

Modelling and Analytics for Supply Chain Management

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Lecture 13

Transportation Decisions: How to Take Decisions on Different Issues with Transportation? [Contd.]

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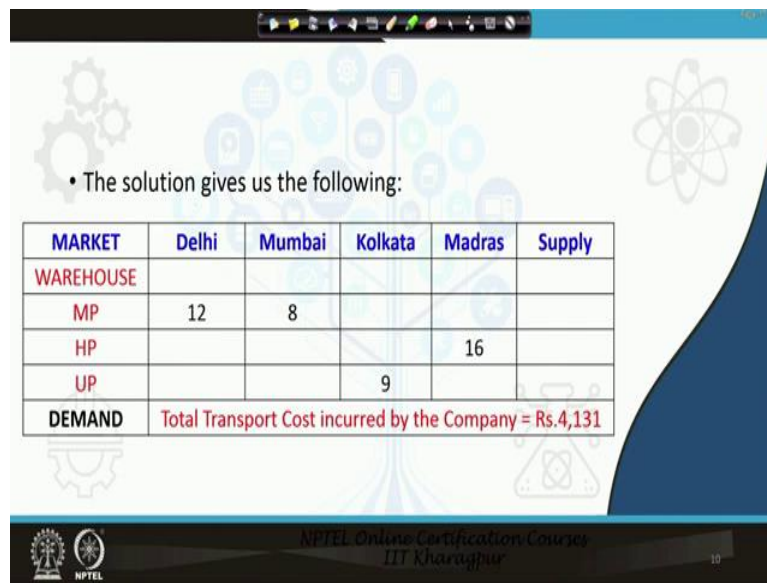


The slide features a blue header with two logos: the Indian Institute of Technology Kharagpur logo on the left and the NPTEL logo on the right. Below the header, a blue banner reads "NPTEL ONLINE CERTIFICATION COURSES". The main content area is white and contains the following text: "Modelling Coordination in Supply Chain", "Dr. Anupam Ghosh", "VGSOM, IIT KHARAGPUR", "Week 3: Lecture 13", and "Transportation Decisions: How to take decisions on different issues with Transportation? Contd.....". A small number "1" is visible in the bottom right corner of the slide.

Hello, in the previous week, we had learned about how to model transportation decisions for the purpose of supply chain and we had taken up the first model that we can use that is the total cost model.

And we had said there were 3 Origin Points for suppliers Madhya Pradesh, Himachal Pradesh and Uttar Pradesh and you had 4 markets Delhi, Mumbai, Kolkata and Madras or Chennai. Okay. And given the cause, how many units should be transported from which supplier to which factory or which warehouse to which market so that my total transportation costs remains minimal that model we had learned Okay.

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• The solution gives us the following:

MARKET	Delhi	Mumbai	Kolkata	Madras	Supply
WAREHOUSE					
MP	12	8			
HP				16	
UP			9		
DEMAND	Total Transport Cost incurred by the Company = Rs.4,131				

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Now, if you just recollect this was the metrics that had the results okay. This metrics told us that 12 units should be transported from Madhya Pradesh to Delhi, Madhya Pradesh 8 from Madhya Pradesh to Mumbai nothing from Madhya Pradesh to Kolkata and Chennai, for Himachal Pradesh nothing to these cities and Himachal should send only to Madras, Uttar Pradesh nothing only to Kolkata Okay. So, this is 1 this is the 1 with which we ended right. But the question that as a modeller as a supply chain modelling expert you should ask is that if you see for all the cities what is happening is Delhi is receiving supply only from Madhya Pradesh, no other place.

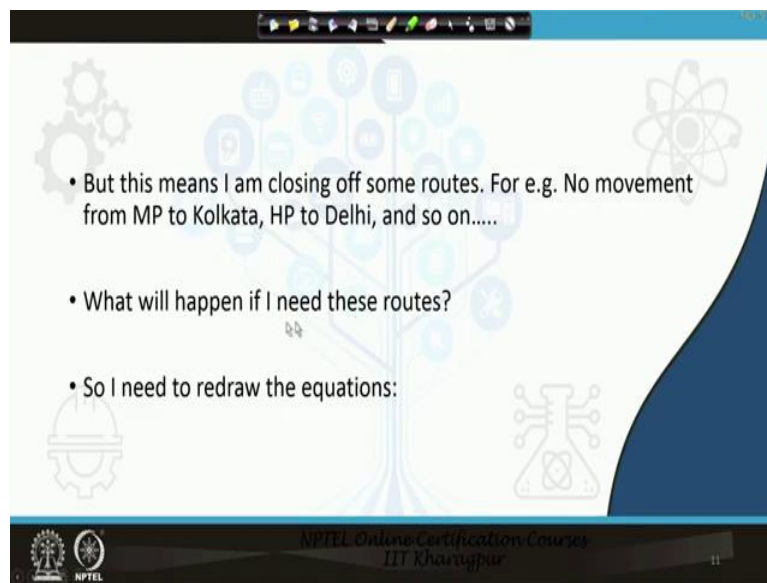
So tomorrow by any chance, if the Madhya Pradesh unit is not able to supply to Delhi, maybe there is a machine break down, maybe there is a transport strike whatever. So tomorrow if Madhya Pradesh unit is not able to supply to Delhi. Delhi will receive no supplies from any place because your transportation cost, total cost model says do not supply from anywhere except Madhya Pradesh. But tomorrow if Madhya Pradesh is not able to supply, Delhi will be starved right, Delhi will be starving, Delhi does not have any supply from any other place, okay.

Same applies to Mumbai, Mumbai is also receiving from Madhya Pradesh. Tomorrow if Madhya Pradesh is not able to supply what will happen Mumbai will also starve and starving means what? It is a lost sale. Kolkata same thing only from Uttar Pradesh. Chennai or Madras same thing only from Himachal Pradesh. So, this though this is the total costs the lowest cost model, what is happening is my supply chain is becoming very, very vulnerable, very, very vulnerable. Tomorrow if 1 route closes down, my supply chain is gone. That market will not

be serviceable, and markets not serviceable means less profit, loss of reputation, goodwill, local competitor coming in, local competitor coming in and taking your place Okay.

So, though the question is as a modeller you should ask is, though my total cost is minimum in this model that you are seeing total cost is minimum in this model that you are seeing. But then is it practically advisable? Should we not have other alternate routes also in place? That is what you should ask okay right.

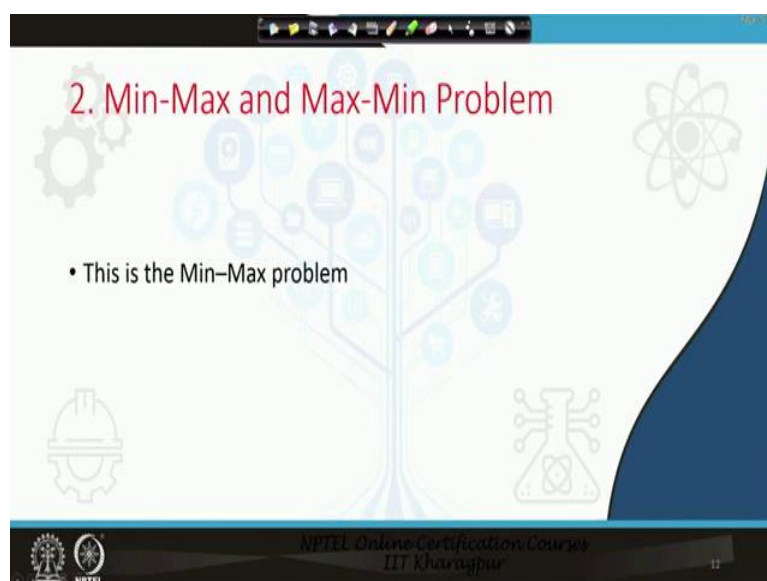
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- But this means I am closing off some routes. For e.g. No movement from MP to Kolkata, HP to Delhi, and so on.....
- What will happen if I need these routes?
- So I need to redraw the equations:

As you were mentioning, what will happen if I need these routes? So, I need to redraw the equation I need to redraw I need to remodel my business of transportation decisions.

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2. Min-Max and Max-Min Problem

- This is the Min-Max problem

This is what is called as a min max problem. Okay.

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• The solution gives us the following:

MARKET	Delhi	Mumbai	Kolkata	Madras	Supply
WAREHOUSE					
MP	12	8			
HP				16	
UP			9		
DEMAND	Total Transport Cost incurred by the Company = Rs.4,131				

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- If we feel that more routes should be there to enable a disruption-free transport movement across the country, we need to have more routes
- But that means we are not at Minimum cost of Rs.4131.
- The company may say that they are willing to accept 10% increase in cost, so the new allowable cost becomes Rs. 4544.

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What it says, if you remember the earlier model, this 1, the last line the total transport costs incurred by the company is rupees 4131, okay is rupees 4131. What it says is if we feel that more roads should be there, to enable a disruption free transport movement across the country, we need to have more routes. But that means we are not at minimum cost of 4131 because my minimum cost means only those places should be served, all the other routes are closed because they have more cost. So, if you want more routes, that means we are not operating at the lowest cost now, because my lowest cost model was this 1.

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• The solution gives us the following:

MARKET	Delhi	Mumbai	Kolkata	Madras	Supply
WAREHOUSE					
MP	12	8			
HP				16	
UP			9		
DEMAND	Total Transport Cost incurred by the Company = Rs.4,131				

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My lowest cost model was this 1. Now, what I want, I want this route. I want this route. I want this route. So, when I want these there is extra cost involved. Okay. These are not the minimum costs; this is what we are wanting to say okay. That means we are now at... we are not at the minimum cost of rupees 4131.

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- If we feel that more routes should be there to enable a disruption-free transport movement across the country, we need to have more routes
- But that means we are not at Minimum cost of Rs.4131.
- The company may say that they are willing to accept 10% increase in cost, so the new allowable cost becomes Rs. 4544.

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Now, what the company says that no we should have more routes for that even if 10 percent extra cost is required let us have it, but let us have let us plan or design a disruption free supply chain. So, the company may say that they are willing to accept 10 percent increase in cost. So, the new allowable cost becomes 4131 plus 10 percent that is rupees 4544. Okay, so, 4131 is the optimal cost that we got from yesterday, the last week. So 4131, they are willing

to accept 10 percent increase in cost. So, the new allowable cost because 4544 Okay, hope you have understood this part.

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• We introduce a variable 'S', where 'S' is the maximum quantity possible to be shipped on any route (e.g. MP to Mumbai, MP to Kolkata etc.)

• $X_1 + X_2 + X_3 + X_4$ etc. are the ideal quantities that should be shipped on any route to keep the total cost to a minimum (also meaning that in some cases X_1 or X_2 etc. will be zero)

• So ideally, $S_1 = X_1$; $S_2 = X_2$; $S_3 = X_3$, and so on.....

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Now I am going very slowly through this model, okay, hope you remember the previous model right hope you remember the previous model this was our previous model minimize the total cost there is on top, minimize the total cost.

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• Min: $81 \cdot x_1 + 92 \cdot x_2 + 101 \cdot x_3 + 130 \cdot x_4 + 117 \cdot x_5 + 77 \cdot x_6 + 108 \cdot x_7 + 98 \cdot x_8 + 102 \cdot x_9 + 105 \cdot x_{10} + 95 \cdot x_{11} + 119 \cdot x_{12}$;

• Subject to:

- $x_1 + x_2 + x_3 + x_4 \leq 15$;
- $x_5 + x_6 + x_7 + x_8 \leq 16$;
- $x_9 + x_{10} + x_{11} + x_{12} \leq 9$;
- $x_1 + x_5 + x_9 = 12$;
- $x_2 + x_6 + x_{10} = 8$;
- $x_3 + x_7 + x_{11} = 9$;
- $x_4 + x_8 + x_{12} = 16$;
- $x_i \geq 0$;

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Min: $81x_1 + 92x_2 + 101x_3 + 130x_4 + 117x_5 + 77x_6 + 108x_7 + 98x_8 + 102x_9 + 105x_{10} + 95x_{11} + 119x_{12}$;

Subject to:

- $x_1 + x_2 + x_3 + x_4 \leq 15$; 20
- $x_5 + x_6 + x_7 + x_8 \leq 0$; 16
- $x_9 + x_{10} + x_{11} + x_{12} \leq 0$; 11
- $x_1 + x_5 + x_9 = 12$;
- $x_2 + x_6 + x_{10} = 8$;
- $x_3 + x_7 + x_{11} = 9$;
- $x_4 + x_8 + x_{12} = 16$;
- $x_i \geq 0$;

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Okay, subject to these were my supplies and these were my demands right? Okay. So this was my total cost, this was the sorry this is 2016 that is it is that some type of graphic error here so 11 I think okay, so anyway, whatever you just check up.

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We introduce a variable 'S', where 'S' is the maximum quantity possible to be shipped on any route (e.g. MP to Mumbai, MP to Kolkata etc.)

x_1
 x_2
 x_3

$x_1 + x_2 + x_3 + x_4$ etc. are the ideal quantities that should be shipped on any route to keep the total cost to a minimum (also meaning that in some cases x_1 or x_2 , etc. will be zero)

So ideally, $S_1 = x_1$, $S_2 = x_2$, $S_3 = x_3$, and so on.....

$S = \max \text{ qty}$
 $x = \text{ideal qty}$ } $S = x$

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And so, that was my cost right. Now what we want to say is look at this so that was my original model right? And you are adding a 10 percent extra cost. I am going through this slide very very slowly, we introduce a variable s, we introduce a variable s, where S is the maximum quantity possible to be shipped on any route that is Madhya Pradesh to Mumbai, MP to Kolkata, Madhya Pradesh to Kolkata, etc. Earlier what was the variable X 1, X 2, X 3 right? This was the quantity that was getting shipped right. Now we are saying we are

introducing a variable s , where S is the maximum possible quantity that can be shipped. Right.

What was X_1, X_2, X_3 , the quantity that was getting shipped X_1, X_2, X_3, X_4 are the ideal quantities that should be shipped to keep the total cost to a minimum, agreed? X_1, X_2, X_3, X_4 , are the ideal quantities that should be shipped to keep the total cost to the minimum that is why in some cases X_1 and X_2 are 0 that is why in some cases X_1, X_2 are 0 so many blank boxes in the matrix that we just now drew that means they are 0 right X_1, X_2, X_3 are 0. So, X_1, X_2, X_3, X_4 are ideal quantities that should be shipped repeating again X_1, X_2, X_3, X_4 are the ideal quantities that should be shipped what was S we have introduced, S is the maximum quantity that is possible to be shipped.

So, ideally S should be equal to the x s okay. S is the maximum quantity that should be shipped, X is the ideal quantity that should be shipped. So to have a proper optimization, my S should equal to x . Right? So S_1 is S equal to X_1 , S_2 is equal to X_2 , X_3 is equal to S_3 . That is ideal. Ideally, this should happen. If I am introducing a variable X , then this should... to keep the total cost minimum, S should be equal to X . Right? I am stopping for a second, you understand this first, understand this very carefully. I am stopping, look at the slide, read it and you understand okay, right.

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- But since we are not following the optimal solution for Minimal Cost and allowing for some quantity of product movement along some more routes, the value of S_1 may not equal X_1
- For e.g., say for route MP to Kolkata, X_3 quantity should be sent, which, from the optimal solution, we have obtained as 0
- So no units should be sent from MP to Kolkata

But since we are not following the optimal solution for minimal cost and allowing for some quantity of product movement along some more routes, the value of S_1 may not equal to X

1, the value of S_i may not equal to X_i . For example, in the previous model if you remember, if you remember, okay. Sorry.

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• The solution gives us the following:

MARKET	Delhi	Mumbai	Kolkata	Madras	Supply
WAREHOUSE					
MP	12 X_1	8 X_2	X_3	X_4	
HP	X_5	X_6	X_7	16	
UP			9		
DEMAND	Total Transport Cost incurred by the Company = Rs.4,131				

If you remember what was this thing, X_1 was 12, X_2 was 8, but X_3 is 0, X_4 is 0, X_5 is 0, etc, right? X_2 , to X_7 at least is 0, right? X_2 to X_7 at least is 0, right okay.

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• But since we are not following the optimal solution for Minimal Cost and allowing for some quantity of product movement along some more routes, the value of S_i may not equal X_i

• For e.g., say for route MP to Kolkata, X_3 quantity should be sent, which, from the optimal solution, we have obtained as 0

• So no units should be sent from MP to Kolkata

$X_3 = 0$ $S_3 = 0$ $X_5 = 0$ $S_5 = 0$
 $X_4 = 0$ $S_4 = 0$ $X_6 = 0$ $S_6 = 0$

So, what we are saying now is what we are saying now is that since we are not following the optimal solution for minimal cost and allowing for some quantity of product movement along some more routes, value of S_i may not equal X_i . Remember, X_3 was 0 just now we checked up in the table. So, ideally, if it is a minimal cost model, S_3 should also equal to 0

because I am keeping the cost at the minimum, my X_4 was 0 so S_4 should also be 0. My X_5 was 0 so S_5 should also be 0, right? X_6 was 0. So $X_6 S_6$ should also be 0. But now, what we are saying is what we are saying is that let us remove some of them then you will understand my X_3 Okay.

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• But since we are not following the optimal solution for Minimal Cost and allowing for some quantity of product movement along some more routes, the value of S_i may not equal X_i

• For e.g., say for route MP to Kolkata, X_3 quantity should be sent, which, from the optimal solution, we have obtained as 0

• So no units should be sent from MP to Kolkata

$X_3 = 0$ $S_3 > 0$ $S_3 \neq X_3$
 $2 \neq 0$

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Now what X_3 was 0, now what we are saying is that no X_3 route that is Madhya Pradesh to Kolkata, MP to Kolkata X_3 route should not be 0 we should use that route. So and we are saying so my S_3 will now not be 0. S_3 will be what, S_3 will be S_3 will be greater than 0 right, so what we are saying is there will be situations when S_3 is not equal to X_3 . Since we are not following the minimal cost model X where X_3 is 0, S_3 should also be 0 ideally if X is 0, S is also 0. If X is 0 S is also 0 because S is the quantity ship but now you are saying that no, we should ship some quantity. So there will be situations when X_3 that is the ideal quantity to be shipped is not equal to S_3 ideal quantity to be shipped was 0. And now we are saying that okay, no, you send at least 2 units, so definitely X_3 is 0 and 2 units is not equal to 0. So there will be situations when s_3 will not be equal to X_3 right? Okay, have you understood right okay.

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• But we are saying that some quantity may (should) be sent

• So $S_3 \neq X_3$, rather $S_3 \geq X_3$

• This applies for other routes also

• But more the quantity sent on a high-cost route, more will be my total cost. So my OBJECTIVE is to MINIMIZE 'S'

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So, the values of S_i may not equal to X_i okay right. As we mentioned, so, S_3 is not equal to X_3 rather S_3 is greater than equal to X_3 because X_3 is 0 so we are sending some units right so S_3 will be greater than equal to X_3 , right, this applies for other routes also. So, S_3 is not equal to X_3 , rather S_3 is greater than equal to X_3 , but more than quantity sent on a high cost route. So, you were sending more quantities on a route that should have 0. So, you are sending it in a high cost route. So, more the quantity sent on a high cost route more will be my total cost. More will be my total cost. So, my objective function is to minimize this S . What is S ? The quantity that has being sent on all the routes right okay.

So, again My S_3 is not equal to X_3 ideally they should be equal if it is an ideal value point ideal method, but we are saying that some quantity has to be sent by that route to enable that market to be serviced during times of emergency okay because if you are depending only on 1 source then it will then that that source may stop producing, stop delivering. So, so you have to have an alternate source ready, alternate source for supplier selection also you may say this is the model for supplier selection also that we learned in the previous model okay. So, some quantity has to be sent so S_3 is greater than equal to X_3 . This applies for other routes also, more the quantity sent on the high cost route more will be my total cost. So, my objective is to minimize the value of S okay right that is my objective.

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• So my new Optimization Model becomes:

• Min S ✓

s.t.

(1) Demand Constraints remain the same (same demand from the markets hold)

(2) Supply Constraints remain the same (same supply from the suppliers hold)

(3) New Total cost of transportation <= Cost under pure optimization model (+) 10% addition

(4) $S_i \geq X_i$ (from previous slides)

Cost 4131
(+) 10%

New Qty sent on a route \geq Ideal Qty

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So, my new optimization model becomes minimize S, my new optimization model becomes minimize S, subject to demand constraints remain the same, supply constraints remain the same, new total cost of transportation, it will increase right. New total cost of transportation is what? We had said that we will have the earlier cost of rupees 4131 plus 10 percent that was what the company allowed me for enabling this multiple routes movement right. So, my new total transportation cost should be less than equal to cost under pure optimization model plus 10 percent addition. This is the maximum right, this is the maximum allowable limit given by the supply chain planners of the organization that look you have multiple routes, but my total cost should not be more than 10 percent of the previous model. So, my previous model was cost, cost under the pure optimization model and 10 percent addition. So, my new transportation cost should be less than equal to this earlier cost model, right, my new transportation cost should be less than equal to my earlier cost plus 10 percent.

And definitely this last one, the amount the new quantity to be sent on every route is either greater than or equal to the ideal quantity, okay. The new quantity sent on a route is either greater than or equal to the ideal quantity okay the new quantity sent on a route is greater than or equal to the ideal quantity, this is my constraints right.

(Refer Slide Time 19:32)

• Min: S ;

• Subject to:

- $x_1 + x_2 + x_3 + x_4 \leq 20$ (handwritten: 20)
- $x_5 + x_6 + x_7 + x_8 \leq 16$ (handwritten: 16)
- $x_9 + x_{10} + x_{11} + x_{12} \leq 11$ (handwritten: 11)
- $x_1 + x_5 + x_9 = 12$;
- $x_2 + x_6 + x_{10} = 8$;
- $x_3 + x_7 + x_{11} = 9$;
- $x_4 + x_8 + x_{12} = 16$;
- $81x_1 + 92x_2 + 101x_3 + 130x_4 + 117x_5 + 77x_6 + 108x_7 + 98x_8 + 102x_9 + 105x_{10} + 95x_{11} + 119x_{12} \leq 4131 + 10\% \text{ of } 4131$ (handwritten: 10% of 4131)
- $S_1 \geq x_1; S_2 \geq x_2; S_3 \geq x_3; \dots, S_{10} \geq x_{10}; S_{11} \geq x_{11}; S_{12} \geq x_{12};$
 $S_n \geq x_n \quad S_i \geq x_i$ (handwritten)

LiPS L.K.1
Excel

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So, when we model it again let us see what equations we have got, minimize s subject to there was a typographic error here So, 20, 16, 11 supply constraint, demand constraint, this is my cost constraint and this will be 4131 plus 10 percent of 4131 right this is my total cost should be less than the earlier cost plus 10 percent escalation okay, total costs should be less than the earlier cost plus 10 percent escalation and the S_s should be greater than equal to the X_s or S_n greater than equal to X_n or S_i greater than equal to X_i . So, basically quantities sent should be greater than equal to the optimal quantity, okay right. So, this is the new model that comes up. And again when you solve this model using LiPS 1.11.1 or any Excel Based linear programming, you will get the changed direction.

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• Min: S ;

• Subject to:

- $x_1 + x_2 + x_3 + x_4 \leq 15$;
- $x_5 + x_6 + x_7 + x_8 \leq 0$;
- $x_9 + x_{10} + x_{11} + x_{12} \leq 0$;
- $x_1 + x_5 + x_9 = 12$;
- $x_2 + x_6 + x_{10} = 8$;
- $x_3 + x_7 + x_{11} = 9$;
- $x_4 + x_8 + x_{12} = 16$;
- $81x_1 + 92x_2 + 101x_3 + 130x_4 + 117x_5 + 77x_6 + 108x_7 + 98x_8 + 102x_9 + 105x_{10} + 95x_{11} + 119x_{12} \leq 4131$;
- $S_1 \geq x_1; S_2 \geq x_2; S_3 \geq x_3; \dots, S_{10} \geq x_{10}; S_{11} \geq x_{11}; S_{12} \geq x_{12};$

12	8	8
8	18	7
8	8	8
9	8	8

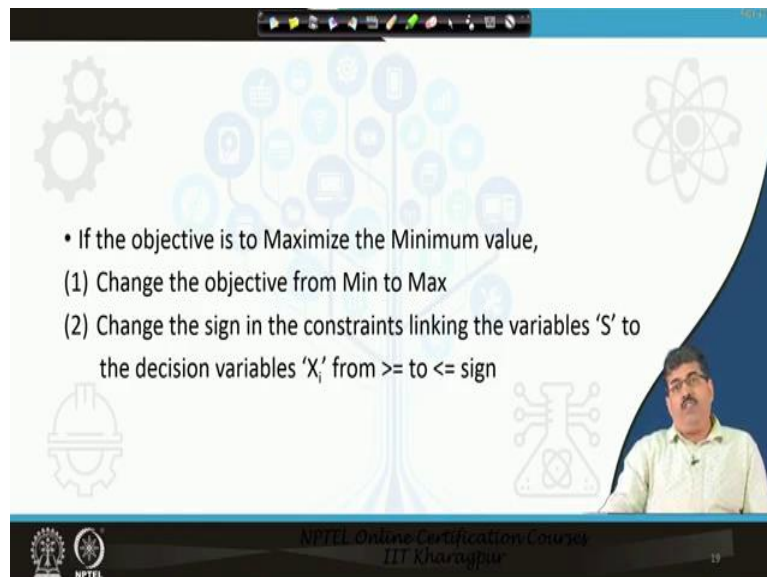
$8 > X \quad 8 = X$ (handwritten)

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Now, the question that comes in is, in this new model this was my earlier model right. Some cases are all blanks maybe 7, maybe 9 ok this was earlier model, and the new model we are saying these are my Ss right these are my S. New model will all the cells be full with some quantity means will all the S i be grater than X i that is the question, will all the cells be filled with some quantity, because we are saying that there should be alternate additional routes available question is. Answer is no, all the cells will not be filled up with more quantity at least some cells will be filled up.

So, all Ss will not be greater than equal to Xs in some case, in some cases S will be equal to X ok, for some routes the Ss will be greater than Xs okay. So, this is so, this is the model okay this is called as a min-max or max-min model.

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• If the objective is to Maximize the Minimum value,
(1) Change the objective from Min to Max
(2) Change the sign in the constraints linking the variables 'S' to the decision variables 'X_i' from \geq to \leq sign

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Now, okay if the object is this was minimization of the maximum because we are sending the maximum quantity in any route and we are minimizing this quantity. So, this was a min-max problem. Next is max-min, if the objective is to maximize the minimum value, change the objective from min to max and change the sign of the constraints linking the variables S from greater than equal to less than equal sign okay, that will solve your problem right. Now, we will end this section here. In the next section, we will deal with the maximum flow, the minimal and the minimal spanning tree problem. Okay. Thank