Management of Inventory Systems
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Lecture – 24
Dynamic Inventory Problems under Certainty (Contd.)

During this session we will continue our discussions on determination of say another EOQ with price discount.

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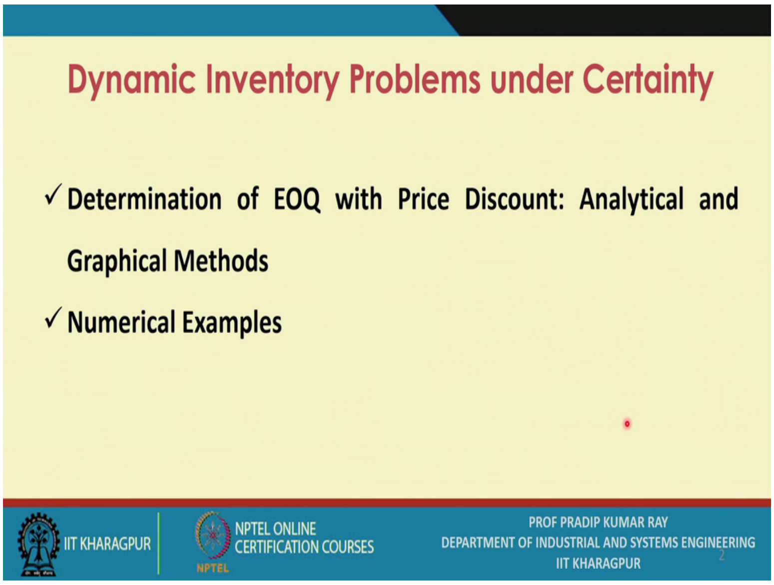

So, already we have explained this concept and in the last lecture session. Now, what we have mentioned that in order to determine this economic order quantity with price discount are the two approaches you may opt for, the first one is analytical approach and the second one is the graphical methods. So, in this particular lecture session we will be referring to both the methods or both the approaches and will take off a few important numerical examples. So, this will be our coverage during this lecture session.

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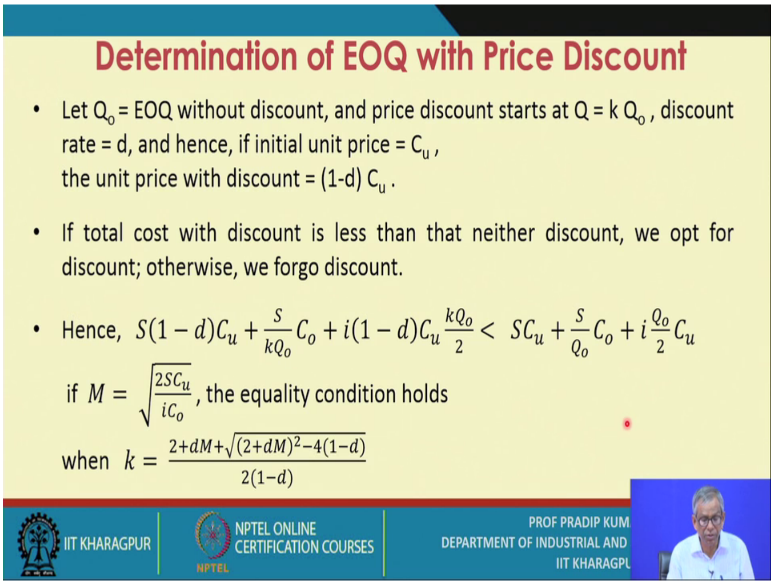

Now, continuing our discussion on say the EOQ determination with price discount. So now, we have explain the phenomenon and we have mentioned that there 2 types of you know the situations you come across. The first one is single price break and the second one is the multiple price break. So, let us first consider the single price break situation and we take of analytical approach; we will take of the graphical methods.

And when you follow when you understand this analytical approach as well as the graphical methods with respect to the single price break situation, you will come to know you will have a very good understanding of the concepts ok. The later on obviously, the same concepts we can apply for some multiple price break situations. So, while say the modelling this particular problem we need to use several kinds of say, we need to consider several kinds of variables the parameters and you are trying to explain the rational at the behind such modelling at each step.

So, let the Q 0 is equals to EOQ; Economic Order Quantity without discount ok. So, that is the first derivations we have the original EOQ formula, sometimes this is a fertilizer classical EOQ formula and the price discount counts starts at Q equals to k into Q 0 right. So, what you are trying to do? That means, the Q 0 or the optimal order quantity or EOQ stands as the base and the other quantities that we need to consider are expressed in terms of the EOQ quantity ok; so EOQ. So, that is why we say that the price discount starts at an ordering quantity Q and this Q you must explain in terms of Q 0.

So obviously, there is a multiplying factor that is small k. So, the Q equals to small k into Q 0, discount rate is d and hence if initial unit price is C u, the unit price with discount will be 1 minus d into C u; that means, suppose the discount rate is 10 percent, it means what? That means, suppose 10 percent discount and original unit price is 100 rupees; that means, unit price with discount will be so, the 0.9 into 100; that means, 90 rupees. Is it ok? So, 1 minus d into C u is general notation were used.

If the total cost with discount is less than that with without discount ok, we opt for discount otherwise we forgo discount. That means what you try to do? When you accept discount first you first you calculate what is this total cost and supposing the discount is not offered, so what is the total cost? And then you check whether the total cost with discount is less than that without discount then, you opt for discount otherwise you do not opt for discount or you forgo discount. So, this is the basic logic we establish. So, hence now if you have say if you discount with the discount offered at a discount rate of d.

So, what will be the total the purchase cost? So, the S is actually the, your yearly demand and the purchase price is 1 minus d into C u with discount. So, this is your purchase cost. Next what is your ordering cost? So, ordering cost per order is C o, capital S is your you know your yearly demand and how much you are ordering that is k Q into Q 0; that is your the ordering quantity with discount. So, S by k into Q o is the number of orders for each orders your incoming a cost of C o.

So, this is the total ordering cost with discount and now what is the average inventory you hold? That is k into Q o by 2. Why 2? Because we are assuming that the demand is uniform and the corresponding price is; so, the average inventory this is in physical units corresponding prices 1 minus d into C u. That is the discount price and the inventory carrying cost will be i proportion of that; which is the average inventory which you hold. Now this total cost with discount must be less than this one; that means, when you do not opt for discount. So, this is S into C u plus S by Q o into C o plus i into Q o by 2 into C u. Is it ok?

So, if now this inequality expression you simplify you know you manipulate and with this substitution we use a term called capital M; capital M is root over twice S C u by i C o. So, the equality condition holds; that means, this the expression has got similarity, with the original EOQ special, but please note down or the difference between this expression for M and the EOQ expression. Is it ok?

So, it is M if M is root over twice S C u by i C o; the equality condition holds when k when that is the unknown. So, I need to determine the value of k. So, the expression for k is 2 plus dM plus root over 2 plus dM whole square minus 4 into 1 minus d divided by 2 into 1 minus d. Actually you will be getting a quadratic equation and 1 root of this quadratic equation will be this one. So, quadratic equation in terms of k small k and the k is the unknown. So, these expressions you get.

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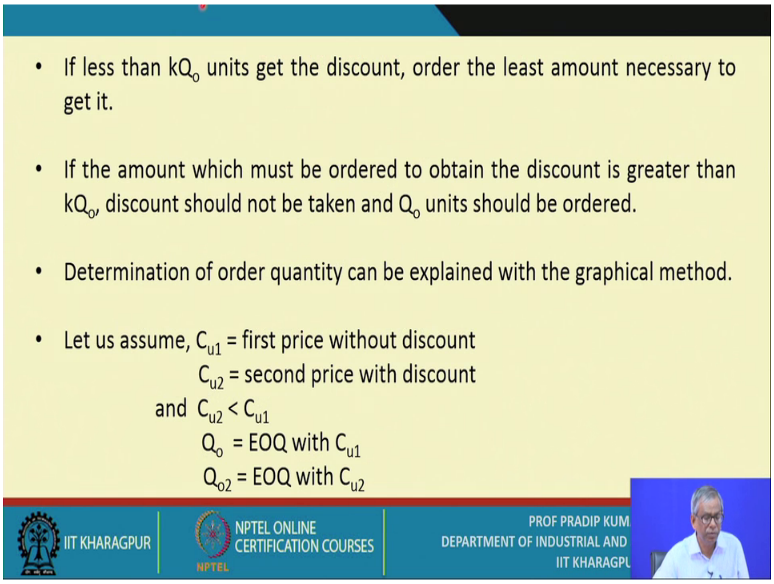

So, if less than k into Q 0 units get the discount, order the least amount necessary to get it. Is it ok? So, that is your decision. So, first you determine the value of k. If the amount which must be order to obtain the discount, please follow this the logic, follow this logic. If the amount which must be order to obtain the discount is greater than k into Q, k Q 0 discount should not be taken and Q 0 unit should be ordered only; that means, original EOQ the forgo discount ok.

Determination of the order quantity can be explained with the graphical method also. So, this is we have just proposed a simple analytical approach to decide if or the discount can be accepted or discount cannot be accepted ok. So, this logic you just follow. Now if you opt for a graphical method, possibly you know you will understand fully that what is the basic logic? Your logic will be made very very clear.

So, let us assume that the C u 1 is the first price without discount. We are that discussing the single price break situation and the C u 2 as the second price with discount. So obviously, C u 2 is less than C u 1 right. Now we use Q o; that is EOQ with C u 1 right and Q o 2 is equals to EOQ with C u 2. Is it ok? So, this notations we are used.

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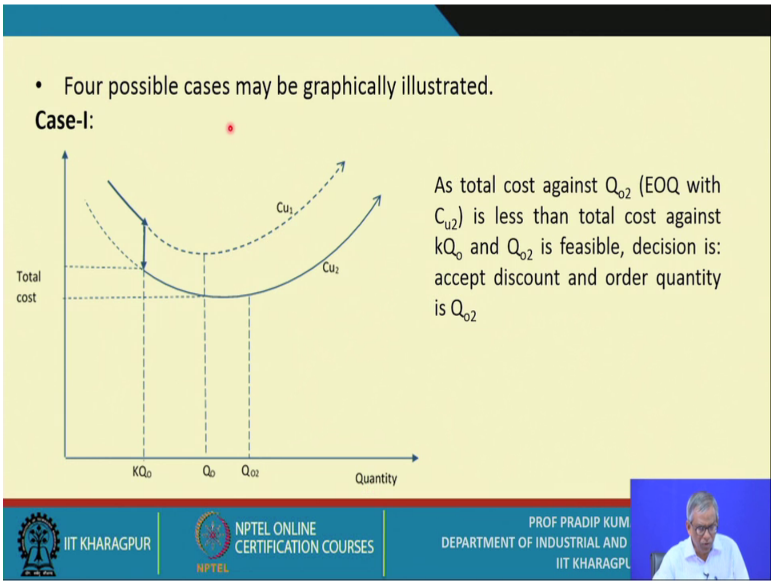

Now, 4 possible cases may be graphically illustrated right. And for each of the case of the cases what you need to determine? You need to determine or you need to decide whether you will accept discount or not. If you accept what will be your the order quantity? That is it. So, what is Case-I? Case-I is just you have the total cost Curve. So, the total cost curve with C u 1; C u 1 is without discount. Whereas, if you opt for you know so, discount with the unit price C u 2, then the total cost curve is this. Is it ok? So, the vertical axis represents the total cost and the horizontal axis you represents the quantity. Now what you try to do?

Suppose you are here. So you move like this and at this point k into Q o. Is it ok? You have Q o over here that is a EOQ with C u 2, but before you reach Q o you are you are getting a discount offer with C u 2. So, immediately you move to the curve of the total cost curve with C u 2 and then you proceeded in this direction. Is it ok? And so, now, the question is and when you move in this direction you come across a point on the cost curve which is the minimum for which the order quantity that is the economic order quantity with C u 2 that is Q o 2. Now the question is whether you will accept discount or not? So, as total cost against Q 0 2; that means, this one ok.

So, now, right now your here you moving along this line is a real one, is a feasible alternative. So, you reach over here. So obviously, the total cost against Q o 2 is less than the total cost agent's k Q 0. So, k Q 0 the total cost is this, whereas, if you traverse on this path you get a order quantity Q 0 2 which is a minimum. So, and Q 0 2 is feasible, decision is accept discount and order quantity is Q 0 2. It is clear?

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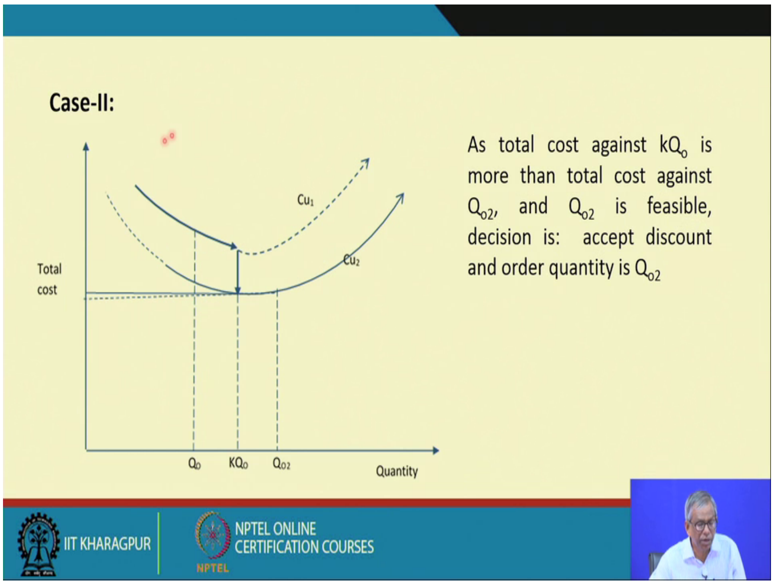

Next one is the Case-II. Now you are moving on this line without discount, C u 1. You reach here and you the cross Q o also over here over here Q o, it is a minimum line. Now you reach over here and you are offered a discount at k into Q o. Is it ok? So, here obviously, the value of k is greater than 1, in the previous case the value of k was less than 1; so, 8 is greater than 1. Now from here you would you reach this second curve and the second curve represents the price discount with the price of C u 2.

Now what will be the decision? that means, you can path like this and has a total cost against k Q 0 is more than the total cost again Q 0 2; again the Q 0 2 is a feasible ok, you can reach Q 0 2; the decision is accept discount and order quantity is Q 0 2. So, that is the second case.

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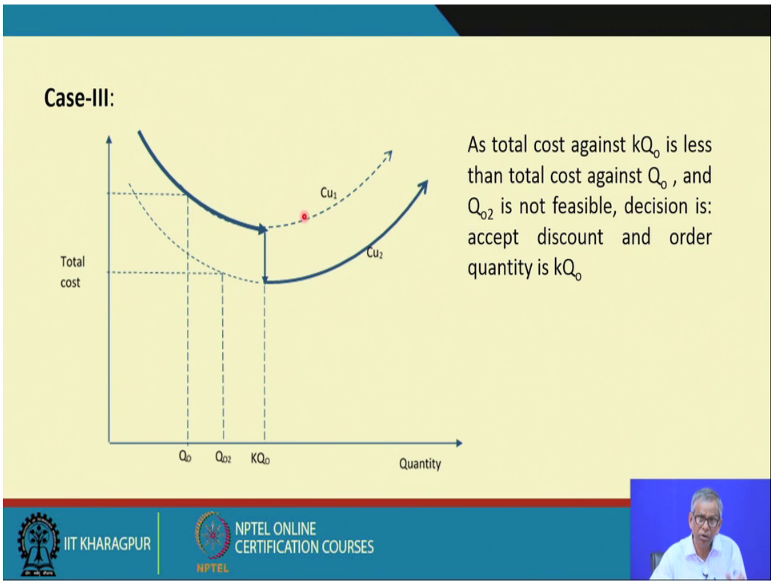

Now, you can come across Case-III also; that means, now you are moving along this is the first curve with C u 1 without discount now, when you cross Q 0 2 over here, ok, then the second curve. Now at this point you are offered a discount. So, you move to the second curve representing the total cost with discount and the discount the prices C u 2; unit price is C u 2. So, you reach over here.

So, here as the total cost again kQ o is less than total cost against Q o and Q 0 2 is not feasible because it cannot go back that is why there this portion of the curve is shown as dotted. So, the decision is accept discount or the order quantity is this cannot be Q 0 2; it is k into Q o because against k into Q o the total cost is this, but against Q 0 2 the total cost is higher. So, that is why you accept the discount for the order quantity is k into k 0.

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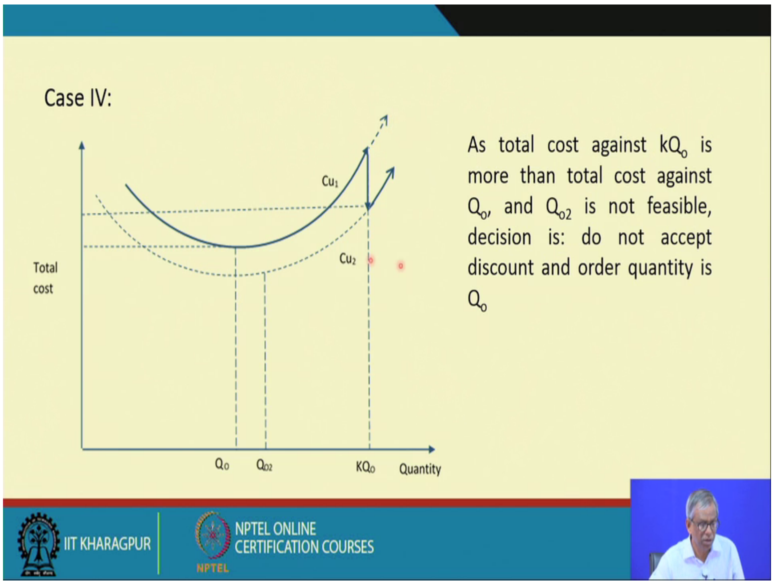

Now, comes the forth case. So, the Case-IV what do you find? So, you move along this line and at this level k Q 0 ok, you are offered the discount. Is it ok? So, as total cost against k Q 0 is more than the total cost against Q 0 and Q 0 2, Q 0 and Q 0 2 is not feasible ok. So, we cannot get this Q 0 2 over here, these on a dotted line; decision is do not accept discount and order quantity is Q 0 because Q 0 is already your traverse. So, this is feasible. So, Q 0 is feasible. So, here the Case-IV, do not accept discount, order quantity is Q 0. So, these are the 4 cases you come across.

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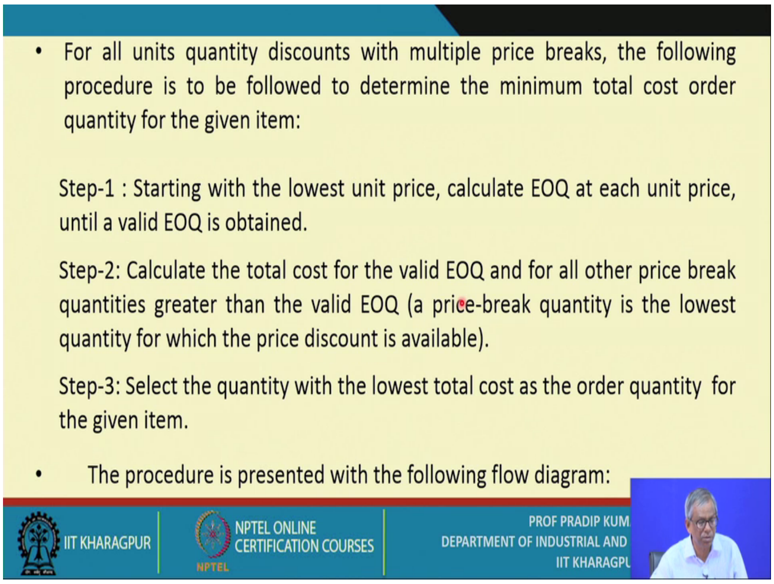

For all units discount quantity discounts with multiple price breaks, the following procedure is to be followed to determine the minimum total cost order quantity for the given item. So, when the multiple the price breaks offered, we have already explained: what is this multiple price break situation. Now, the certain steps you have to follow. The Step-1: Starting with the lowest unit price, calculate the EOQ at each unit price, until a valid EOQ is obtained that is step 1. Step-2: Calculate the total cost for the valid EOQ and for all other price break quantities greater than the valid EOQ.

A price break quantity is the lowest quantity for which the price discount is available ok. So, we will take up one example, it will be made very clear and in the Step-3, in the last step what do you do? Select the quantity with the lowest total cost as the order quantity for the given item. So, you just follow these 3 steps and you will know how to calculate or how to determine the order quantity in the multiple price break situation. So, what we have done to the entire procedure? We present with a flow diagram.

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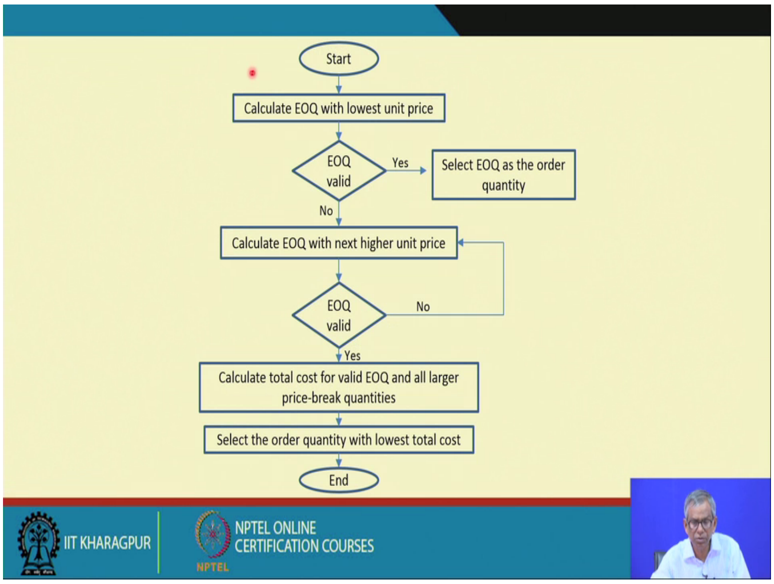

So, what is this flow diagram? You start calculate EOQ with the lowest unit price. Is it ok? So, I just find out lowest unit price and then you find out EOQ ok. Is EOQ valid? That means, when you determine EOQ with this price, now against this price the order quantity is mentioned. Now, whether this EOQ falls in that falls in that range or not? If it falls in that range it is called the valid EOQ, if it does not fall in that range it is referred to as invalid.

So, supposing EOQ is valid, select EOQ as the order quantity. If it is it is not then calculate EOQ with the next higher unit price. Suppose the lowest unit price is rupees 10 ok, at the next higher unit price maybe rupees 11. So, you calculate again EOQ with rupees 11. EOQ valid the same question you ask, if it is no then you go to the next one; next one could be 12, next to next one could be 13. So, this process continues till you get a EOQ quantity or EOQ which considered valid. So, if it is yes, now what you do? Calculate total cost for valid EOQ and all larger price break quantities. Is it ok?

So, maybe beyond EOQ the valid EOQ you may have the 3 or 4 larger, so the price break quantities. Is it ok? That is the minimum quantity against a particular price. Is it ok? Offered so, that is basically called the price break quantities. So, suppose the 3 more price break quantities you have, so all together you have 4 say the quantities. So, against each quantity specified, you determine the total cost and select the order quantity with the lowest total cost that is it and just you remember that the total cost includes the total purchase cost also. So, this is a numerical example.

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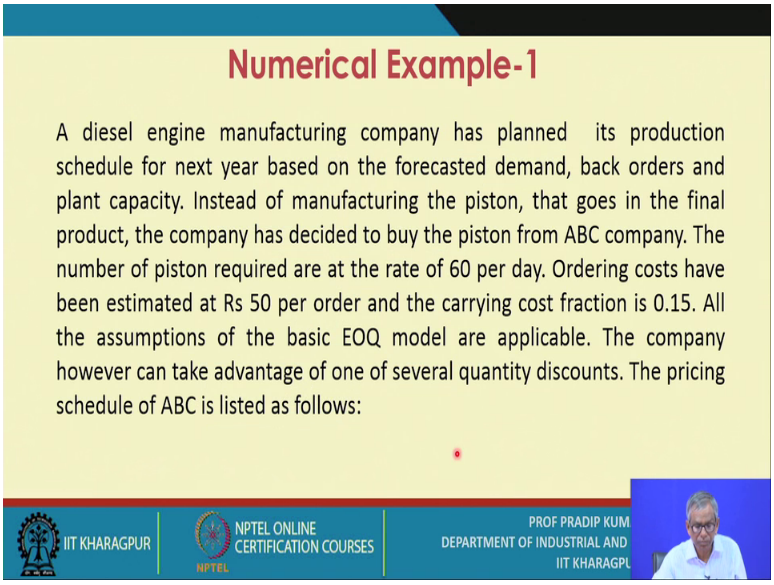

So, you just go through these numerical examples and you will you will have all the data and all the assumptions of the basic EOQ module are applicable that we are assuming, there are 10 assumptions. Now, just to make a note that that EOQ model is widely applicable even if there are say the 10 assumptions; the question is why? Obviously, you will find that the majority of the organisations. So, the hundreds and thousands of organisations there, they are been using EOQ model original EOQ model classical EOQ model, when fully well there are 10 assumptions.

So obviously, someone might say that if you have 10 assumptions; obviously, you know you are in an ideal situation. But still many companies they have been using EOQ model original EOQ model and as and their getting a positive results and that is why they are accepting it. Now, the question is why? Even if there is a 10 assumptions, why do you use your EOQ model and what benefit? So, this point will be discussing later on. This is one of the key questions the raised in any discussions on inventory management ok. So, this is the quantity is numerical example the quantity order is this is your multiple price break situation.

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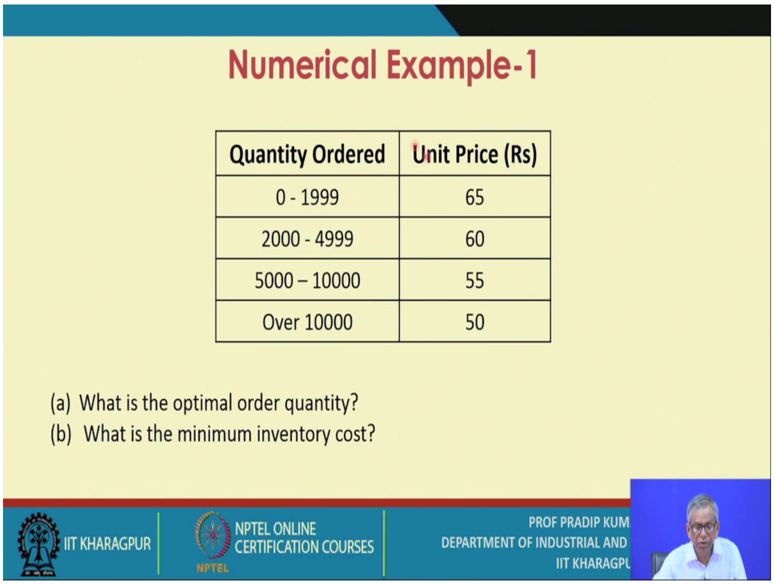

So, if the quantity order is between 0 to or maybe say 1 to 1999, it should be 1 ok; 1 to 1999, it is 65, between 2000 to 4999 it is 60, like this and you have this table with all the data and over 10000 it is just 50. What is the optimal order quantity? What is the minimum inventory cost?

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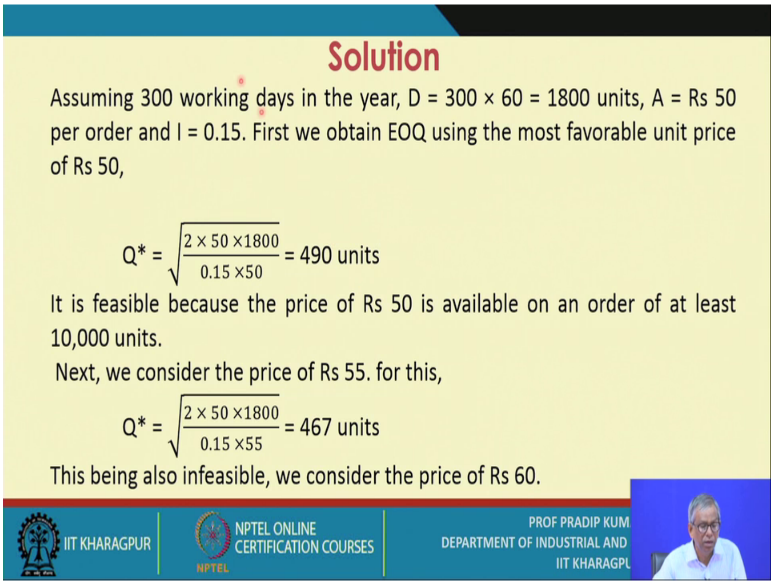

So, assuming 300 working days in the year, so, you have this annual demand that is d, A is also known ordering cost and small i is 0.15 percent small i. So, first we obtain EOQ using the most favorable unit price that is the lowest price it is 50. So, it is Q star 490. Now it is feasible because the price of rupees 50 is available on an order of at least 10000 units. So, it is ok.

So, next we consider the price of rupees 55. So, again you determine Q star 467 units, these being also in feasible. We will consider the price of rupees 60. So, this is in feasible actually 490 with rupees 50 you cannot get. Is it ok? So, rupees 50; so, rupees 50 over 10000 only so; obviously, you can’t get. So, this is infeasible, this is also infeasible.

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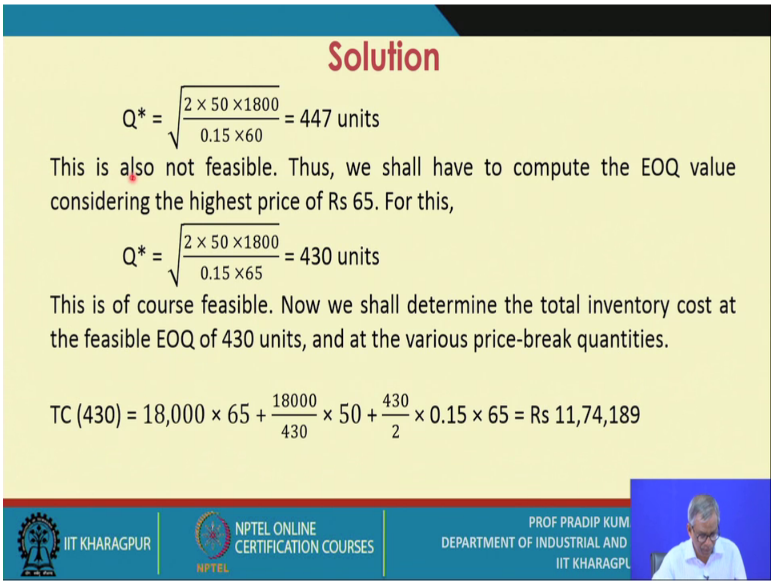

And the next one is next we try with 60, this is also not feasible. Thus we shall have to compute the EOQ value. Consider highest price of rupees 65. So, when you consider the highest price 65, the EOQ becomes 430 units.

So, this is of course, feasible. So, you just refer to the table. Now, we shall determine the total inventory cost at the feasible EOQ of 430 and at the various price break quantities as we have already pointed out. So, total cost at 430 comes down to this one and like this the next price break quantity is 2000, next one is the 5000, you calculate the total cost for 2000 or 5000 or 10000.

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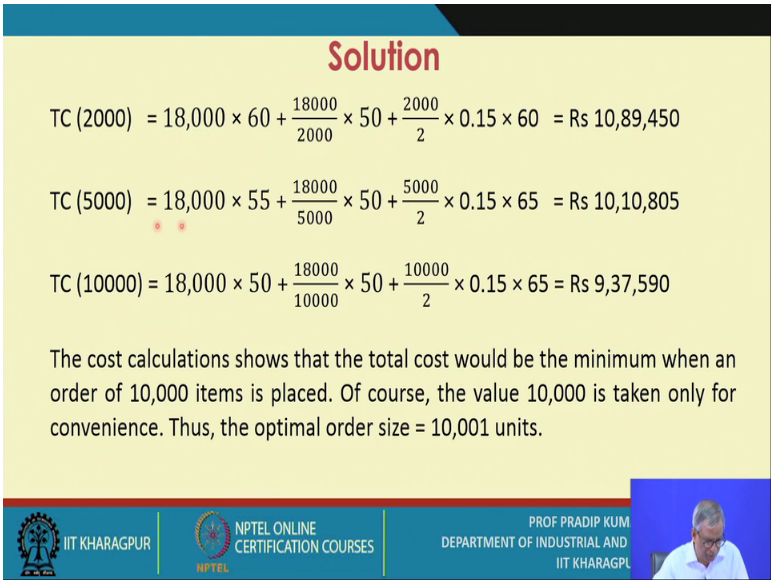

These are the so, the higher the price break quantities. So, these are the values and so, the cost calculation shows that the total cost would be the minimum when an order of 10000 items is placed. Of course, the value of 10000 is taken only for convenience thus the optimal order size is 10001units ok.

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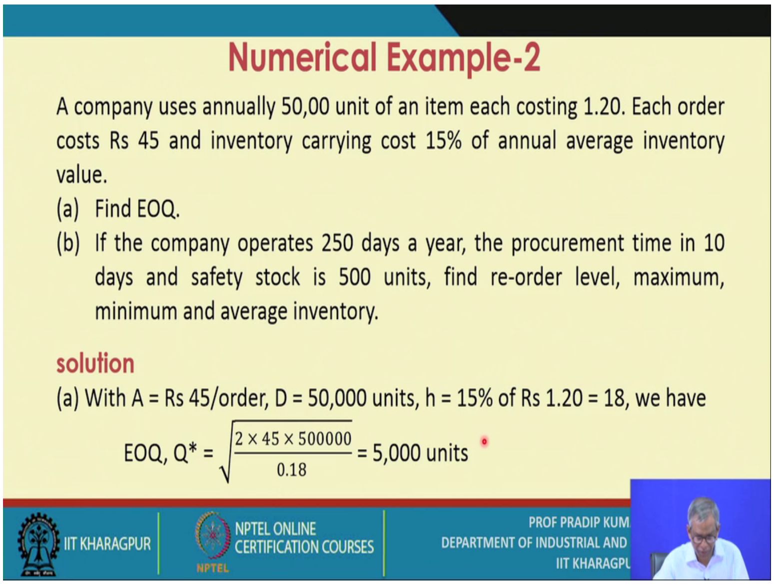

So, follow the steps. This is the second examples the related to say the EOQ and reorder level. So, here also what you have; that means, you have to find EOQ and the operating days per year of the company that the data are given that is 250 days, the procurement time is it in 10 days and the safety stock is 500 units. So, you need to determine the reorder level, maximum, minimum and average inventory. So, you have all these data. So, please follow the steps all the data are given. So, you can use the EOQ expression Q star, is it ok? That is the 5000 units.

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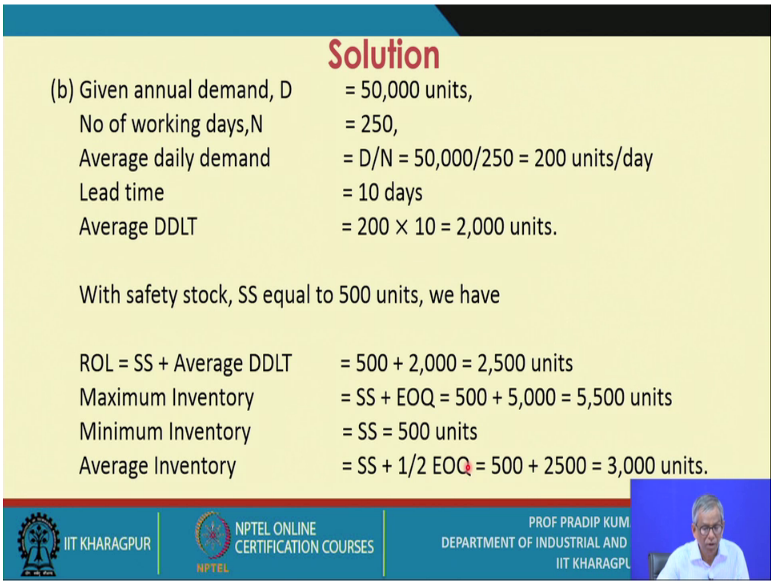

And then; so, annual demand 5000 units, number of working days 250 days, average daily demand that you calculate. The lead time is the lead time is a 10 days and the average the demand during lead time; obviously, it is the demand is uniform the demand is known with certainty. So, the average demand you need lead time is 200 into 10 that is 20000 units so, with the safety stock equal to 500 units.

So, we have the reorder level that is the safety stock plus average demand during lead time. So, that is 500 plus 200 that is 2500 units. So, what is the maximum inventory? Maximum inventory is safety stock plus EOQ, so, 500 plus 5000; 5500 units. So, minimum inventory is a safety stock that is 500 units and the average inventory is safety stock plus half of EOQ; that means, Q by 2; that means, 500 plus 5000 divided by 2 that is 3000 ok. So, this is your EOQ is 5000. So, this is 3000 units is your average inventory.

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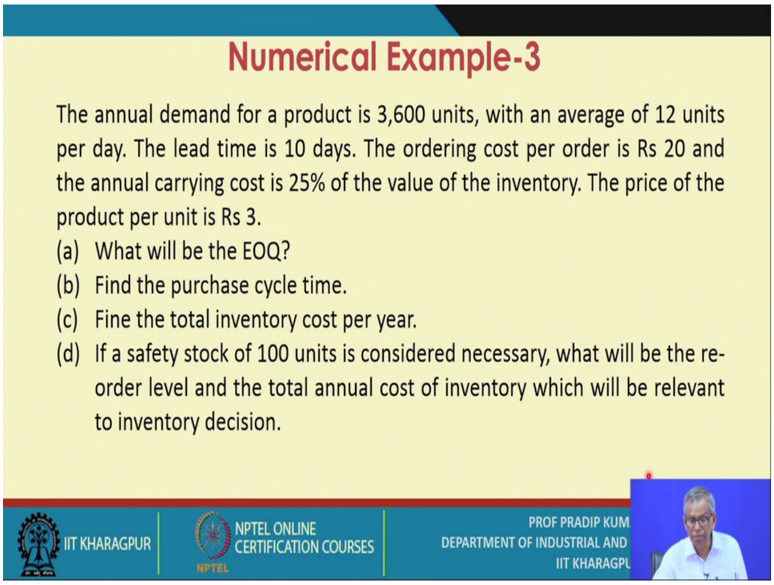

Now, so let us the discuss the third example numerical example. Let me read it out. The annual demand for a product is 3006 units with an average 12 units per day. The lead time is 10 days, the ordering cost per order is rupees 20 and the annual carrying cost is 25 percent of the value of the inventory is considered to be very high, 25 percent. The price of the product per unit is rupees 3. What will be the EOQ? Find the purchase cycle time; find the total inventory cost per year. If a safety stock of 100 units is considered necessary, what will be the reorder level and the total annual cost of inventory which will be relevant to inventory decisions?

Now, here just you make a note that you have one parameter called ROP or ROL. Now this ROP that means, reorder point or reorder level, consist of 2 parts; the first part is the average the demand during lead time and the second part is the safety stock. Now, usually you know the safety stock in the when you have a probabilistic the demand then, only you need to determine the safety stock and because of the fluctuations of demand during lead time as well as the fluctuations of the lead time itself. So, you need some extra stock, buffer stock we have already explained. So, here in this case as the problem is under certainty; that means, demand is known with certainty, then obviously, the safety stock determination is not at all a problem.

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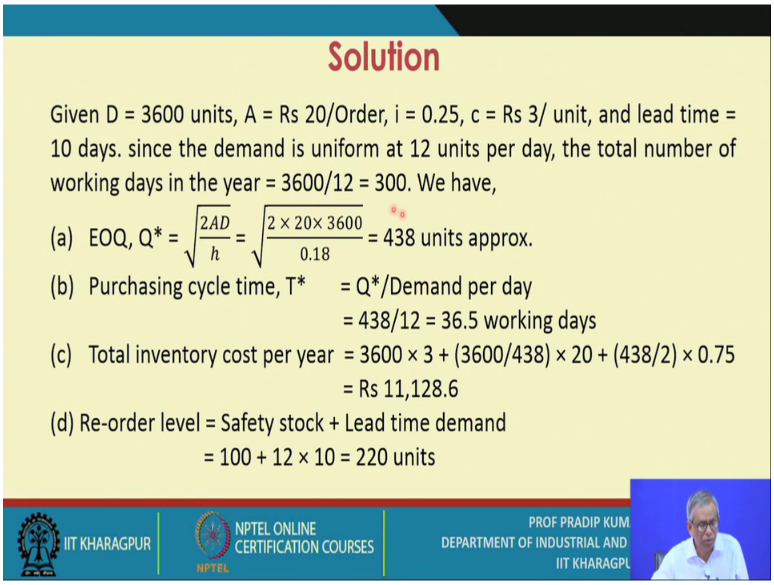

So, what you have? So, we have all these values. So, corresponding values we have. So, EOQ you can determine root over twice 2 A D by h. So, 438 unit. Purchasing cycle time; that is Q start for demand per day, is it ok; that is 438 by 12 that is 36.5 working days. This is just make a just you check that how it is calculated and all. Total inventory cost per year, is it ok?

So, this is 3600 into 3 plus 438 into 20 plus 438 by 2 into 0.75 that is this one and the reorder level is a safety stock plus lead time demand. This point already we have elaborated that is 100 plus 12 into 10; that is 220 units.

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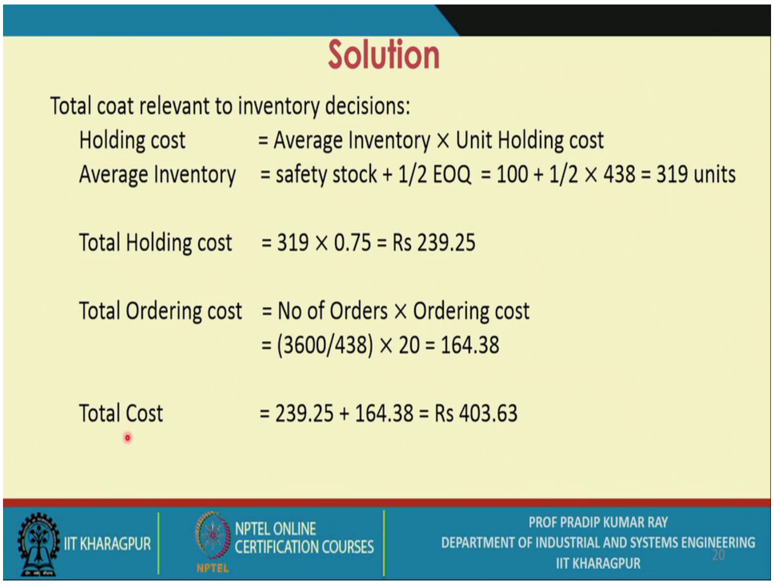

So, the total cost for the total cost relevant to inventory decisions the holding cost average inventory unit holding cost is the average inventory you can calculate. The total holding cost also you can calculate, total ordering cost also you can calculate. Is it ok? That means, the total number of orders into the ordering cost per order, is it and the total cost just you add. So, you can also calculate, 239; total holding cost and total ordering cost if you had them you get the total cost.

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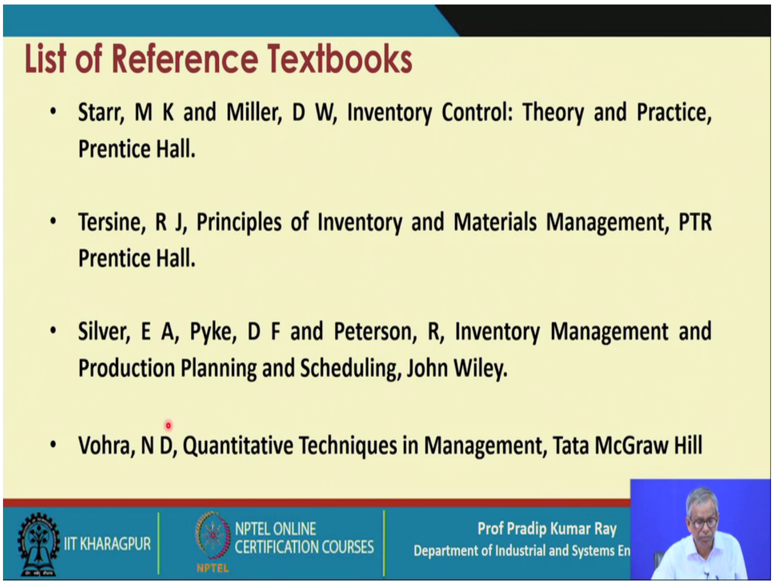

So, this is so, these are the text references. So, you go through and what we have what we have actually explain, what we have described, what we have you know in dwelt weight is that what are the what are the EOQ models and available under different conditions; like original EOQ model, the EPQ model, EOQ with the shortages and EOQ with the discounts.

Now all these problems you frequently come across and the typical you should be also aware of the typical numerical problems. So, we have sited if you are representative numerical problems. So, please go through them and I am sure that with this a the basic the knowledge in inventory the dynamic inventory problem under certainty. So, till now till this now we have assume the there is no constraints. In the next lecture session, what we will do? We will try to determine that order quantity or optimal order quantity. Always this is referred to as economic order quantity or economic production quantity under constraints, is it that. So, that treatment will be slightly different.

We will discuss the problem formulation and how to determine the relevant; so, good how to get an expressions or how call to derive the expressions for the optimal order quantity under constraints ok. So, more or less we have covered all the important aspects related to dynamic inventory problem under certainty.

Thank you.