Transcriber's Name Maria Priya

Applied Econometrics

Prof. Tutan Ahmed

Vinod Gupta School of Management

Indian Institute of Technology, Kharagpur

Module No. # 02

Lecture No. # 17

Expectation, Variance, Covariance

Hello and welcome back to the lecture on Applied Econometrics. So, we have been talking about probability distribution. So, we have learned the basic concept of what we call a probability distribution, how we constitute a probability distribution, what are the probability mass functions, probability density functions and so forth. Now, we are going to deal with specific types of probability distributions. However, to understand probability distribution, it is very important that we have ideas about these tools and concepts like (refer time: 00:51) expectation, variance and covariance that we will often use.

Perhaps you already know some of it, but let me again sort of give you a brief summary of what these concepts mean. So, expectation; we will start with expectation. So, expectation as a term, it means, you know, to expect something, right, it is sort of like, you know, like, I do not know what exactly is going to be the result, but I am expecting this. And we we remember the idea of double structure of variable, we we remember the idea of how a frequency distribution with probability distribution differs.

So, in frequency distribution, we actually get the numbers, whereas in probability distribution, you sort of sort of it is sort of predict, right? So, here, uh we use the term expectation just as opposed to the term average or mean, which we use for a frequency distribution. So, for a probability distribution of a random variable, we use the term expectation to denote the mean or average. Okay. Now, how we do that? So, simple; in expectation, we always use this, we use the value of the variable x; we use their probability, probability of that particular value of that variable; we can write x i, probability x i.

And we basically sum them up. So, where I multiply; so, the steps are, I multiply x i and P x i; and then I sum up, summation of x i P x i. Here P x i, probability of x i is nothing but the weight, we the weight that we use for any sort of, you know, averaging. So, here also we are using the weight. And we do that for all the possible values, okay, for all the possible values of the random variable; so, over all i. Okay. So, this is what we call as the expectation of; let me write down; let me use a different colour, so that; this is something we call expectation of X.

All right. This is how we derive the formula, uh expectation of X. And if we sort of try to write all the components, so, it is going to be; so, I can write x 1; x 2, x 2; x 3, x 3; and so forth. Okay. So, here, uh the notation usually, because it is the realised number, so, you already, we are getting this number, so, these are small x's that are used and the corresponding small p's. Whereas, the X value, when we write expectation of X, that is going to be a capital X, because that is the random variable that I am denoting. All right.

So, with this, uh let us do an example. uh We already have uh done the example of throwing of 2 dice. So, just in order that, (refer time: 03:54) we do not have to uh again redo the whole thing, I have written down the numbers for you. So, this is taken from the previous example. So, let us calculate expectation from this particular problem, and we will write down expectation of X. Let me use a different colour. Expectation of x is going to be basically then the value of the variable and the corresponding probabilities.

The value of the variable is 2 here, and the probability is; let me; 1 by 36. Okay. Then the second one, the value of the variable, the value of the ba variable is 3 and the probability is 2 by 36. Third one, the value of the variable X is 4, and the probability of 4 coming is 3 by 36. The next one, the value of the variable is 5, the probability is 4 by 36. Similarly, this is going to be 6 into 5 by 36. The next one, 7 into 6 by 36. Then the next one, 8 into 5 by 36; 9 into 4 by 36; 10 into 3 by 36; 11 into 2 by 36; 12 into 1 by 36. Okay.

Now, if we sum these things up, so, we will get basically, we will have 36 in the denominator, and here we will be adding 2 plus 6 plus 12 plus 20. So, 2 plus 6, 8; 12; 20 plus 20, 40; plus 30, 70; 70 and 42, 112. Then this is 40; 112 and 40 is uh 152; 152 and 36 is going to be 188. And then 30; 188 and 30 is uh 200 and uh what? 218. 218 plus 22 is going to be, 218 and 22 is going to be 240. And then 12; 252. So, we will have 252 by 36. Okay. So, if we get 252 by 36 and if we cancel uh things out; so, you can divide numerator and denominator by 6; 4, 2.

And then, you can again do it; so, 7. So, this is going to be the expected value of the sum of the 2 dice. Okay. So, when you basically, all you are doing is basically averaging the values. Whereas, the average, the weight that you assign is the corresponding probabilities. All right. So, the same way, we can also, (refer time: 06:47) we can actually simplify the problem for you. And I can ask you; okay, so, let us have 1 dice, and where, you know, you have to calculate the expected value appearing uh after a throw of a dice.

So, in in that case, if it is a, you know, if you are throwing only 1 dice, so, all the possible outcomes are going to be 1, 2, 3, 4, 5, 6; that is it, right. And the (corres) probability for each of these is going to be 1 by 6, right? Now, if you have expected value of X is going to be summation of x i P x i; so, then I am going to have; over all i, over all i; so, then I am going to have 1 by 6 plus 2 by 6 plus 3 by 6 plus 4 by 6 plus 5 by 6 plus 6 by 6. So, which means it is going to be, I think 21 by 6. So, that is nothing but 3.5. Okay.

So, this is how any problem you sort of deal uh wi; you basically take the probability value as well as the value of the variable, okay, and then you get the average. Now, in the same way, you can actually think about, you know, you can actually use the expectation formula for much wider use. So, instead of X having just the value, you can actually represent X as a function. Okay. So, how we can write that? If perhaps, let us say you can write it as expectation of f X, okay; instead of X, you write f X. All right.

And when you write f X, so, then what you do is, instead of x here, you write f x here. Okay. So, basically, we will be writing f x i into P x i here. Okay. And if you sort of f x 1, f x 1, probability of x 1; f x 2, probability of x 2; and so forth f x n, probability x n, okay. Now, to give you an example, let us say the previous example only, if we just use uh here, f X is equal to, f X is equal to x square, let us say. Now, if or; X square; so, if I use this; X square; so, if I use this formula and for this particular example, so, then what I will get is, let me write, let me add another column here.

Let me use this. So, the value of x square is going to be 1, 4, 9, 16, 25, 36, 49, uh 64, 81, 100, 121 and 144. Okay. Now, if I want to sort of get the expectation, expected value of X square, so, what I will get is, what I will get is; so, perhaps it will, it is a better idea to use here, the same screen. So, then the value of expectation of X square is going to be 4 by 36; then I will have 9 into 2 by 36; then I will have 16 into 3 by 36; then I will have 25 into 4 by 36; I need more space. So, then I will have 36 into 5 by 36; then I will have 64 into 6 by 36.

Just a second. Let me see if I am messing. 1 by 36, 2 by 36, 3 by 36, 4 by 36, 5 by 36, 6 by 36. Oh, I just missed out one thing here fortys 49, 49. Okay. Then I will have 64 into 5 by 36; then I will be using; uh so, this is a 8; 81 into 3, uh sorry, 4 by 36; then I will have 100 into 3 by 36; then I will have 121 into 2 by 36; and then finally 144, 144 into 1 by 36. Okay. Now, if you; I am not really adding now, but if you add these things up, the value that you are going to get is 54.83, 54.83.

So, that is basically uh how you calculate the expectation for any functional form of X. Instead of just the X, you can represent the random variable in terms of any functional form.

Okay. So, that is how you calculate uh this expectation, expected value. Now, one last thing about expectation, sometimes that people may confuse is the difference that we see (refer time: 11:57) between expectation of X square and expectation of X of whole square. So, just note that these 2 are not same.

So, here you are taking expectation of this function, where the function is represented by X square. And here, you are actually squaring the whole expectation value basically. And we will see the uh application of these differences uh in the next lecture. So, in the next lecture, we are going to talk about variance and covariance, and we will also talk about the properties of variance and covariance. So, with this, we end this lecture here. Thank you.