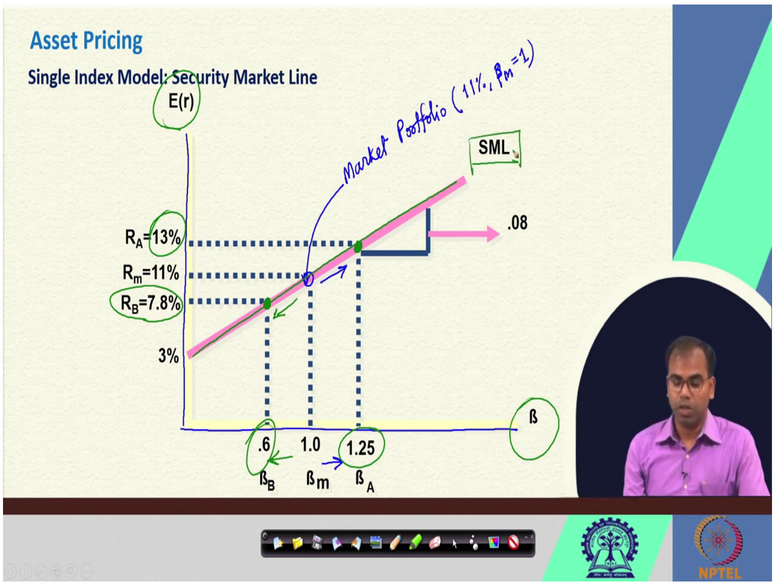
This asset is generating 13 percent return. And if the beta is less than 1 which is the case here which implies that it is less risky than the market asset and if it is less risky then it should generate less return than the market asset. Here market is generating 11 percent and this asset is generating just 7.8 percent.

(Refer Slide Time: 26:47)



So, this way we can fit it into the security market line.

(Refer Slide Time: 26:52)

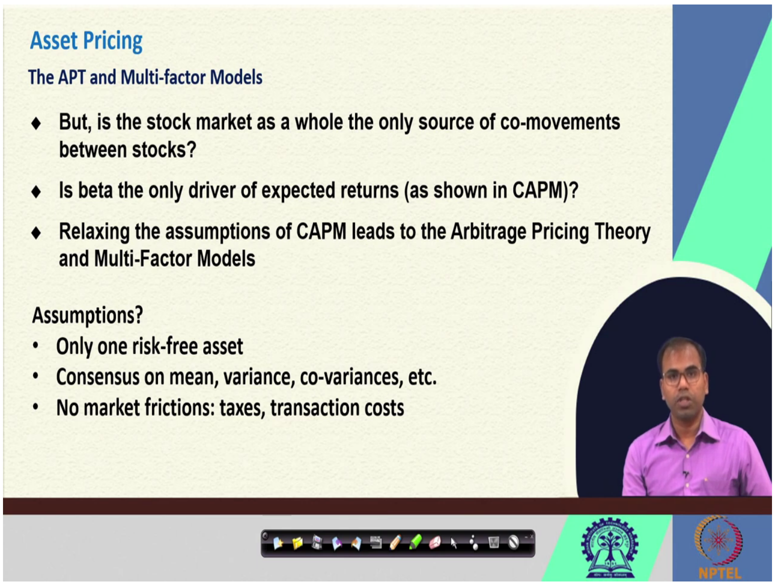


And if you try to continue to understand the security market line for an asset here, we can see that if there are two axis return and risk we know that there is this asset called market asset. So, this is the market asset lying here or market portfolio we can say where we have 11 percent of return and a beta value of 1.

So, any asset that is lying above here with a beta value of more than 1 should be generating more return and if an asset is lying somewhere below than this asset and having less than beta value of less than 1 then it should be generating less return, like in this case here we just calculated with beta value of 0.6 this asset is generating 7.8 percent return and with beta value of 1.25 this asset is generating 13 percent of return which is higher than the market return.

So, this particular relationship between return and risk of the asset is expressed to be securities market line or SML.

(Refer Slide Time: 28:22)

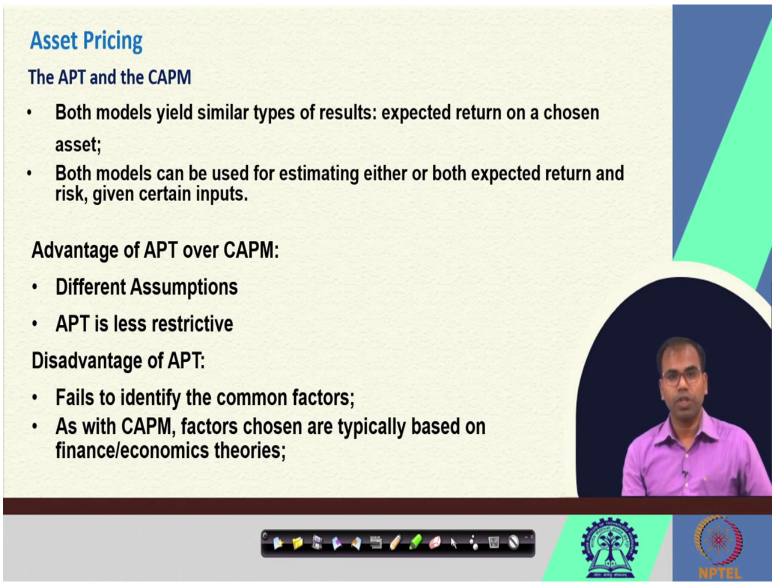


If we infer our understanding from security market line, we know that beta is the single factor that is effectively affecting the return generation process in all cases. But in the real world is the stock market as a whole the only source of co-movement between stocks, if we try to understand this or even if we try to think about the beta being the only driver of expected return as we have seen in capital asset pricing model where expected rate of return on any asset is a function of risk free rate of return plus beta into market risk premium.

So, is beta the only driver of expected rate of return. And at the same time, we understand that capital asset pricing model is based on several assumptions. So, if we relax those assumptions it may lead to the development of what we known as multi-factor model or to certain extent arbitrage pricing theory.

Now, assumptions here for CAPM is there is only one risk free asset, there are consensus on mean, variance, co-variances and other factors all investors agree to that, there is no market frictions such as there is no taxes, there are no transaction costs. With these assumptions capital asset pricing model was developed. And if we try to relax these assumptions, we can find multi-factor asset pricing model or arbitrage pricing model.

(Refer Slide Time: 29:53)

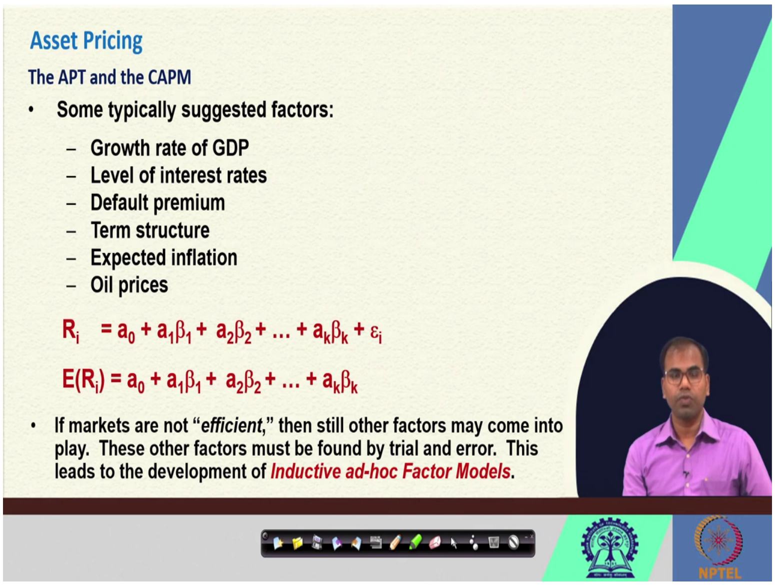


So, just to touch upon the arbitrage pricing model, we know that arbitrage pricing model also yield similar type of results as capital asset pricing model where we are able to generate expected rate of return on a chosen security for given inputs and CAPM does the same job, when we try to understand at the advantage of arbitrage pricing theory over capital asset pricing model.

We know that arbitrage pricing theory is based on different assumptions where not all the investors need to agree on the same risk factor or risk driver which is beta in case of capital asset pricing model, but in arbitrage pricing theory it could be anything as perceived by investors or set of investors. And that is why it is also known as arbitrage pricing theory is less restrictive it does not restrict investors or analysts to consider only certain risk factor in that sense it is more flexible.

However, given the advantages arbitrage pricing theory has some disadvantages as well because it does not identify common risk factor which is available which is common for all market participants. Because every investor might have unique set of risk factors and that is what arbitrage pricing theory suggests. As with capital asset pricing model factors are chosen typically based on finance and economic theories.

(Refer Slide Time: 31:21)

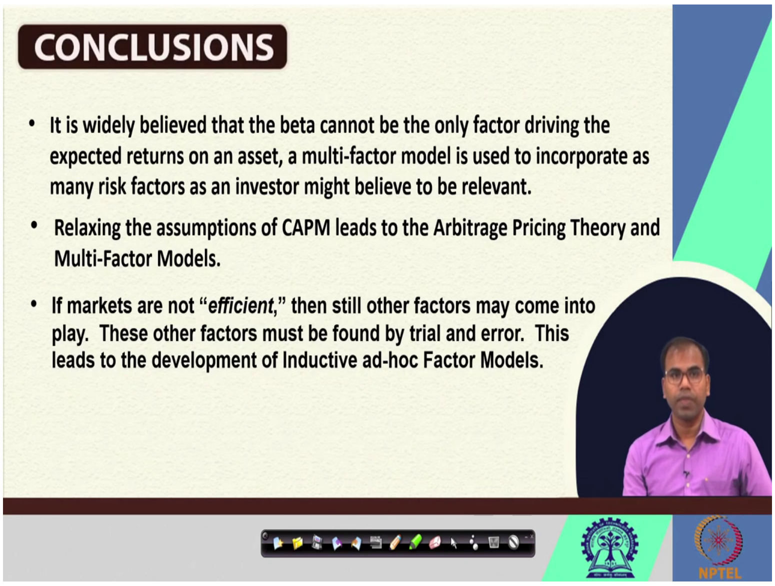


For example, if we see some typically suggested factors that might act as risk factor in arbitrage pricing theory these are growth rates of GDP, level of interest rates, default premium, term structure, expected inflation, oil prices, currency exchange rate, investor sentiment could be anything.

And with this we can develop a model like this where we have expected rate of return on asset i as a function of beta 1, beta 2, beta 3 till beta k where every beta represents unique risk factors and this can help us in generating expected rate of return on any asset.

However, if the markets are not efficient then still some factors may come into the play and these other factors must be found by trial and error. And this approach of finding risk factor by trial and error leads to the development of inductive add-hoc factor model which is beyond the scope of the coverage.

(Refer Slide Time: 32:22)



So, we stop here, we know that the beta cannot be the only factor driving the expected rate of return on asset and that is why multi-factor model is more suitable where we can include as many factors as investors can to incorporate in the return generation process. Here we need to relax the assumptions based on which capital asset pricing model was developed and that is why arbitrage pricing theory or multi-factor model seem to be working better than other typical model. And with this I conclude this session.

Thank you very much.