Six Sigma
Prof. Jitesh J Thakkar
Department of Industrial and Systems Engineering
Indian Institute of Technology, Kharagpur

Lecture – 52
Operating Characteristic (OC) Curve for Attribute Control Charts

Hello, friends. Let us go ahead in our six sigma journey and I welcome you to lecture 52, Operating Characteristic Curve for Attribute Control Chart. So, I hope now you have understood the overall mechanism that we design develop the control chart and we also check the performance of the control chart using operating characteristic curve and average run length, these are the performance indicators of the control chart. So, we are moving ahead in the last phase of our DMAIC cycle and we are discussing the control face of our DMAIC six sigma cycle and this is our lecture 52.

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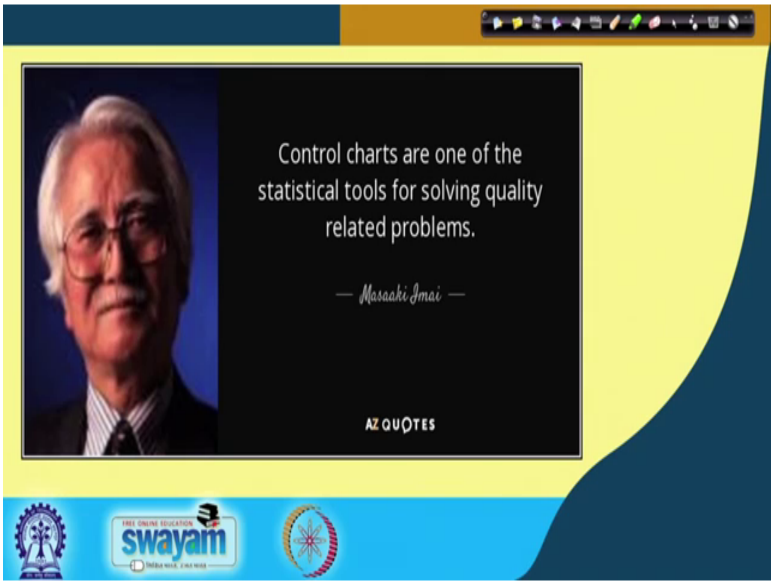

So, once again I will keep emphasizing that control chart and associated concepts are extremely useful to do the fact based analysis and establish a statistical control on the process.

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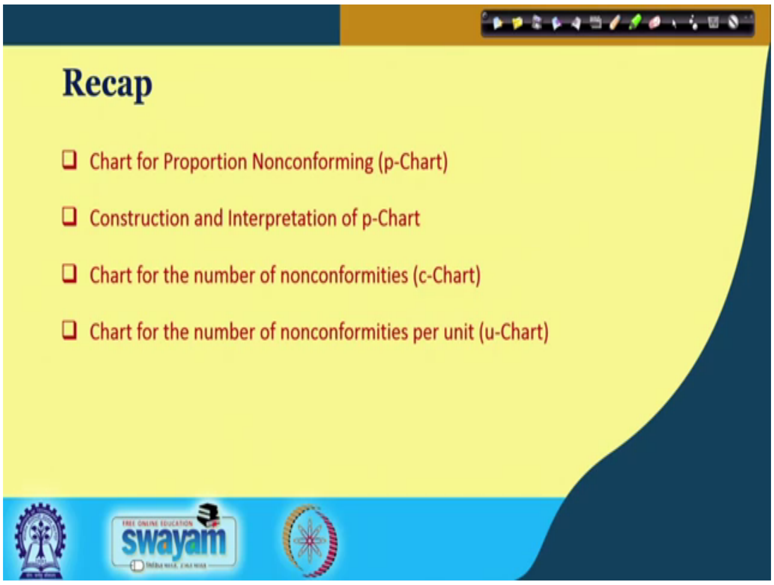

So, we will try to discuss the chart for proportion nonconforming p-Chart, construction and interpretation we have done this p-Chart, chart for the number of non conforming c-Chart and u-Chart. So, typically we have talked about three different categories of chart p-Chart also np-Chart you can add, c-Chart and u-Chart we have seen with the illustrative example in the last lecture that how this charts can be constructed and they can be interpreted.

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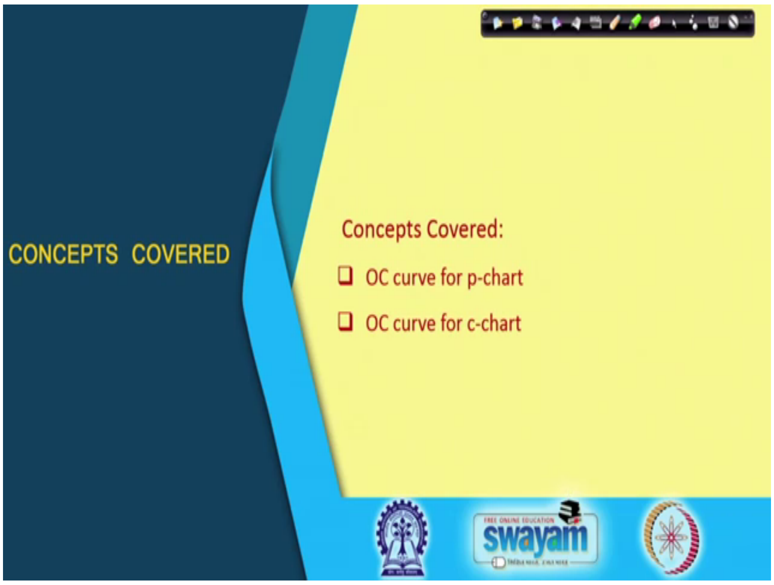

Now, in this particular lecture we will focus on OC curve for p-chart and OC curve for c-chart.

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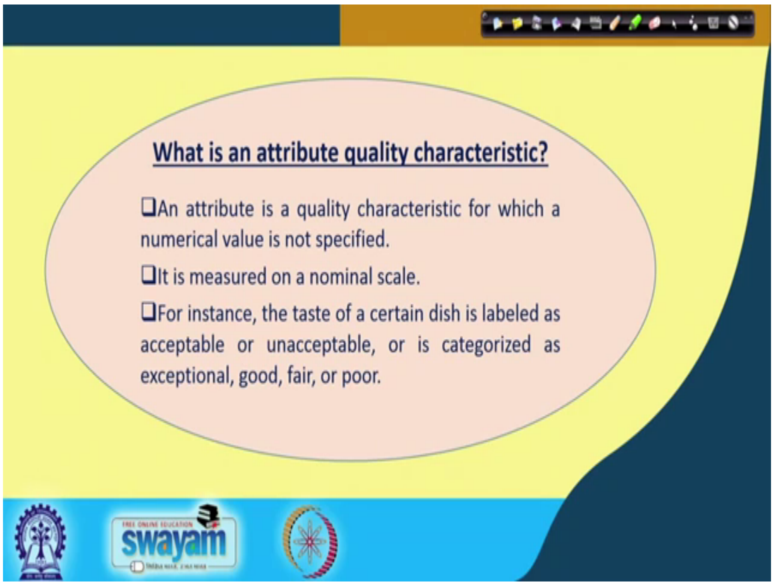

So, let us try to appreciate first once again, what is an attribute quality characteristic? So, this is something which we measure in terms of good or bad, acceptable or not acceptable and we do not say assign any numerical value to this. So, it is measured on a nominal scale.

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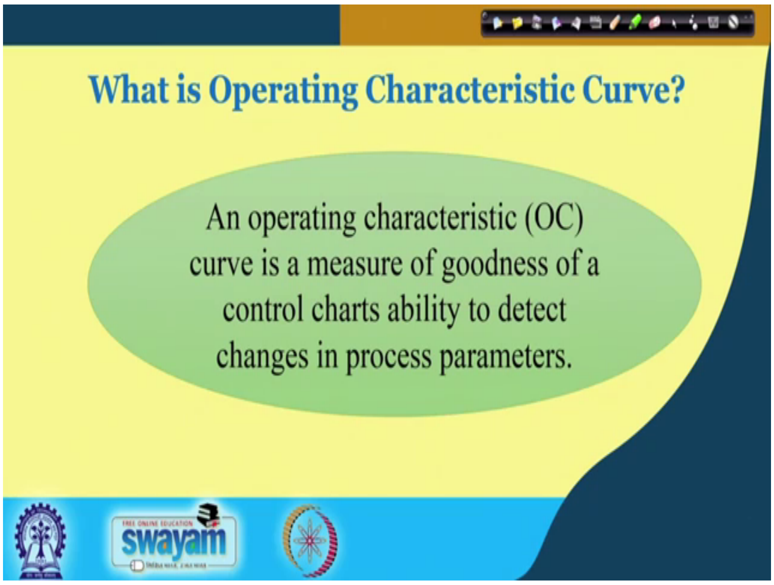

Now, OC curve, again I will repeat that it is a goodness of a control chart my chart should be able to detect when there is a change in the process either mean or maybe the variance and if this change can be detected well, then my control chart really has some goodness in terms of its discriminatory power. So, this is what we have seen.

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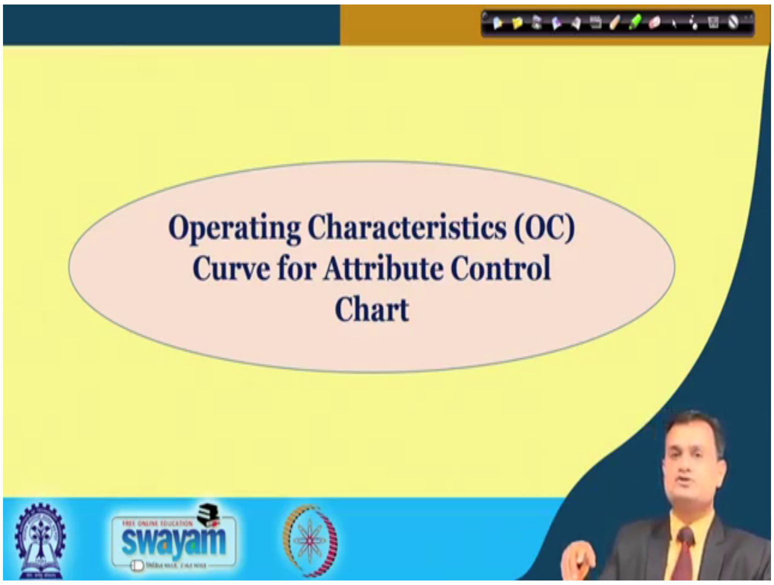

Now, let us try to see the OC curve for attribute control chart and the first one is let us say p-chart.

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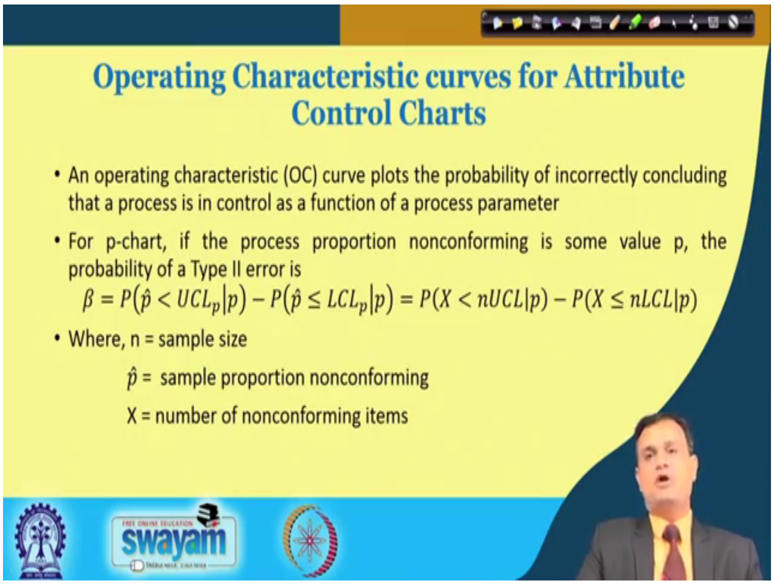

So, here again I will say that the moment you talk about the OC curve we need to have a knowledge about the type II error and type II error in the simplest way I will say that I will say accept the null hypothesis when it is of reject, I will say declare the guilty as the innocent means I will exempt I will not punish. So, this is my type II error and type II error is indicated as beta.

So, we always compute the type II error in order to have the development of the OC curve and here because it is for p-chart what you can see here that probability that p hat is less than UCL p. So, it is upper control limit for p-chart for given P proportion nonconforming minus p p hat is less than or equal to LCL p for given p and if I multiply both the sides with n here also n there it comes out to be say probability X is less than nUCL given p minus probability X less than equal to n CL given p.

So, p hat is the sample proportion nonconforming n is the sample size and X is the number of non-conforming items. So, obviously, when you have proportion nonconforming when you multiplied with the n you get number of non conforming. So, proportion is converted into the number. So, this is my type II error.

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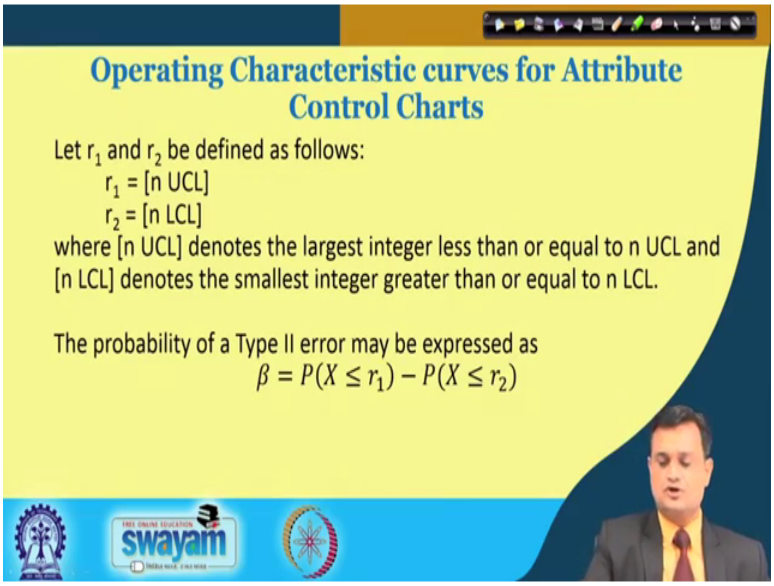

And, I can also do little bit substitution. So, I will say that r 1 is equal to n LCL and r 1 is equal to n LCL r 2 is equal to r 1 is equal to n UCL and r 2 equal to n LCL. So, I can just put it here and my beta will be probability X less than equal to r 1 minus probability X less than or equal to r 2. So, we have discussed two – three times in detail in the previous lectures, how to interpret the type II error when there is a shift in the mean and what is that particular area, which indicates the type II error, when my process has shifted from mu 0 to mu 1. So, now, with that understanding you can easily compute the probability of type II error for my control chart that is p-chart.

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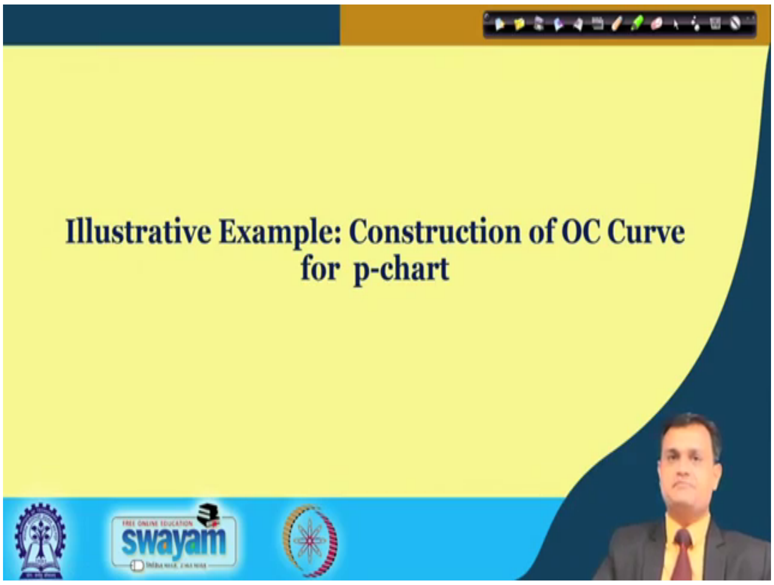

So, now let us try to appreciate the interpretation part and construction of the OC curve for p-chart.

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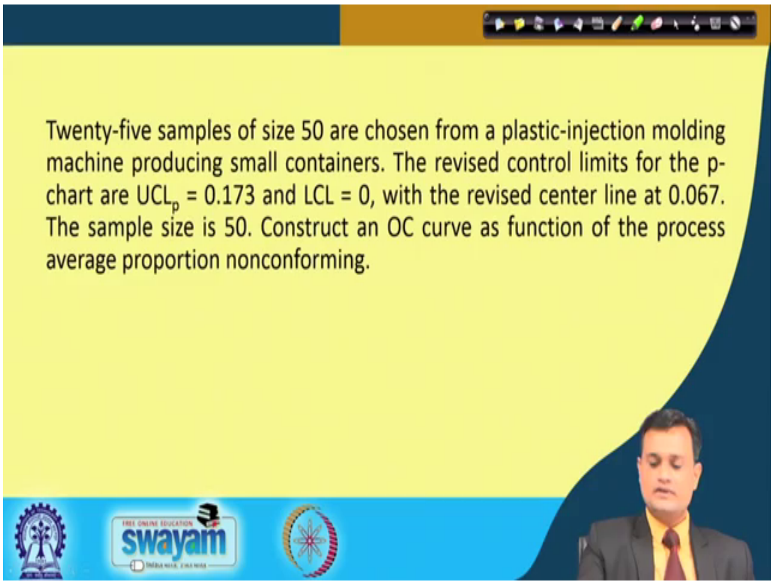

Let us say I have an example like this the twenty five samples of 50, so, each sample is of 50 size and they are selected from a plastic injection molding; the same example we referred last time producing small containers and the revised control limits for the p-chart are now UCL p 0.173, so, directly this limits are given LCL is 0, it was negative if you recall so, I converted it into 0. With the revised central line at 0.067 by eliminating the point going outside upper control limit I got the revised central line that is 0.067.

So, sample size is 50, construct an OC curve as a function of the process average proportion nonconforming. So, this is what we want to do.

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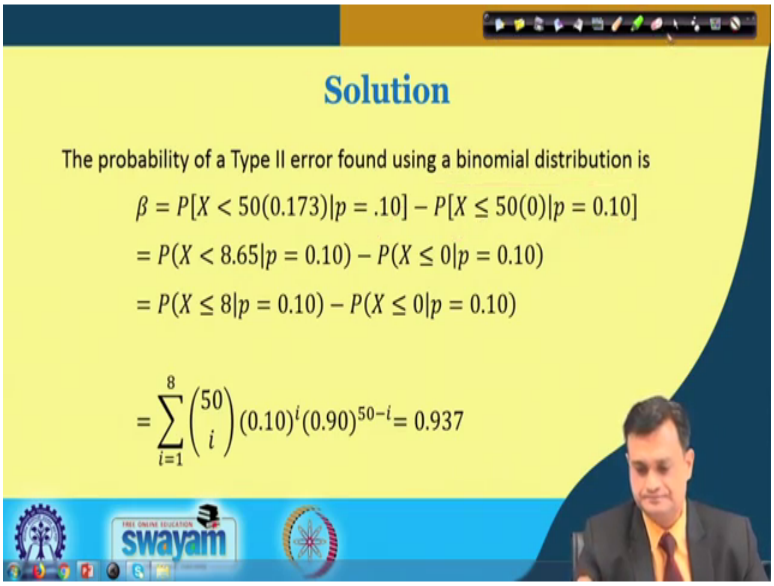

So, obviously, I need to compute the type II error beta, I will just try to show the type II error for one example and we are basically using the binomial distribution. So, for this the distribution which is useful is binomial distribution. So, type II error is beta is equal to probability X is less than 50, 0.173 you multiply and into p for a given p and you have 0.1 as the given p minus probability X less than equal to 50 into 0. So, your LCL is basically 0 for a given p. So, p is equal to 0.1.

Now, when you use the binomial distribution you can plug in the values in the binomial distribution equation or you can refer the binomial distribution table. So, both way it is and you can find the probability that is 0.937. So, this is what you have determined.

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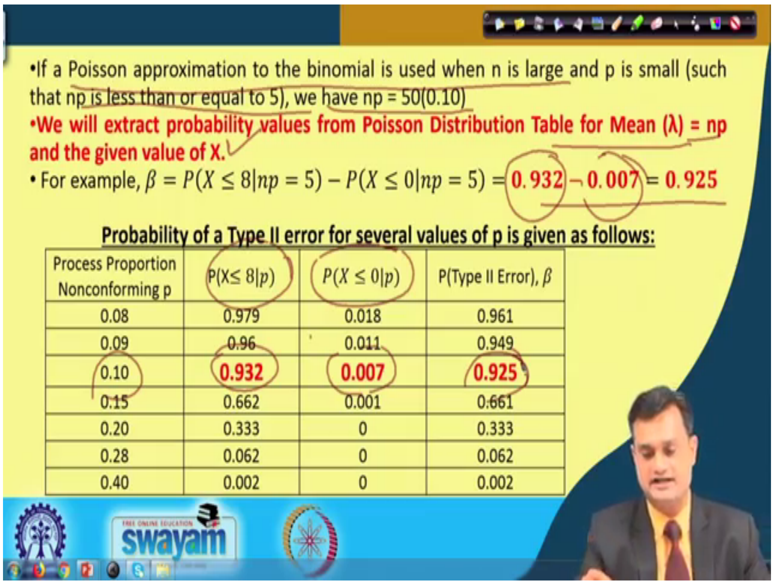

Now, once it is done then you can construct the table and here I will just try to show you the example that you can use if the binomial is the combustion you can use Poisson approximation also.

So, here I have developed the table with poison approximation and more or less it will give you the same probability value quite close closer to the binomial and I can use the Poisson approximation. So, Poisson approximation to the binomial is used when n is large and p is small so, such that np is less than equal to 5. So, we have np 50 into 0.1. So, there is a condition when I want to use the approximation Poisson approximation for binomial. So, I must have np less than equal to 5. So, this condition is satisfied.

Now, here for I am just showing one example remaining thing you can just compute using the Poisson distribution table. So, what we have done here, we are extracted the values for mean lambda is equal to np for the given value of X from Poisson distribution table. So, you should be very much familiar with the use of Poisson distribution, binomial distribution, normal distribution tables so that you can read it very easily and the probability values are readily available. So, you have lambda is equal to np and for given value of X, I am extracting the values from Poisson distribution table.

So, for example, say probability X is less than or equal to 8 and this is basically computed like this. So, probability X is than equal to 8 for a given p and probability X is less than or equal to 0 for a given p this you can find 0.932 this is 0.932, 0.007 from the

Poisson distribution table. So, this values you can easily find for the given X and the mean value here your mean is lambda is equal to np. So, once you have the lambda is equal to np mean value and you refer the column, you refer the row with the X value and you will get the probability value. So, I have just shown the example for one case that is 0.1 process proportion nonconforming and condition X is less than or equal to 8 for given p; given p is 0.1 for this case and 0.932 and 0.007 you can compute.

So, type II error is nothing, but the subtraction of these two and this comes out to be 0.02, point 0.925. So, this is what you can extract from the Poisson table.

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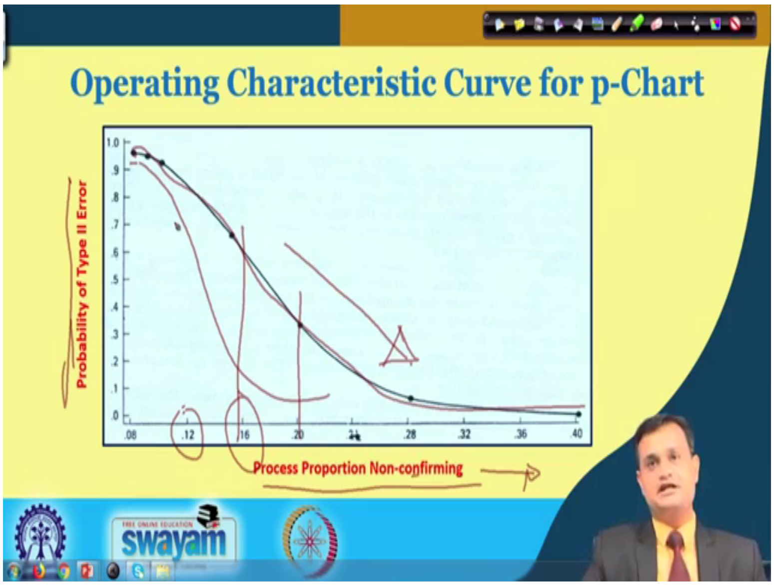

And, now, what you do you plot it. So, my OC curve would like look like this. I have process proportion nonconforming this is here process proportion non conforming and I have probability of type II error here; probability of type II error process proportion nonconforming and what you get you get an OC curve like this.

So, how to interpret? You have 0.08 and you have some value of probability of type II error you have 0.12 you have some value you have 0.16. So, basically the values I put I found in the table from the Poisson distribution table I just plotted it here and you can see that as my process proportion nonconforming increases, what is happening; my type II error is basically reduces. So, this says that as the non conforming item will increase in the process by type II error means saying that, it is acceptable not nonconforming reduces. So, I will have the better quality, better goodness of my control chart and up to let us say 0.16 you can see that for up to 0.20 my type II error is high. If you want to make it steeper then obviously, you can choose the higher sample size and you will have say steep OC curve and this would give you better discriminatory power.

So, this is what we can say about p-chart.

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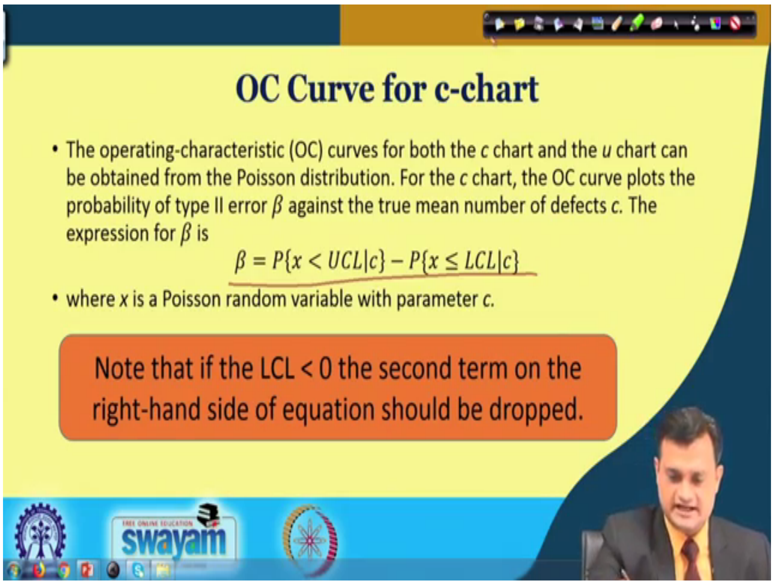

And, now similar way you can construct the OC curve for c-chart you have to compute the beta that is the type II error and here the logic remain same that beta is equal to probability X less than UCL for a given c minus probability X less than or equal to LCL for a given c and once again you refer the Poisson distribution table.

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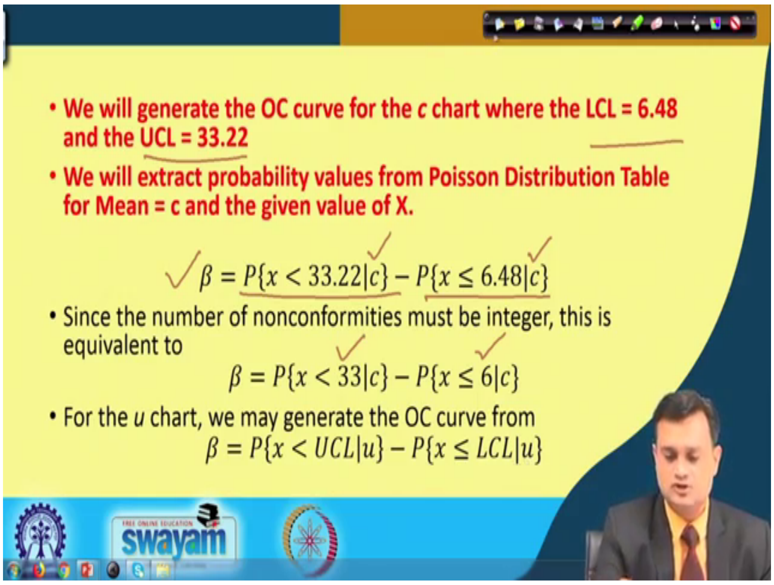

So, this is how it is constructed you refer the Poisson distribution table and extract the values, I will just show near the sample calculation. Let us say for beta I want to compute probability that X is less than 33.22 because UCL is 33.22 minus probability X is less than equal to 6.48 and I want to compute this for a given value of c. So, you have LCL 6.48. So, I am finding it now because it is not possible to go for the decimal I am just converting because it hardly makes difference when I say 33.22 nonconforming, it is better logical to say 33 nonconforming or 34 or 6 or 7. So, I am just choosing the lower integer value that is 33 and 6 and you can compute the beta.

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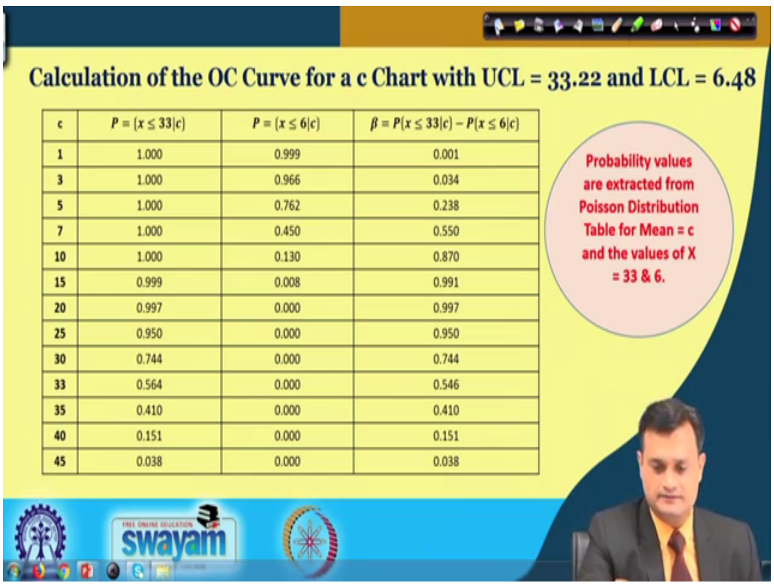

So, now, you refer the table of Poisson distribution in this is where I have tabulated the values which are extracted basically from the Poisson distribution. So, Poisson values are extracted probability values are expect extracted from the Poisson distribution table for mean c your column you just open it and you will get the idea and the various values of X that is in the row. So, 33 and 6 for 33 for c is equal to one you will get one value, for 3 and 33 you will get another value, for c is equal to 5 and 33 you will get another value and similar way you can have the values for your this as well as this.

So, I have just tabulated this in order to get the data point for my operating characteristic curve.

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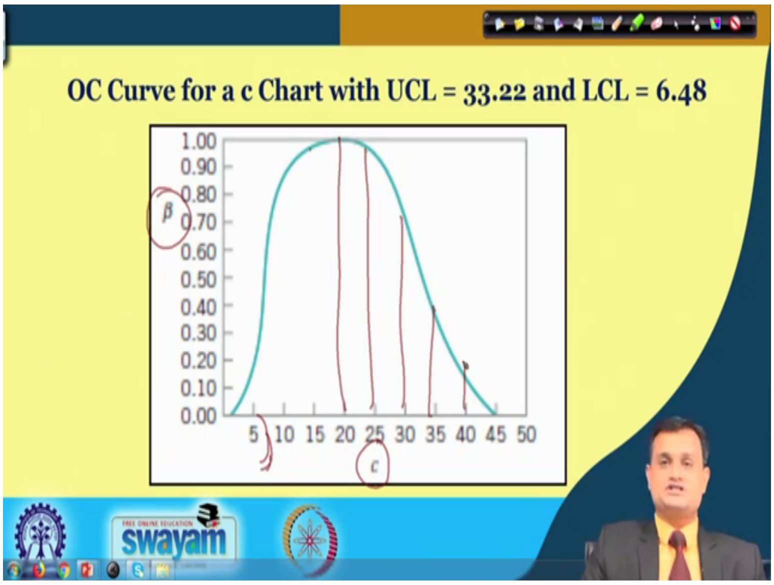

And, once you have this table available then you can plot the OC curve and it looks like this. So, you have the type II error on the y-axis that is beta and this beta I have already completed in the previous table I have the value of c. So, I will just plot the value of beta for a particular c and I will get the OC curve.

So, now again this will tell me or as a quality manager I will get the idea that suppose let us say I have the number of non conformity up to 10 or 15, then my type II error is very high, if it is 20 it is very high, if it is 25 it is going down, if it is 30, 35, 40. So, I will get an idea that suppose my particular has a process is producing less number of nonconformity or very high per unit then in that case a given OC curve will tell me that, what is the goodness of my control chart. It means it is ability to detect the non conforming nonconformity in a given sample when I have a particular slope or steepness of the OC curve.

So, basically it tells me about the discriminatory power of my OC curve and this is for UCL 33.22 and LCL 6.48.

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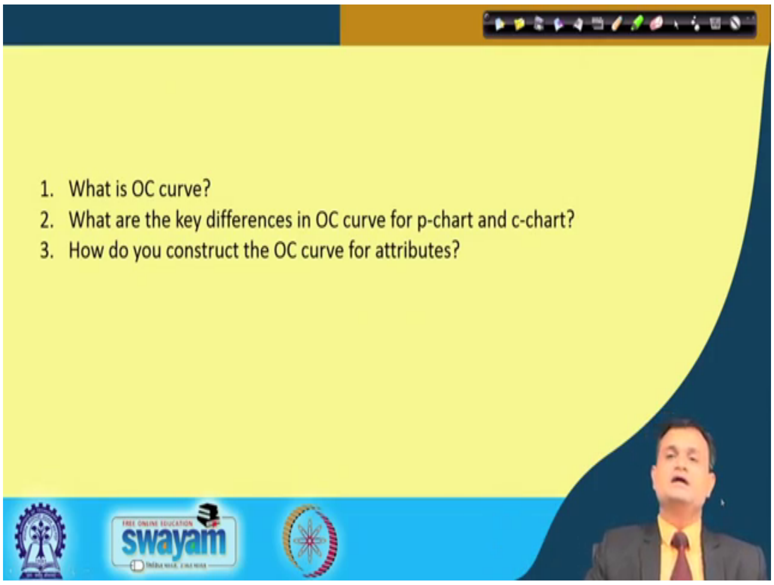

So, before we end let me just plot some simple think it. Once again, ask yourself what is OC curve? What are the key difference is in OC curve for p-chart and c-chart? How do you construct the OC curve for attributes and what do you interpret from this about the goodness or the discriminatory power of the OC curve?

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So, as I mention Mitra and Montgomery are very much useful books for understanding the concept of OC curve and I have referred these books.

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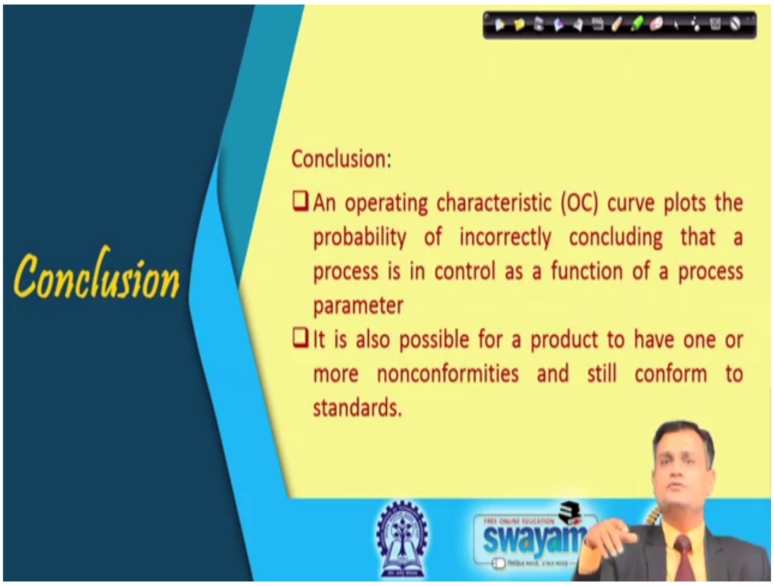

So, OC curve plots the probability of in correcting, concluding that a process is in control as a function of process parameter. So, wrongly I am saying that process is in control, but actually process has gone out of control and it is also possible for a product to have one or more nonconformities and still confirm the standard.

So, this extreme is also possible thank you very much for your interest in learning the OC curve for attribute control charts. I hope now this session would have lecture would have given you the complete idea and if you refer the last couple of lectures then control charts for variables OC curve for variables, control chart for attributes and OC curve for attribute, it is a complete understanding on the statistical control chart.

So, one is the development design of the control chart and then once you have the properly stable control limits you implement them and, then you would like to also check the performance of your control chart using the operating characteristic curve average run length and these are basically the performance measures.

So, with this I wish you say great introspection and application of the concepts in real life application so that you can internalize the concept and you can apply them in day to day life or real life in your industry setting. So, please keep revising, keep reading the suggested reference material, strengthen your concepts, be with me. Enjoy.