Industrial Safety Engineering
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Lecture – 29
Quantification of Basic Events – Exponential Distribution

Hello everybody, today we will discuss Exponential Distribution, which is a part of Quantification of Basic Events.

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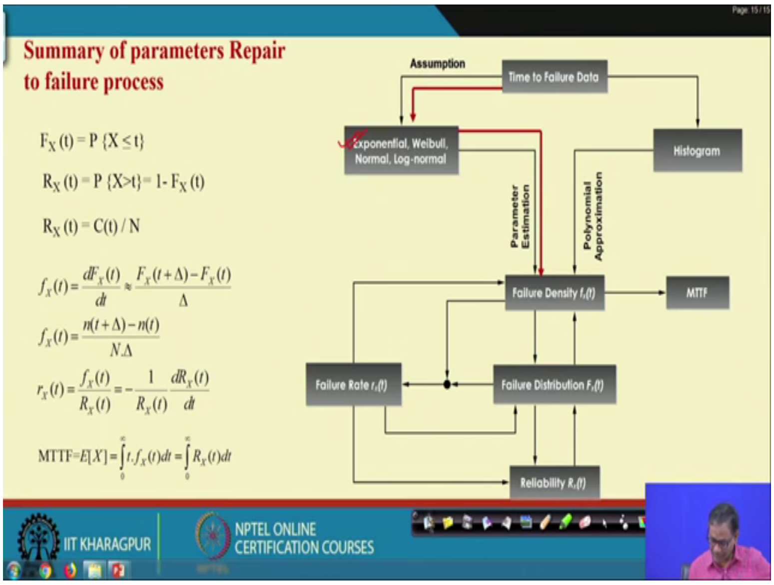

Understand, what we have discussed in last class? We have discussed this, we have discussed this part fully and then I have shown that the if you have the time to failure data. How this side you ultimately go for polynomial approximation or from the histogram? Although, we have not given you any numerical problems and it is solution for histogram to polynomial approximation, but it is possible and I am sure that you will be able to do it.

So, today we are basically seeing the other side of this figure, because we are interested to know all those distributions. Now, at the other side is that, given the physical condition, as well as the unit under operation.

So, what happen it may it may show you may find you may assume certain distribution, because you have first experienced that if these are the symptoms then ultimately the particular random variable and if exhibits certain symptoms. So, then it is basically a coming from a set or non-probability distribution.

For example or many electronics a equipment or component like transitions and others in their useful life. What they will they basically follow the, that constant failure rate that is basically constant hazard rate. And, if under constant hazard rate situation, you will find out that exponential distribution time to failure data we will follow the exponential distribution.

So, when we have beforehand some a primary knowledge that the time to failure data, will be following certain distribution then in that case it is better to assume that data coming from that particular distribution. And, you just test when a test through certain means and whether the exponential distribution is fitted or not.

Now, it may so, happen that you are not sure that whether it is a particular distribution or there can be many distribution, which may each one may be the possible candidates. So, then what happened you feed the data to many distribution and through particular certain test like hypothesis testing, you will see that whether the parameters coming from that are from that proper particular distribution or other way the data coming from that particular distribution or not.

So, with this background in mind and we will be discussing today exponential distribution. And, this is also coming under quantification of basic event, because most of the component useful if it is in the useful life. It usually it is considered that there coming from exponential distribution, it will not ultimately deviate much from the reality, but at the same time your mathematical complexity is reduced.

So, exponential distribution is extensively used in safety as well as reliability study when particularly when, we are interested in component level failures.

So, now let us see that what is exponential distribution, how it is derived and with some example we will also show you the calculations.

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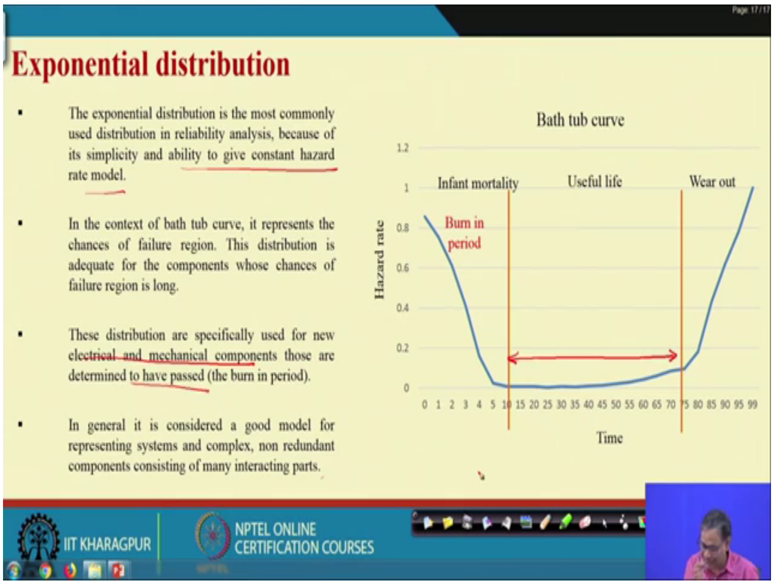

So, I again rely on this hazard rate curve, because the because exponential distribution and the constant failure rate, they have huge relation. Basically, if the hazard rate is constant which is basically the time of useful life. And, then you will see later that the failure distribution density will be exponential distribution.

So, let me read out some of the important points. So, it is basically the most commonly used distribution in reliability as well as in safety analysis. And, because it is of it is simplicity and ability to give constant hazard rate model this constant hazard rate, it will explained, if you go by exponential distribution. And, second one it is a constant hazard rate is the useful life means that is the maximum period of over life in the component or unit will be under operation.

So, that mean if it is exponential constant hazard rate, then the useful life is more. So, you can understand the value of this particular distribution now. And, the distribution is specifically used in new electrical and mechanical component.

So, and usually those have all passed the early period burning period that is already past. In general it is considered good model for representing system and complex non redundant component consisting of many integrating parts.

Another one is this basically, suppose you have a complex system, many integrating parts are there, they are basically definitely non redundant parts. Then if you, when you sum up thus all failures, then ultimately it exhibits constant failure rate, the sum aggregated failure rate will be almost constant failure rate. So, that is another interesting point. So, that mean you can use the exponential distribution there.

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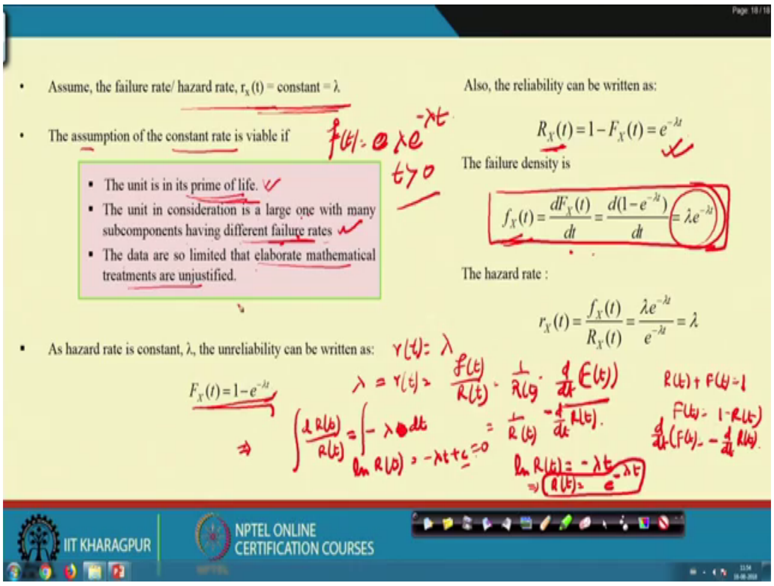

So, if failure rate is constant, that mean r x t is lambda, then you will see that from here using this r x t you will be able to device all those things. So, I will write down here. So, our r x t, I am not writing x I am only writing r t so, this is lambda. Again, you know that r t is nothing, but f t by capital R t.

So, this can be written like this that 1 by R t and small f t is d by dt capital F t, you all know what is this, capital F t means CDF. Another issue is that R t plus F t equal to 1. So, capital F t can be written as 1 minus R t, then d by dt capital F t this is nothing, but d by dt 1 minus R t, which is basically minus d by dt capital R t.

So, you can replace this and then put the an R t value. So, it will be minus d by dt capital R t. So, now, what is this is nothing again lambda. So, lambda equal to this. So, you can write here that d R t by R t capital R t equal to from this minus lambda into dt, lambda into dt.

So, if you now integrate; obviously, you know that this will be with reference to R t 0 2 1 and this will be reference to 0 to infinite. So, if you integrate what you will get here, you will get log of R t here equal to minus lambda t plus constant and under boundary conditions c will be 0.

So, you are getting log of R t equal to minus lambda into t. So, then R t will be anti log of minus lambda t this is basically e to the power minus lambda t, e to the power R t equal to e to the power minus lambda t.

So; that means, if your hazard rate is constant, then reliability will be e to the power minus lambda t. Now, then if I know reliabilities of these then you know this one, what is the CDF 1 minus this. So, 1 minus this then what will be F t? F t will be small f t small f t will be del f t by dt. So, what is this day by dt 1 minus e to the power lambda t, which is lambda e to the power minus lambda t.

So, we have now defined the exponential distribution pdf of exponential distribution, which is f t equal to it equal to lambda e to the power minus lambda t. Obviously, t greater than greater than, if it is t equal to 0 it is lambda t greater than 0.

So, this is what is our theoretical exponential distribution, which one this. So, what are the assumptions for this case? As such then the exponential distribution will be followed, unit is in it is prime of life, that mean useful life. The unit in consideration is a large 1 with many subcomponent having different failure rates.

So, this also possible suppose first one is these this may be under say this kind of constitution the data are. So, limited, that elaborate mathematical treatment are unjustified.

So, when you go for exponential distribution? First is you know it is in the prime period of life component level or you have a system with many parts with different failure rates and your aggregating all those failure rate. So, then also exponential distribution is useful. Or, otherwise what happened you do not have data to understand that, what will be the actual failure distribution, under such cases it is better to use that exponential distribution, because higher level mathematical or complicated distribution even, if you use that may not solve the purpose because you do not know exactly from the data what is the distribution.

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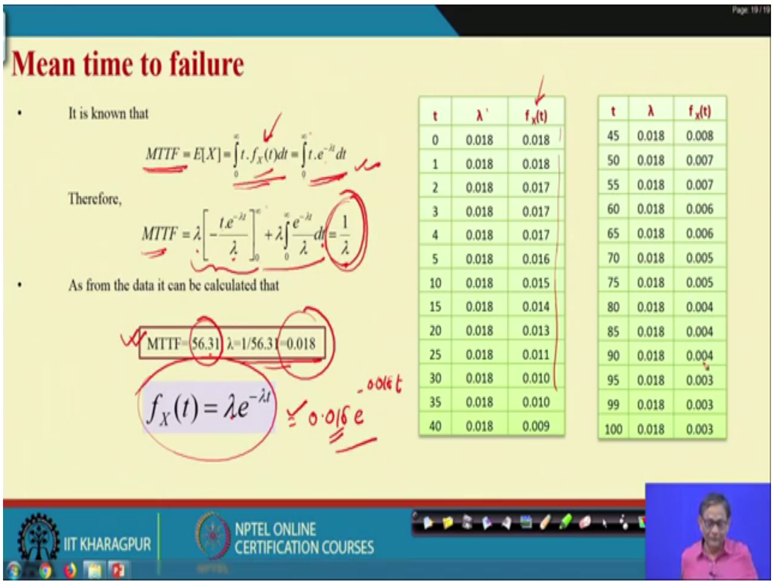

So, now let us see the example is a the is the same what we have discussed in last class last, but one that class also. We have given you some 100 data points and then from that data what we have calculated? We have calculated the mean time to failure mean time to failure.

So, the mean time to failure for that data set is 56.3 1. That mean time to failure has a relationship with the parameter of the exponential distribution that is lambda, we have already seen what is exponential distribution? Now, we also know what is the theoretical formula to compute mean time to failure is nothing t f x t dt, which is t e to the power minus lambda t dt, because our f x t is e to the power minus lambda.

Now, if I, if you integrate this one, you will be you will be getting this. Now, the value is (Refer Time: 13:23) value range changing from 0 to infinite put 0 to infinite, and then your ultimately you are getting this value. So, first one lambda t e to the power minus lambda t by lambda, then plus lambda e to the power minus lambda where lambda into dt integration, integration by parts we have done. So, that this 2 ultimately this result in to this quantity 1 by lambda.

So, that mean time to failure is 1 by lambda or other way lambda is 1 by meantime to failure for exponential distribution. So, we assume that this followed exponential distribution. So, mean time to failure is computed. So, lambda is also computed. What is the value of lambda 0.018? Then, what is our f x t? That these is lambda is 0.018 e to the power minus 0.0 1 8 into t.

So, if I put the value of lambda this will be 0.018 e to the power minus 0.018 into t.

Now, we have computed this putting all lambda value is 0.0 1 8 and f x t value is putting into this formula, you are getting these variety t values, that density values.

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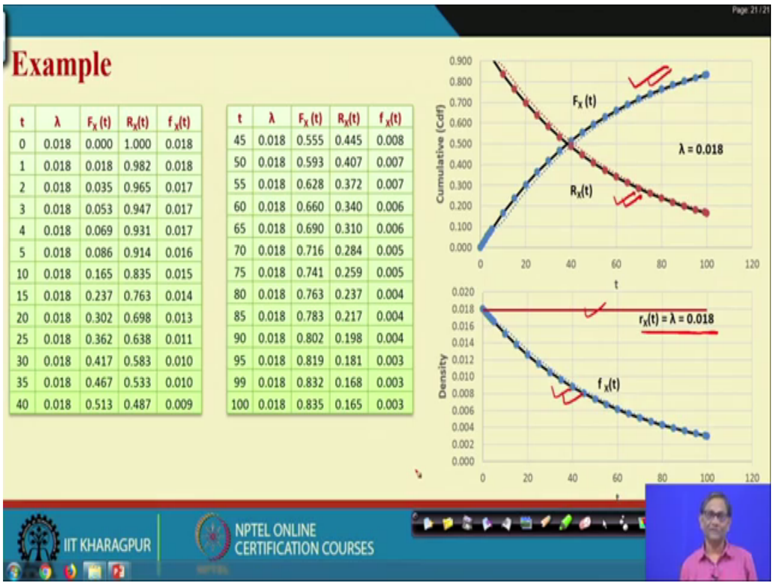

So, we have the lambda, then theoretical reliability, theoretical density, theoretical cumulative distribution, functions all can be derived, all values and we have done it. And, you need excel sheet and do it and get the values like this one is the, this is your F x t cumulative CDF. This is your survival function, this is your failure function, this is the density function, and we know it is exponential distribution.

So, the hazard rate or failure rate will be constant, which is this life. So given data you have to draw such diagram and you have to interpret also. Suppose you are given a that some kind constant hazard rate function. So, you must you must know what will be it is density in that in this case it is exponential distribution density.

So, those you must be able to write down in the assignment as well as during the exam.

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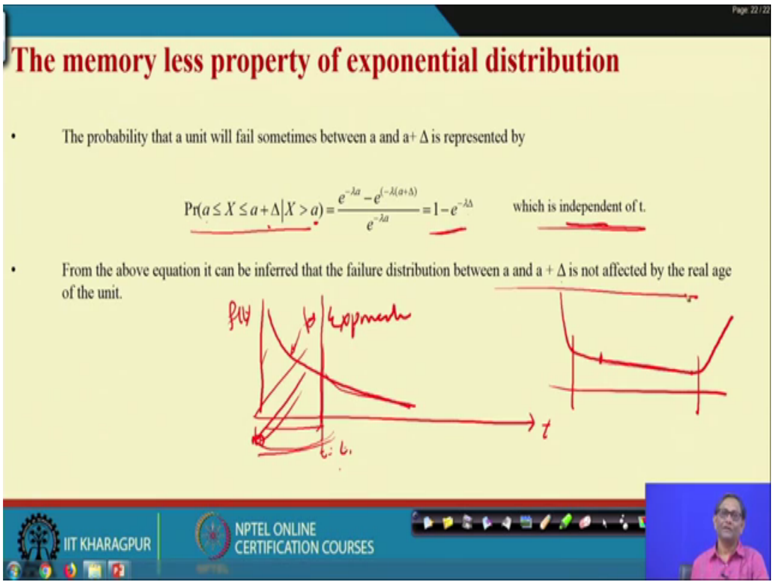

So, exponential distribution has a very good property, which is known as memory less property. So, it is something like this suppose you started with time t equal to 0 and you have a distribution like this. This is exponential distribution this one is f t and this exponential distribution.

So, f t is exponential. Now, suppose you have passed certain time let 30 equal to 80 equal to 0. So, you are at t equal to t 1. Now, you want to know the which distribution this unit follow after time t equal to t 1 it will be the same exponential distribution, because it does not remember the past of the his distribution.

So, that is what is the memory less property mean, it will again, it will be suppose if I forget if I forget this part then again it started with exponential distribution. So, that mean it can be again if you think it is 0 you can it will bring back to this same thing happened. So, it will not the past does not depend on in exponential distribution. The past does not matter, where ever you are starting that if it is coming from exponential distribution it, because you have seen the hazard rate function.

Suppose, you this is the useful life suppose this much work is time has elapsed, still it is constant failure rate with the same parameter values.

So, this that interpretation is given mathematically like this, that probability that X will be X in between a and a plus delta given that X greater than a, this is again you see that 1 minus e to the power minus lambda delta, which is independent of t.

So, and that is the from the above equation it can be inferred that failure distribution between a and a plus delta is not effected by real age of the unit.

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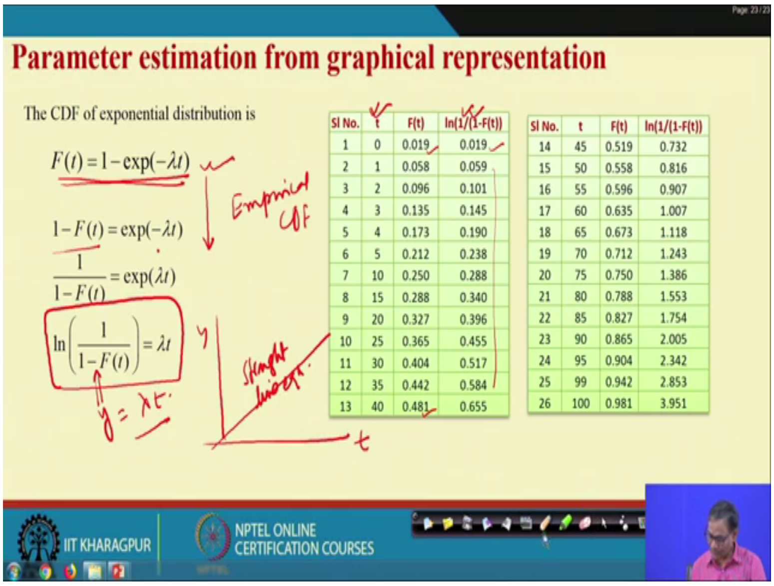

So, it is our, it is one of the another advantage of exponential distribution. Now, suppose a you have data and you want to find out the lambda values graphically. From theory you have seen that the lambda value is 1 by mean time to failure, the reciprocal of mean time to failure or a, but here what happened. Suppose, you have data and you want to find out the, what will be the lambda value (Refer Time: 19:39) from the from plots.

So, this was given here. So, we know the distribution of cumulative distribution of exponential distribution will be like this. So, you just do this manipulation. So, 1 minus this equal to this and this and then finally, log this is the equation. What is this equation? If, I say the this left hand side is y, then this is nothing, but y equal to lambda t.

So, if I want to plot now y versus t, then lambda what is this y versus t. So, you will be getting a straight line.

Now, whether the what will be the what kind of straight line that depending on the value of lambda. So, you get it is a straight line equation; it is a straight line equation. Interestingly this straight line will pass through the origin, because there is no intercept.

So, you have data from data you compute this cumulative empirical CDF you find our empirical CDF. And, then what happened you find out that 1 minus that CDF and 1 by of this N log and get the values like this we have done it, when t equal to 0 our CDF is this t equal to 40 CDF is this.

So, then log of 1 by 1 minus F t. So, all those things are known because F t is known this is this. So, in this manner we hardly got we got all the log of 1 by 1 minus F t values. What is required to be plot? We require to plot these versus t. So, if you plot these versus t you must get a straight line and by theory this straight lines should go through the origin. So, let us see that what is the plot.

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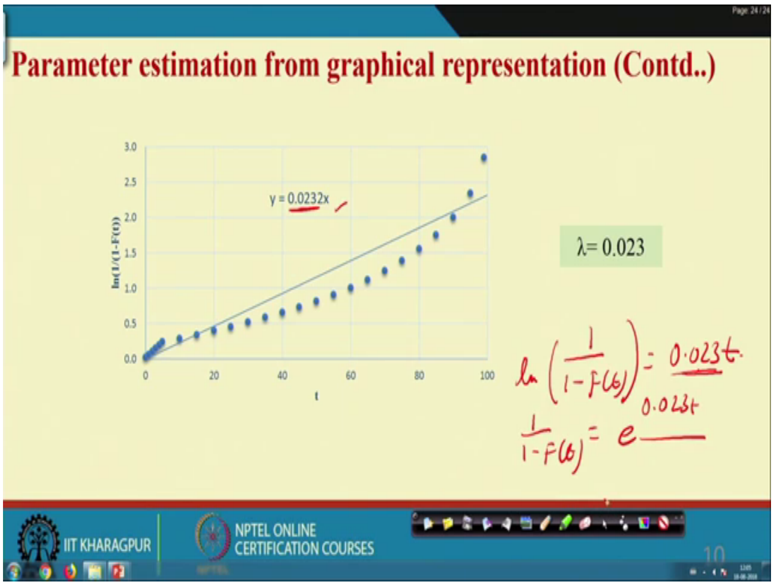

You see, we have given the plot all those points and it straight line is drawn through origin and we got the value y equal to this. So, y is nothing, but what y is log of 1 by 1 minus F t. This equal to 0.0 2 3 into x z t, this is your lambda.

So, then 1 by 1 minus F t equal to e to the power this 1 0.023 t just e to the power minus this. So, now you can you in back calculation, you can find out, what you will find out? You may find out that small f t, what is this one, this one you can find out.

So, our lambda is this and lambda is this and it will be e to the power minus this I think you it will come like this only.

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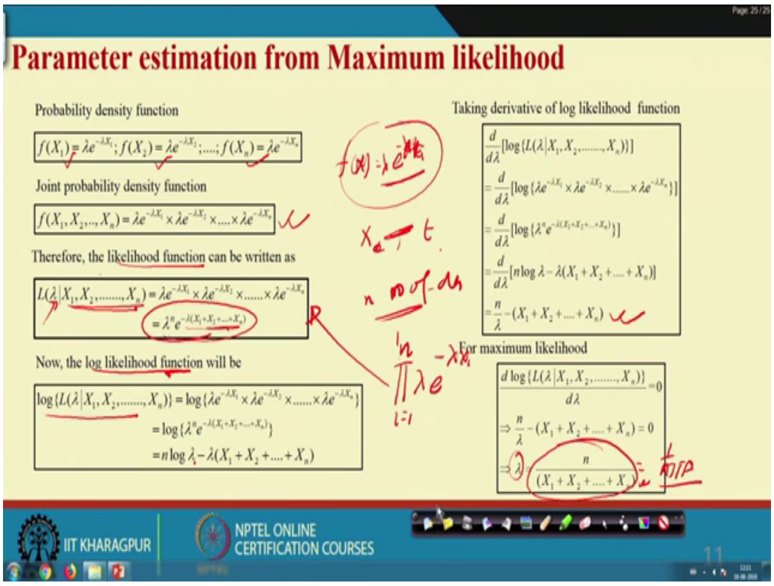

Then another issue is that, suppose you will not go by that graphical method you can use, but at the time we have shown you that the, you can use lambda equal to 1 by MTTF. So, how this 1 by MTTF is coming that when through maximum likelihood estimation you can find out this one.

So, in order to use maximum likelihood estimation you first know that what is the distribution? Distribution is here we are using X, because X we consider as time to failure. So, this is x is e to the power lambda e to the power minus lambda t, here lambda x 1 I am we are writing like this.

So, this X 1 and t X and t you can use interchangeably here. So, then suppose you have collected n number of observation; n number of data, small n. So, 1 2 all those t time to failure data n number of data is available to you. So, is data coming from this exponential distribution. So, that mean each data will follow a observation, each observation will follow exponential distribution and it is a density function will be like this.

Now, what you want to, we want to find out a parameter lambda. Who is actually makes this n number of observation mostly likely? What this is what is the maximum likelihood estimation? Maximum likelihood estimation means, we have a data set and what we want to know what is the parameter of that distribution that makes this data set most likely. So, there can be many that parameters you can choose, but there will be 1 parameter which will value which will make the data set most likely.

So, that is why what is required, that mean you are not dealing with 1 density function, you are basically dealing with n number of density function of same type that mean the lambda value will not change only the x value is changing. So, you are taking them to putting them together and you require to find out the joint probability density function here. As the observations are independent so, they are joint distribution will be multiplication of the density function. So, that is what is done here. So, that mean the joint distribution if X 1 to X n is multiplication of the individual distribution, density functions.

So, then you take the log this is basically we are talking about likelihood function. So, likelihood function is nothing, but thus multiplication I equal to 1 to n, then your lambda e to the power minus lambda X i or t i. So, this is nothing, but this; that means, given observation, observed data what will be the value of lambda.

So, then this one is lambda to the power n e to the power minus this. Now, what happened to make it simplify you take the log of this function likelihood function. So, when you take the log. So, lambda n will become n log lambda and all those e to the power term will become the sum of this.

So, what do you want? We want to maximize the that one, what I guess the log likelihood, the reason because if likelihood if we maximize that will be the most likely value of lambda will be there. So, you can in order to find the maximize values maximum value, what you require you take the derivatives. So, taking derivatives with respect to lambda you are getting this equation.

Now, all and you can go for the second order equations, say for to check that maximum that or minimum which quantities, we are not going to that level why it will be definitely a maximization case. So, then this value if you put to 0, then you are getting lambda equal to this. Now, you lambda equal to this X 1 to X n means t 1 t 2 or t n or X 1 to X n this is nothing, but 1 by MTTF, Mean Time To Failure.

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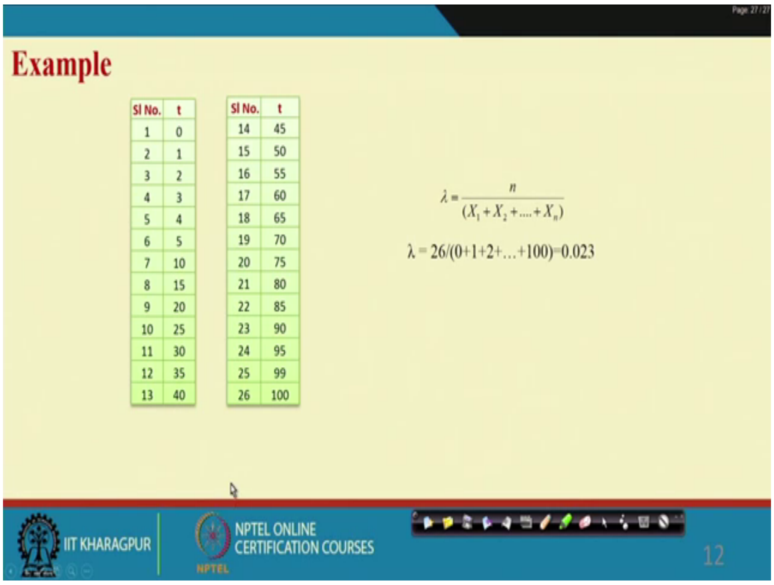

So, this is a with reference to this example, we have computed lambda equal to this value 0.02.

Thank you very much, I hope you have understood it.