Industrial Safety Engineering
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Lecture – 38
Systems Safety Quantification: Minimal Cut and Minimal Path Representation Using Structure Function

Hello everybody, have a nice day today. We will continue Structure Function. Today's topic is Safety System Safety Quantification Minimal Cut and Path Representation Using Structure Function.

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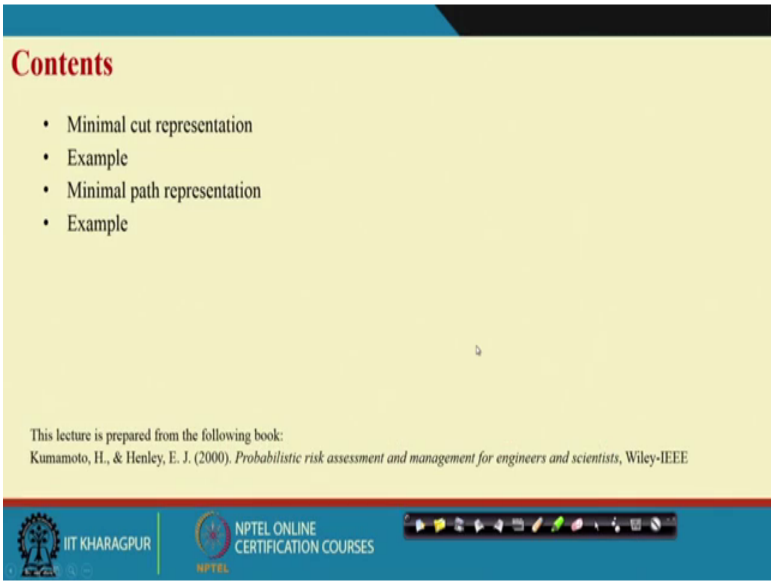

The content of today's presentation, we will start with minimal path set and then, see the structure function using minimal path sets. Then, a couple of examples we will discuss then, we will go to minimal cut sets, where the same fault tree will be converted in such a manner that, you will get the minimal path sets. And then, with set of a we see the structure function using minimal path sets and then same examples number of examples will be discussed.

And now, we have taken material from this book Probabilistic risk assessment and management for engineer and scientist, written by Kumamoto and Henley. Hope you will enjoy it and I understood also that in the last class when we have discuss the structure function, the fundamental mathematics behind each also been understood by you. So, this is in continuation with the previous lecture.

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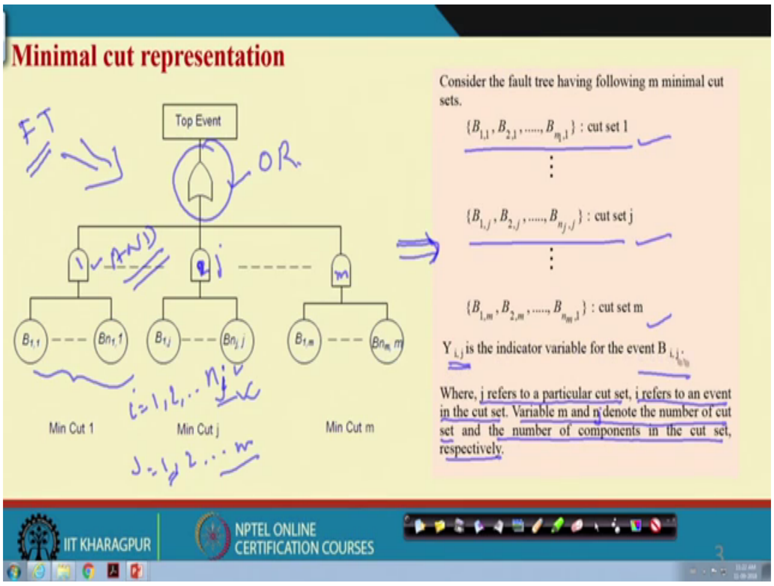

So, all of you know cut set. So, you go back to that lecture, where we have develop the cut sets for a fault tree using Mukhas algorithm. And, you will see that, the cut set particularly is a set when and if it is a minimal cut set then the all the events of that cut set if occurs then, the top event top failure event will occur.

So, as a result, this is one cut set and with an this cut set with an AND gate linked with the top event using OR gate also, so the and because, all those basic events would occur for the top event occur. So, this is cut set one similarly, second one cut set 2 cut set 3. So, like when you have the cut sets, minimal cut sets with you for a particular top event then, you can represent that in this form of fault tree. So, that mean why it is OR gate, any one of the cut set if occur lead to the top event to occur. Why and gate, all the event in the cut set must occur for that of event to occur.

Now, this explanation you have already seen. So, now what we well just formally go for the structure function of this particular fault tree. Then, we will define the basic events in the cut sets So, one is that how many cut sets we have let the, this is cut set 1, cut set 2 and this is cut set m.

So, we will basically, use that J equal to 1, 2 m to represent the number of cut sets. And then and against under each cut set, there will be n number of events. So, for the first cut set there is n 1 events, second cut set there will be n 2 basic events like J th cut sets there will be n j basic event. So, if I represent this as j, so, this is n j basic events.

So, that will be represented by i. So, i equal to 1 to n i ok. So, 1 2 n j, if it is the jth cut set 1 2 nj this is the notation. Using this notation, we are defining this cut sets. What is the first cut set? B 1 1 B 2 1 to B n 1 j equal to 1, j equal to 1 So, B n 1 into 1, this is the first cut set, similarly, the jth cut set, similarly m th cut set.

Now, if we want to define indicator variable for each of the event then, we also require to use i and j. So, Y i j is the indicator variable for event B i j; i stands for the event basic event, j stands for the particular cut set. So, that is what is written here that, j refers to a particular cut set, i refers to an event in the cut set, and m and n that variable m and n denote the number of cut set and number of component in the cut set respectively.

So, if you write m is number of cut set then, n j if I write here that the number of event basic number of component in the jth cut set, so that is it is you can write. So, let me repeat, first we have a fault tree, from the fault tree you develop the founded all the cut sets minimal cut sets. So, from original fault tree to this minimal cut set fault tree that, equivalent representation of original fault tree is this where, as we know that a cut set means a set if, all the element of that set occurs then top event occurs. So, if there are n cut sets that mean ways the at least independent ways that, m anyone of the cut sets leads to the top event to occur at least.

So, what will happen that, there will be 1 or 2 cut set, 3 cut sets there can be means m many number of cut sets a many number of combinations So, that is why here OR gate is used and here AND gate is used that is, for any fault tree when it is represented by cut sets then, the equivalent fault tree using cut set is this. And then, they are that for their notation for the basic events and or the cut set and the indicator variable will be like this ok. So, what happened now; that means, we have the fault tree with minimal cut sets and we have defined the notations.

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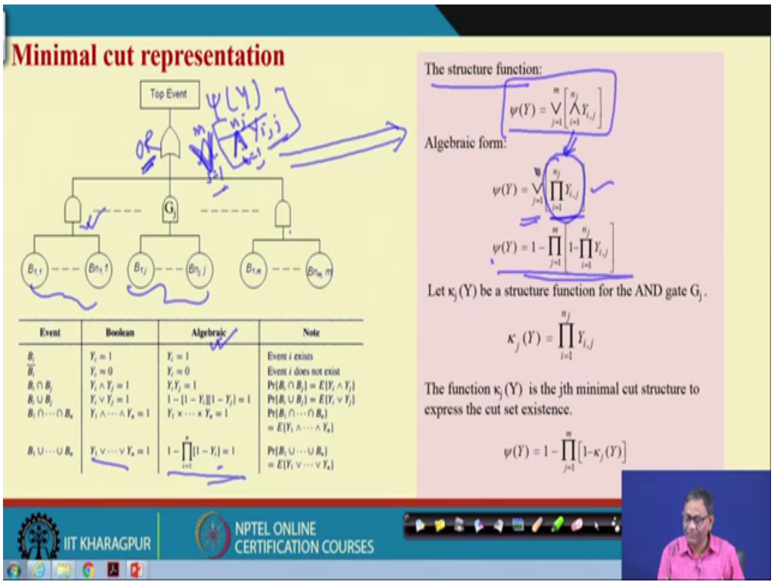

So, now what do we want to see that what will be the structure function for it. So, let us see the structure function for this fault tree or this fault tree. So, structure function for the top event that we are writing psi Y and it is an OR gate, so we will be using this symbol. So, how many inputs are there, there are m inputs, so j equal to 1 to m, m inputs. Now, again each of the AND gates, each of the AND gates there are nj number of inputs.

So, because of AND gates and there are so many, so many AND gates. So, what we will do basically, we will put the bracket and then we will write that intersection i equal to 1 to n j indicator variable is Y ij. Let me repeat because, of OR gate you have to use union symbol ok, so j equal to 1 to m union symbol. Because of n AND gate you are using intersection symbol. AND gate inputs are B 1 j to B n jj. So, that is why i equal to 1 to n j Y ij.

And OR gate in the union symbol, how many inputs m inputs that is why j equal to 1 to m; that is what is your structure function here. So, that is what you written here. So, psi Y equal to j equal to 1 to m union then, intersection i equal to 1 to nj Y ij or this can be represented like this where this union is there this is nothing, but the multiplication i equal to 1 to n j this, so psi is this. Now, when there is union, so you know that, this is basically the union case and when we put down the algebraic form you have to write like this, 1 minus i equal to 1 to n this. So, that is what you written here. So, psi i psi Y equal to 1 minus j equal to 1 to m 1 minus j equal to 1 to m and this quantity will be 1 minus j i equal to 1 to n j Y ij ok.

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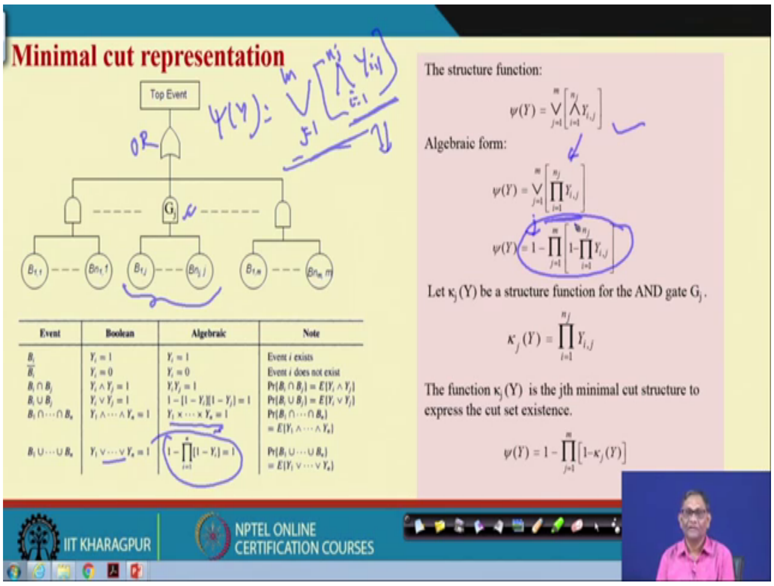

So, let me repeat, so that you will not get confused. First one, we were defining psi Y. What is psi y? Structure function for the top event. There are 2 gates, OR gate, which will be denoted by union, then there are another the gate combination, so the several AND gates ok. So, now, how many AND gates are there? That is j equal to 1 to m, how many inputs to a particular AND gate, that is i equal to 2 to nj. What is the indicator variable? Y i j so, this is my structure function.

When you each type when you go for writing it in algebraic form, this one that the intersection part you say that intersection part is these this is basically multiplication one. So, the intersection part, this part is converted to this multiplication, when you write union you will be writing in this form. So, but then this union part you are writing in this form, that you have seen earlier in last class ok.

Now, what we do, in order to get rid of this clumsy equation, we will put a structure function which is K j Y then, for AND gate for this gate. So, for this gate you know that the AND gate means, it will be the intersection, so that mean K j Y is denoted by like this, how many inputs nj inputs. So, i equal to 1 to n j Y ij. So, this is basically, structure function for the AND gate.

So, then for this what will happen, here this equation will be changed to this, 1 minus j equal to 1 to n then, intersection j equal to 1 to m 1 minus K j Y. ok. So, this is: what is your structure function representation for the minima structure function for the minimal cut set representation a fault tree. So, as in what happened, this is the final form and this one is the structure function for AND gate. So, once you know these now, given a fault tree you can very easily find out the final structure function. So, that is what we will see now.

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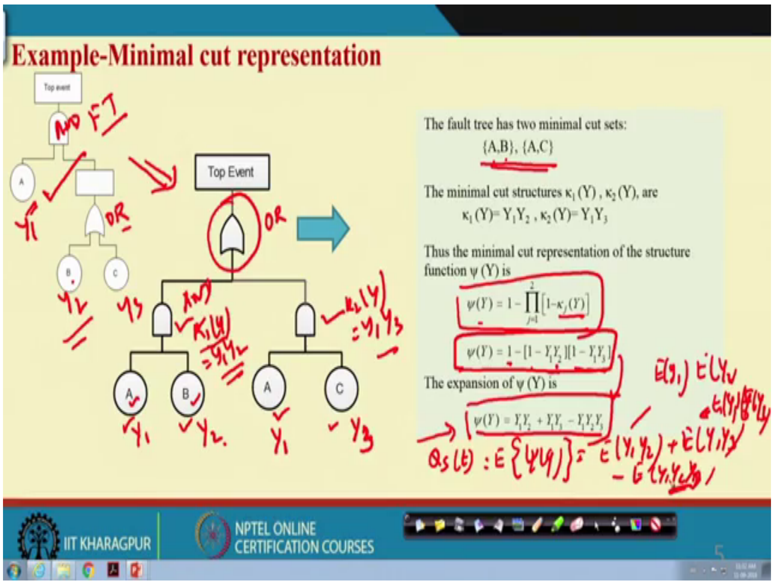

Let us concentrate on this example. So, this example, this example, this is the fault tree, A B C basic events, 3 basic events, it is an and gate, this is OR gate, this is the starting point. We have converted these 2 equivalent fault tree using cut set of this fault tree. What are the cut set, cut set will be what will be there will be 2 cut sets, A must occur, B or C occurrence will lead to so, A B and A C. So, there are 2 minimal cut sets A B and A C.

So, this one is represented by like this, so n we know the cut sets. So, first you will be you must know, how many cut sets here 2 cut sets, so 2 AND gates will be there. And input to the every AND gate will be the element of the cut set, first one is A and B, so A and B, second one A and C, A and C then, these AND gates will be linked with the top event by an OR gate so OR gate ok.

Now, what we will do, we have earlier seen that for minimal cut representation what is the structure function, the minimal cut representation the structure function is this. So, first find out the structure function for this, so this one will be K1 Y. K 1 Y is here A and B so, that means, this is Y 1, this is Y 2, again this is Y 1 and this is Y 2 if we say. So, A B, sorry A B C Y 1, Y 2, Y 3, A is Y 1, Y 2, Y 1, Y ok.

So, now this is and gate, so it will be Y 1, Y 2, K 1 Y is Y 1 Y 2. Now K 2 Y is Y 1, Y 3. So, you have K 1 Y and K 2 Y now, the structure function representation with OR gate will be this. Now, put 1 minus this one j equal to 1, 1 minus K 1 Y and j equal to 2, 1 minus K 2 Y, K1 Y each, Y 1 Y 2 and K 2 Y is Y 1 Y 3, so this is your structure function.

Now, what happened if you do little more exercise manipulation then, you will be getting this expression, So, Y 1, Y 2 plus Y 1, Y 3 minus Y 1, Y 2, Y 3 so, instead of using this structure this minimal representation, if you simply start with this and use the structure function you will also come to this equation ok. So, but when you make minimal cut representation things become easier because, of this simple structure and or an AND gate only. First find out the AND gate structure functions then, use the or formula, algebraic form and get the final structure function.

And now, if you want to find out the QS t at time t equal to 2 this should be expected value of psi y then, expected value of everything. Expected value of Y 1 Y 2 plus expected value of Y 1, Y 3 minus expected value of Y 1, Y 2, Y 3 as, Y 1, Y 2, Y 3 are independent. So, this will be multiplication of the expected value, this will be multiplication of its expected value expected value of Y 1, expected value of expected value of Y 3, similarly this ok. So, I hope that now it is clear I hope so, it should be clear.

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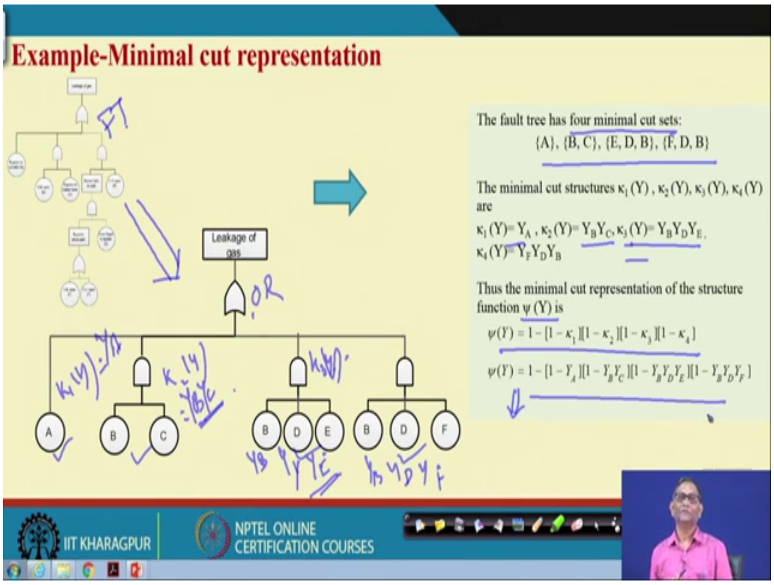

Let us see another example, here you see what is happening, this is my original fault tree. So, we have found out all the cut sets, how many cut sets are there, 4 minimal cut sets. Now, these 4 minimal cut sets this is 1, 2, 3 and 4, first one has no gate it is straight away 1 so directly with this OR gate, here you will be getting K. Suppose if, you write K 1 Y equal to this is basically a K 2 Y, this will be BC, K3 Y this will be BDE like this, so, K1 Y, Y A Y A, K 2 Y, Y B, Y C, Y B and Y C K 3 Y Y D Y B, YD and Y E. We are using the indicator variable, so that is why will not write B and C we will write multiplication of the indicator variable so, this one Y B Y D and Y F.

So, now structure function this is OR gate, you know this formula So, 1 minus K1, 1 minus K2, 1 minus K3, n 1 minus k 4 because, 4 cut sets are there, what happened you just decompose it further to do its lowest form you will be getting the another the final form. And then, you will the expected value and get the Q st and finally, there the availability all those things. Let us see another one.

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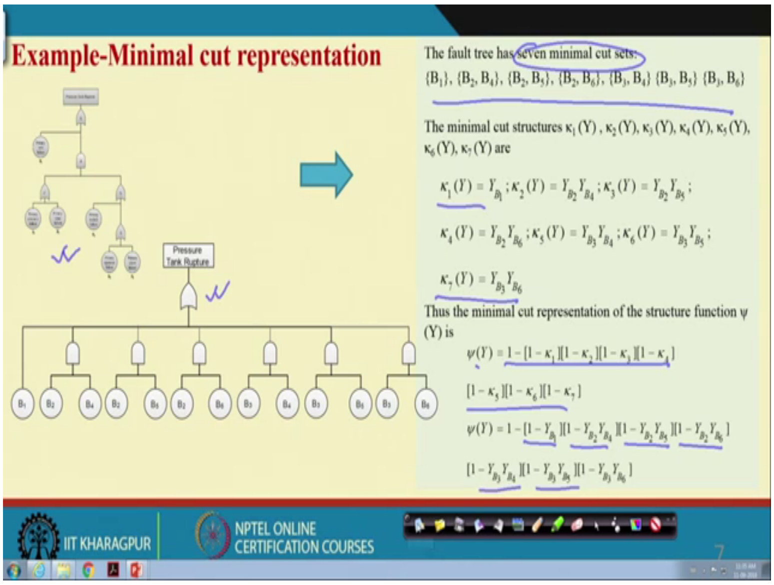

So, here is another example. This is the pressure tank rupture case, so how many cut sets we have, we have so many cut sets, 7 minimal cut sets. So, 7 AND gates, 7 AND gates including this 1, 2, 3, 4, 5, 6 here also one item (Refer Time: 19:49) ok. So, that means, 7 structure functions for AND gate K1 to K 7 and those all those 7 structure function linked with OR gate, used the formula this formula and then finally, you get this and then again go to the lowest level if required, just you check that if in the product term there is repeat events then, you have to go to the lowest level.

If suppose this one B1, here B 2 B 4 here B 2 B 5 yes repeat events are there here B 2 B6, B 3 B 4 so; that means, you go to the lowest level representation and then take the expected value. Otherwise, you have to use some more formula like a partial pivotal decomposition exclusion inclusion formula.

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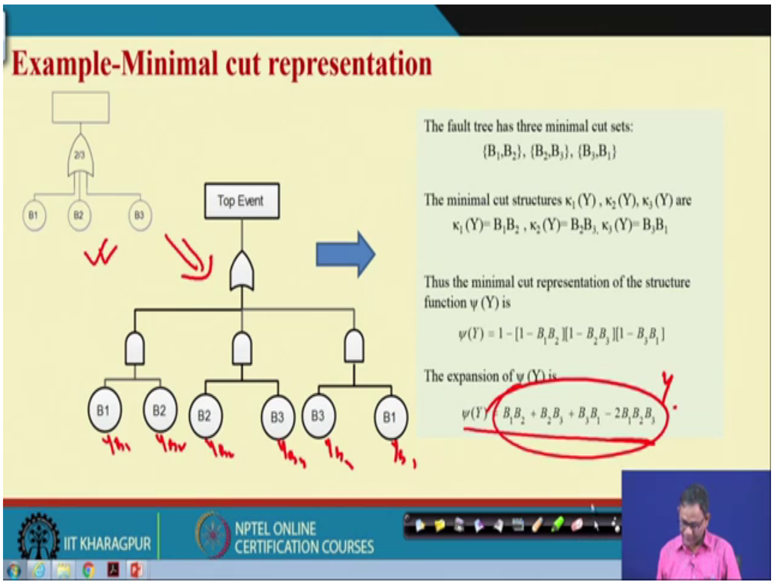

Here you see that voting gate case this voting gate, these voting gate equivalent cuts a representation is this, this structure function you have already seen. Now, here what happened, you will you will you will see that here, let me let me add here just some just some mistake. In the sense of indicator variable, so for indicator variable, this will be we are we are writing like this that this is, Y B1 Y B2 Y B2 Y B3 Y B3 Y B1 So, in that way you will use in terms of Y ok. So, fine so this is not a problem and you can do it very easily you can do it.

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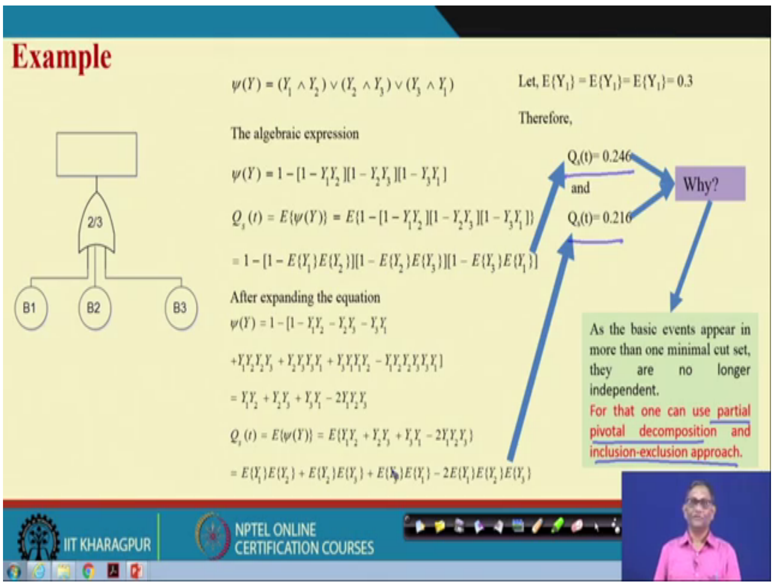

So, now let us see that second one that yeah we are we are using Y 1 Y 2 Y 3 or YB 1 to YB 3 So, if you use YB 1 YB 2 this will again become clumsy. So, that is why straight way we use Y 1 Y 2 Y 3. And this particular example I have shown you in the last class also then, I have shown that the Q st is changing. Basically Q st, if you if you use expected value earlier and then the Q st value and when you go for the lowest level decomposition the Q st value is different and that is what the dependent product terms.

So, wherever there are dependent product terms, so what you will do, you will basically better to reduce up to the lowest level otherwise, you have to use that partial pivotal decomposition method or inclusion exclusion approach method in between to get rid of exploring or digging it into or up to the lowest level of resolution ok. So, I hope that, you have understood it ok. This is what is our cut set tree representation.

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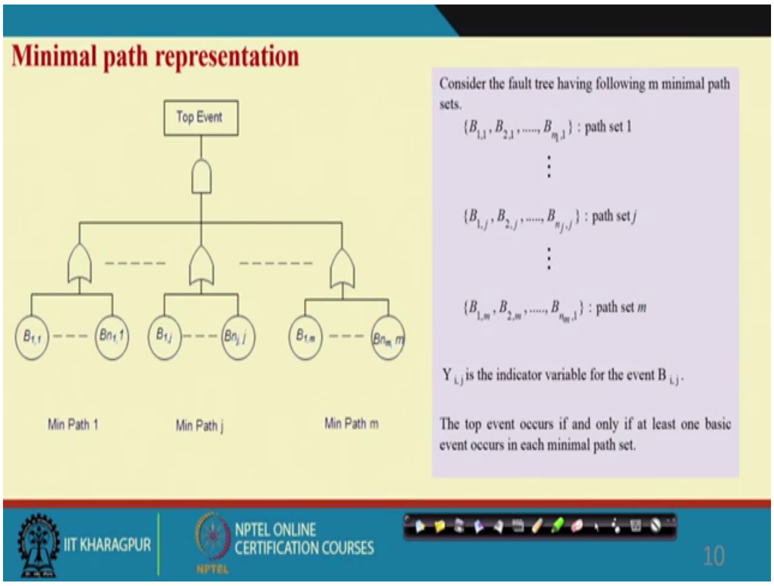

Now, what I will do, I will just go to the second part of to this presentation that the minimal path representation. You have seen that the cut set talks about the failure and path set talks about the success. So, but we are interested in the failure path. So, what will happen if, you create find out the path sets and then and you want to find out the top event failure then, the simple rule is here that in the cut set representation AND gate will be converted to OR gate in the path set representation and cut set representation OR gate will be converted into AND gate in the path set representation, keeping the basic events as failure event and top event as failure event.

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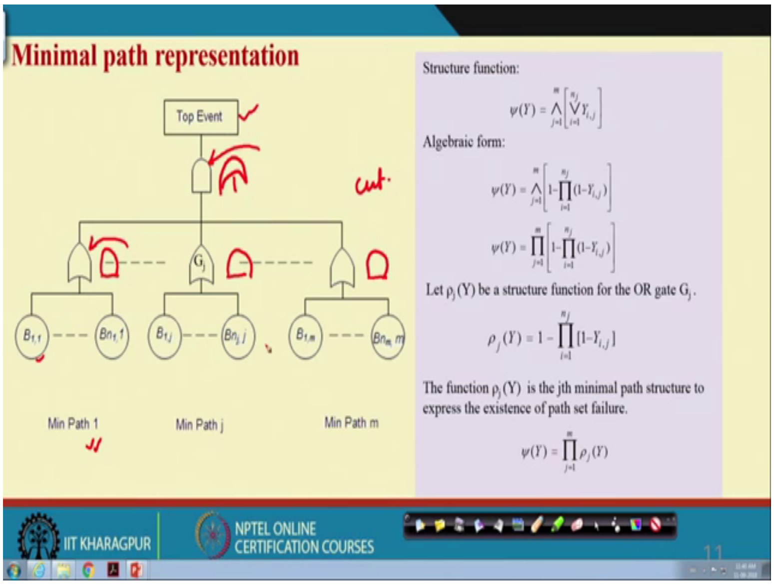

So, as a result if you if you go back and see the minimal path representation in terms of cut minimal cut sets, so you have seen that here we are used sorry, here we have used this OR gate and here everywhere we have used and gate, this is for cut representation So, but the these are all failure events and this is also failure events. in path representation what happened, we keep this failure event and top failure event in tech So, in order to go for path representation, this and OR gate is converted into AND gate, AND gate is converted into OR gate. So, that is basically path representation.

Now, what will happen or when the top event will occur. You see this is OR gate, means anyone of the anyone basic event of the minimal path set here also anyone of this path set. Anyone of these path sets all those things at least one in each of the path set if occur then, top event will occur because this or will be satisfied and finally, and (Refer Time: 25:13) satisfied. So, that is why what happened in the path representation we are basically, saying that anyone basic event in the path set, at least one event leads to the top event to occur but those events for the path sets what you have found out earlier so ok.

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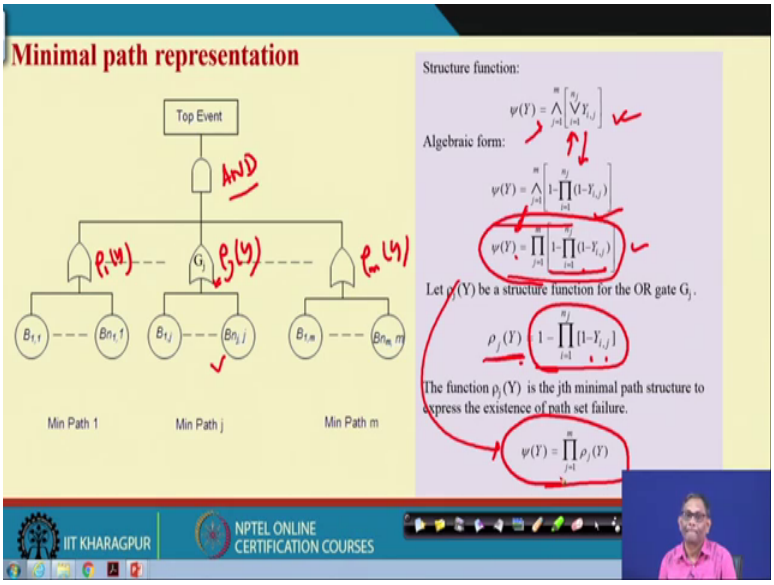

So, with this structure function we have to developed, but structure function looks into the gate what gate, this is my AND gate. So, my structure function the first part is intersection. This is OR gate, so in the second part it is union. So, union i equal to 1 to n j, Y ij is the indicator variable and this is AND gate intersection how many cut path sets m path sets, j equal to 1 to m. So, this is my structure function. So, this one can be written like this now, algebraic form of this is 1 minus this and finally, for this one if we decomposed further, so algebraic form will be this is a final algebraic form.

Now, then, what will happen here now we will we will just find out that structure function here this is for rho 1 Y this is rho jY and this is a rho my that means, structure function for OR gate you find out and then put into this equation So, structure function for OR gate for the jth OR gate we can that like this rho jY equal to this because, it is OR gate, 1 minus that multiplication of all those into 1 minus Y ij, this is written here.

So, now, this one 1 minus this means this, so if I put rho j Y here this function will be like this ok. So that means, if you go for minimal path representation what do you have to do, you have to first find out the structure function for all the or gates, which is basically input to the top event using logic AND gate and then, find out the structure function everywhere and get its algebraic form and then AND gate this is the multiplication form. So, that way it will be written like this ok. So, using this analogy or using this representation we will see all those examples what we have seen for the cut set representation. So, let us see.

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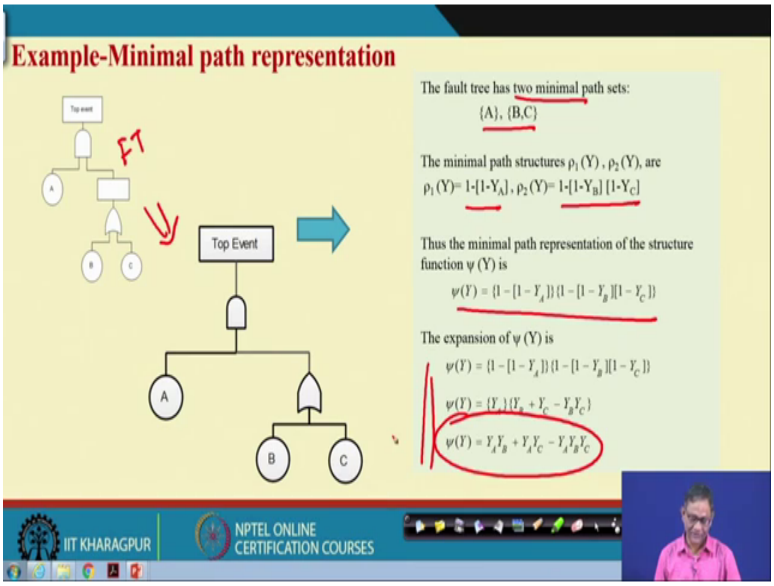

First one, this is my fault tree, this is path representation. So, how many path set, 2 path sets and so here rho 1, rho 1 Y will be nothing, but Y A and rho 2 Y will be nothing, but this ok. So, rho 1 YA rho 2 YA is found and you put into this equation you get this. Final this is my representation for structure function.

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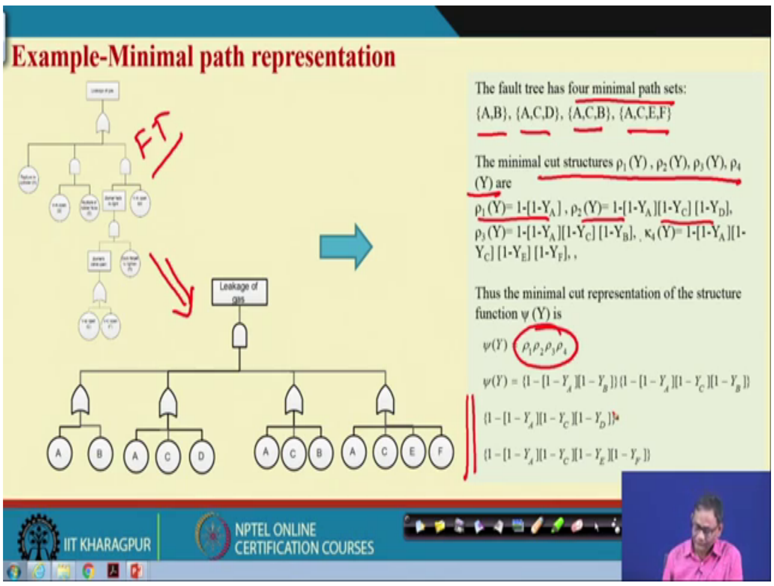

The second example this is leakage of gas, this is original fault tree. Here this is the minimal path set representation, 4 path sets are there A B A CD A CD B and AC E F. And then, we have generated 4 cut set structure function, you found out everyone and then finally, you (Refer Time: 29:14). So, you multiplied all those cut sets structure function and if you decompose it you will be this is the function and further you can multiply and get the values ok. Once you have the psi Y, you know how to calculate the unavailability the same formula will be using.

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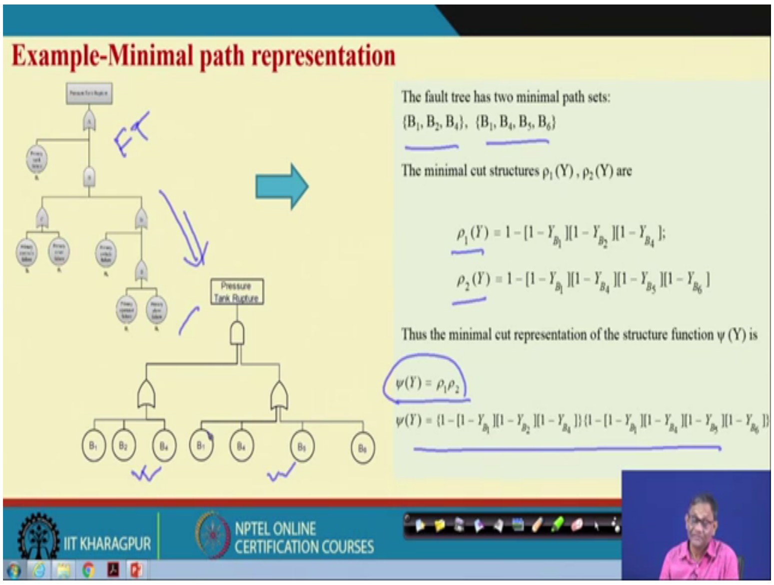

You see this is another one, the set and rapture, this is original fault tree, this is our path representation, how many only 2 paths. But in cut set you have seen that and so many cut sets, here per se 2 paths. So, 2 structure functions for the 2 OR gates then, final structure function it is multiplication and you are getting this equation.

So, I am repeating this for these things because, you will have more example and you will be able to practice and then you find out that, what do you know what way it should be done. It is not a tough task, do not get exhausted by this that different that that equations, like the symbols, do not get over exerted by this. Think for the concept first then, put the equations. So, for your benefit, first is the fault tree then, the path representation, once you have the path sets, the if there are n path sets there will be m or m structure functions for m OR gates and then, final structure function for the top event and that final structure function will be the multiply their multiplication of the or all of all of the OR gate structure functions ok.

And then, rest of the things are algebraic only, you have to just manipulate algebraically and final form you will get. You can use expected value in between if, you find there is no that that terms are not repeating in the product terms or the basic events are not repeating in the product terms. So, otherwise, it is better always better to find out the lowest level of equations ok. By lowest level we are saying that, when that a decompositions top here you cannot further decomposed these things, all those arithmetic operation algebraic operations are finished ok.

So, then another one, so this one you have seen that this voting gate 1 its path sets you found out, 3 path sets impaired because, of the 2 by 3 path and cut sets, having almost that similar structure and you are finally, getting this expression ok. So, this is what is our lecture today.

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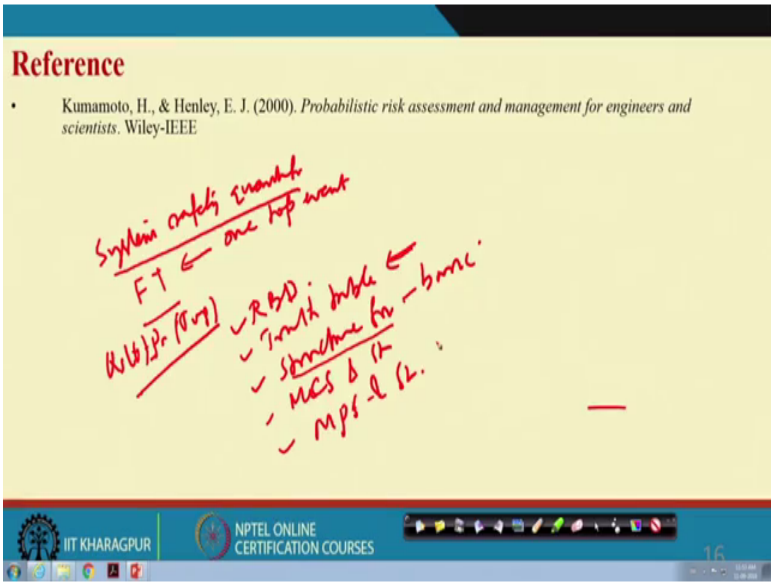

And we have taken the concept from this book and please let me I can say summarize. So, what is this we have started the system safety quantification that is very very important. Now, in system safety quantification, we consider fault tree first. And we have considered that there is a one top event. By unavailability, we say that probability of top event that is the unavailability Q st. Then, once you have unavailability you can find out availability in other things. Our sole purpose in the system safety quantification basically, how to find out the unavailability, that mean how to find out the probability of top event. Probability of top event even your fault tree, there are many methods like get by gate method cut set method all these things are there.

In this particular few lectures last few lectures under system safety quantification, we have started with quantification that RBD, Reliability Block Diagram, then we have shown you that truth table, then we have shown you the structure function, structure function that is the fundamental one the basic one, basic one means from the fault tree only. Then, we have shown you the minimal cut set and structure function finally, minimal path set and structure function.

The sole purpose is basically, make the system safety quantification is to simpler. So, there will be a certain case where RBD will be better, when you have a series system RBD is better, when you have a parallel system RBD is better. But, if there are system like parallel series and parallel combination may be RBD is not that easy. So, there may be a some structure like bridge structure, when you will find out RBD is difficult.

So, there you may find out that truth table is useful. Now, truth table basically is an enumerated one, it will basically give you all possible combinations. So, that is why it is a probably the more accurate one or other ways I can say this is when your knowledge about this total system in terms of the top event occurrence will be much more clear, nitty gritty is known.

Then, what happened then we have gone for the structure function. In structure function, instead of using the basic event probability we have used the indicator variable for all the basic events. Because, the basic events have 2 states either it is event exists or event does not exist. So, that mean then indicator variable Y i equal to 1 or 0 depending or even exists or does not exist. Then with the fault tree AND and OR gate structure what happen the structure function for the top event is possible where the indicator variable Y psi Y you can do 1 or 0 depending on top event exists or does not exist.

Now, when the fault tree will be very big then, you will it will be very difficult for you to find out the structure function using the from the basic fault tree, it is better to convert the fault tree to that equivalent in that minimal cut fault tree or minimal path fault tree. And then, what will happen there you will be having only 2 sets basically, 2 gates OR and AND gate.

So, in case of minimal cut set, you will be finding out the all AND gate structure function first and then the OR gate using OR gate the top event structure function will be created. And for the minimal path set, what you will do you will first find out the OR gate structure function and then final another one m AND gate structure function where inputs will be the all the path sets. So, when there is a AND gate, it is basically simply multiplication when there is OR gate then it will be 1 minus that multiplication of 1 minus all indicator variables.

Suppose and if a that means, there will be product terms in the structure function representation 1 minus some product term and in the product term, if you find out there are dependent, dependent basic event means the event appearing in more than once in different different product terms. Under such situation you it is recommended that you basically, explore or find out the lowest level equation and then use the expected value of indicator variables to find out the top event, top event available, top event occurrence or unavailability of the system.

If the product terms are independent then you do not require to decompose these 2 the lowest level and in between you can use the expected value of the indicator variable and your solution Q st which is the unavailability of the systems will be calculated So, this is what is the what is your system safety quantification. And all of you please go through the book written by that Kumamoto and Henley Probabilistic risk assessment and management for engineers

Thank you very much.