Business Analytics for Management Decision
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Lecture – 50
Prescriptive Analytics (Contd.)

Hello everybody this is Rudra Pradhan here, welcome to BMD lecture series. Today will continue with Prescriptive Analytics and that to coverage is on integer programming. So, we have discussed several problems in the prescriptive analytics, here the main idea is to get the values of the decision variables to address a particular business problems and come with a kind of you know management decision as per the particular business requirement. The thing is that we have discussed several problems, starting with you know linear programming, primal structure, dual structure.

Then there are various methods through which you can solve a problem and then come with a kind of you know optimal solution that is the values of the decision variable, through which we can address the business problem, as per the particular you know management requirement. So, the whole idea is a we have a kind of you know business kind of you know business problem and the process you know what we are doing here in the case of you know prescriptive analytics, that you know having the kind of you know informations and the kind of you know particular problems, we transfer the particular problem to a you know mathematical models, may be in a linear programming format, or in a kind of you know non-linear programming format.

So, the whole idea is a we have to first transfer the particular problem into model format, then through a particular technique we like to optimize the particular structures, and then look for the optimal solutions, where the values of the decision variables can give you some kind of you know strategic decision to a business process. So, that is how we are discussing various you know structure of you know mathematical modeling and that to under the prescriptive analytics to address the business problems, and come with a kind of you know decision through, which you know the business problem can be addressed in a more effective way, or in a kind of you know more efficient way. And in the last couple of lectures, we have solved some several problems starting with you know simple structure to complex structures, I mean we start we start with you know bivariate

structure, to multivariate structure, where the objective functions objective function may be with respect to 2 variables or more than 2 variables.

And then corresponding constants are maybe 2 or more than you know 2. So, the simplest structure to start with the game the prescription analytics structure that to or the optimality structure so, we must have at least you know 2 variables and then we must have a 2 constant means you know the problem which you are you know discussing and, then depending upon the problem. So, it may have a more number of variables and more number of constraints.

Then we look will we like to actually optimize the particular you know process, you know for instance optimizing the resource allocation to get you know you know maximum profit, or the kind of you know minimizing cost, something like that. So, it is a once you understand the problem.

So, we like to develop a kind of you know structure, through which the entire problem can be transferred into a mathematical model, then we look for the kind of you know solutions and that is with respect to some of the you know techniques, what we have already discussed like you know graphical structure and, then primal simplex method, dual simplex methods.

So, depending upon the particular you know problems, we can you know apply a particular technique and look for the kind of you know solution. So, after knowing the linear programming structure, the kind of you know model formulation several kind of you know techniques to work out the a particular you know business problems, agains we have already discussed the sensitivity concept, where you know after getting the optimal solution, we can you know bring some kind of you know alternative options with respect to some of the changes that with the coefficients the objective functions, then the coefficient of the constraints and the addition and subtraction of the particular you know constraints.

And then we can add more variables. So, there are many different ways we can apply the sensitive structures to the optimality a solution and, then against we can have a more more kind of you know flexible options to address the business problem and to come with the kind of you know management decisions. Now taking the clue or you know

over the linear programming problem that to a primal structure, dual structure, primal simplex, dual simplex, then sensitive structures.

So, now we are we are going to address 2 more problems in the prescriptive analytics, and that is the integer programming and the kind of you know goal programming, then after that we will move to a particular structure called as a non-linear programming. So, so, corresponding to primal structure and dual structure, integer programming goal programming are not completely different, there they you know they are the kind of you know tools, you know analytic analytics, where the particular problem must have objective functions, must have constants must have conditions and, then look for the kind of you know solution and in the case of integer programming.

So, the typical structure is more or less same you know, like you know the problem which you have solved in the case of you know primal problem and, we have solved in the case of you know dual problem and, what is more important here that you know, once you know understand the problem and transfer the model transfer the problem into model and look for the optimal solution, then one extra things we are supposed to check there, is whether the values of the decision variable are integer type or not. If the values of the decision variables are coming integer type; that means, the particular optimal solution is per you know is a perfect kind of you know structure through which you can address the business problem.

And there is a high chance that you know after solving the problems, you may reach optimality and we may get actually values of the decision variable through which, you can address the problem, but sometimes some of the business problem in such a way, that you know your values of the decision variable must be you know by default you know integer type. Otherwise you cannot actually address the problem more effectively and, you cannot actually you know streamline the process as per the particular you know requirement.

For instance let us say there is a kind of you know production process and there are you know 2 decision variables say x 1 and x 2. And x 1 needs to let us say a production of you know chairs and, x 2 oh maybe the production of you know tables and, then there is a objective function to optimize the a profit function subject to certain constants, may be manufacturing constant, may be labor constants, may be infrastructure constants. And

after doing the kind of you know process, we will get the optimal solution; that means, typically this kind of you know problems the whole idea is a you know what should be the number of you know you know chairs productions and tables production. So, that you know the firm can achieve you know highest profit. So, now while calculating all this you are fixing a kind of you know problem like this.

So, you know we must have actually the values of the decision variable you know kind of you know integer type. Otherwise a otherwise we cannot actually a get the optimal structure perfectly. So, that you know we can address the business problem as per the particular you know requirement. For instance let us say x 1 optimal in the optimal solution x 1 is coming let us say 3.5 and x 2 is coming 4 point 5 so; that means, tables cannot be you know, 3 and you know half and just cannot be 4 and a half that means.

So, if I am saying that you know chairs are 3 or tables are 4; that means, it is completed. So, the process is completed and it is ultimately you know final product and that to available for the market or available for the customers. So, now the moment you say 3.5 or 4.5; that means, so it is a half product and chair half product and table half product is a kind is a kind of you know meaningless concept to address the business problems so, for as a optimality is concerned. So, you know likewise we have a couple of you know business problems, where our optimal solution you know should be integer types; that means, you know in this kind of you know scenario business problem.

So, our objective is not to get the optimal solution or the values of the decision variable, we have we have another you know kind of you know condition or restriction where, the values of the decision variables you know something like you know integer type so; that means, after getting the optimal solution next step is to check whether the values of the decision variable or integer type or not. If in case we are getting the values of the decision variable integer type of in the optimal stage, then we can stop there and address the problem.

So, there is no role of the typical you know integer programming. So, now if in case a particular variable or all variables or few variables are not integer type, then this is a complicated kind of you know issue and that to you know the a the method will show you that you know, we have reached the optimal solution, but actually it will not practically true or you know so, far as a practical requirement is concerned. So, this

cannot be you know considered as the optimal solution to address the problem or business problem.

So, so again so, there is the next level you know entry agains, you have to restructure the problem and a redesign this typically in a structure that is the iterative process through which actually look for some kind of you know another optimal solutions, where the values of the decision variable will be integer type so; that means, it is a 2 stage process first having the particular you know problem business problems, with objective function constraints and condition, we first look for the optimal solution and then check the values of the decision variable whether you know they are integer type or not.

If they are not integer type then; that means, technically a so, we have not reached the optimal solution even the method is showing that you know we have reached the optimal solution optimality. So, this is this is something called as you know extra kind of you know condition or extra kind of you know constraints, through which you know the optimality structure will be you know change little bit you know; obviously, it cannot be in the in the you know in the case of you know in the type of you know ratio, but it will be the kind of you know integer types ok.

So, that is how the kind of you know structure so; that means, technically. So, we like to know how is the kind of you know integer programming game and, what are the you know areas, or what are the business problem it is strictly the kind of you know requirement and depending upon the kind of you know requirement we look for the you know optimal solutions, where the values of the decision variable will be integer type; that means, now we like to see whether the a methods which we have discussed like you know primal simplex, dual simplex, you know they are in a position to you know give you the values of the decision variable at the optimal stage which are you know integer type. If not then what are the procedure through which the particular optimal solution again, will give you another kind of you know optimal solution where the values of the decision variable will be integer type. So, more or less it is a kind of you know sensitivity structure.

So; that means, after getting the optimal solution, we are putting another kind of you know restriction through which your we will evaluate the problem. Depending upon our requirement and, then it check whether we have reached the optimal solutions and the

particular optimal solution will the you know fulfill our target or the kind of you know requirement, that is the management requirement.

So obviously, we like to know few things here, regarding the integer programming goal programming and you know something like you know non-linear programming. So, this is the coverage of this particular you know unit. So, let us start with the, you know objective of this a unit and, then will go by the kind of you know requirement of integer programming.

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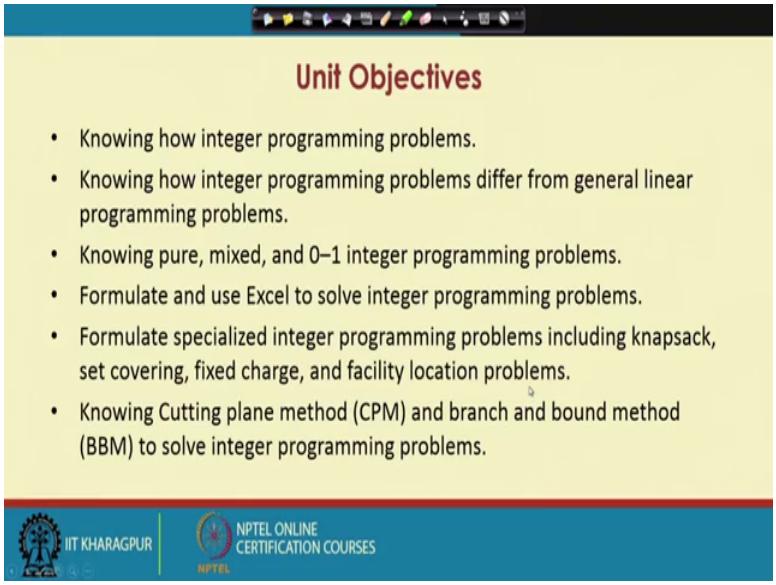

And so, the whole idea about this unit is to know the integer programming problems, which I have already you know already highlighted a slightly to know the kind of you know requirement the know the kind of you know situation, where it is very useful and that to in you know the kind that is that that to that to actually to find out the kind of you know situation, where management decision can be effective in order to more effective. In fact, you can say.

So, now knowing how integer programming problems differ from general you know linear programming problem and in the case of you know integer programming. So, we can have a 3 different structure. So, one is called as you know pure integer programming, then the second one is called as you know mixed integer programming and, then you know 0 1 integer programming it is like you know binary kind of you know structure the way we have discussed in the case of you know predictive analytics, where you know we

have linear probability model or binary search models, where the values of the decision variable will be 0 1 only.

So; that means, here in the case of you know integer programming. So, typical structure will be 3 types pure integers, where all the values of the variables are integer type and the second one is the mixed integers, where some are integer type some are non integer and, then again you know 0 1 integer programming; that means, the values of the decision variable will be only 0 1. So, in that case how you have to address the business problem and what should be the kind of you know optimal optimals you know kind of you know solution. So in fact, you know depending upon a particular business problems.

So, this the kind of you know integer integer programming problems can be more effective and can address the problem, you know business problem efficiently. And we like to know how excel or solver package can help to get the you know integer programming solution. And then we like to you know formulate a specialized integer programming problems like you know set covering, fixed charge facility, location problems and like that you know we have a couple of you know business problem in there like scenario.

And so, for as a solution to this particular integer programming is concerned. So, it is not something you know different, it is a simple kind of you know linear programming structures like you know primal structure and dual structure. As usual normal process we start and we not reach the optimality and, then get the values of the decision variable and the additional check after, you know reaching the optimality you know to see whether the values of the decision variable are integer type or not.

If not then agains there are some methods again we can apply, through which you can get the values of the decision variable that will be the integer type. So, there are 2 standard mechanisms to you know solve the problems and, then look for the optimal solution or the values of the decision variable, where you know they are integer type. So, the first mechanism is called as a CPM that is cutting plane methods. And the second mechanism is called as you know BBM that is branch and bound methods.

These 2 methods are very useful to address the a particular you know problems and, then look for the optimal solution which is actually the integer types so; that means, so typically in the integer programming problems. So, we simultaneously like to use both

primal simplex structure and dual simplex structure. So, in the first part of the particular you know process is in the form of you know primal simplex structures, where we simply solve the problems and try to reach the optimality and, once we reach the optimality, then we will check whether you know we have a feasible solutions and, if we have a feasible solution what is the you know values of the decision variables.

So, now if the values of the decision variables are integer type, then we can stop there and not point to go to the a second you know step where you can use cutting plane method or BBM that is branch and bound method, but if you at least one of the particular decision variable is not actually integer type, then we can look for you know further kind of you know processing that to through cutting plane methods and branch and bound method, where we can get you know some kind of you know, further solution or you know some kind of you know change as you know structure through which you know we can again have the optimal solution.

And the values of the decision variable will be typically integer type and in fact, there is no much variations from you know the original optimal solution, where there is no in there is a question of you know integer or type, and agains there is a another optimal solution where the fulfillment of you know integer type solutions. So, for instance let us say a particular variable say 3.5 so; that means, typically it will give you the kind of you know signal that you know. So, the values of the decision variable will be either 3 or you know something like you know fours, you know typically not exactly with that particular you know variable it may also less or more.

Because it is it is simultaneously connected or you know integrated with the other variables and you know, and that to with you know different constants and the kind of you know conditions, but the particular you know value is not actually useful for the you know the particular you know business requirement. So, that is how we are looking for the alternative solutions and, we will reach the optimal solution and that to the values of the decision variable will be integer type.

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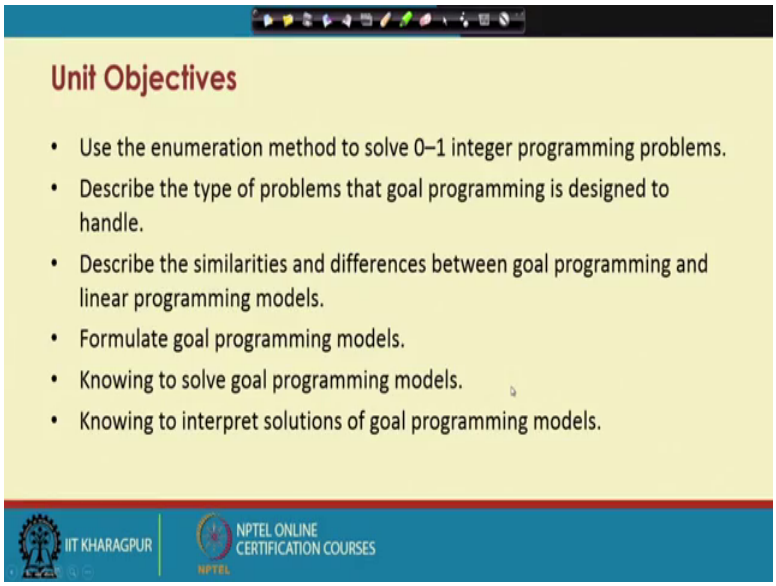

So, we like to know the you know method to solve 0 1 integer programming and, describe the type of you know problems again that goal programming is designed to handle and, agains we like to check the similarity between goal programming and linear programming. And we like to know the formulation of goal programming models and agains we like to know the structure of you know how to solve the goal programming models and how to interpret the solution of the goal programming model so; that means, typically in this unit.

So, we like to touch upon first the integer programming. So, now after knowing the integer programming, then we move to a concept called as you know goal program. This is a kind of you know multi objective kind of you know concept. Till now whatever we have discussed it is a single objective with you know more number of constraints and more number of variables, but there are you know certain business problems, where we have multiple objectives with you know number of constants and conditions and number of variables, where we are looking for the optimal solution. And then looking for the values of the decision variable here, we can actually simultaneously take care all the objectives together.

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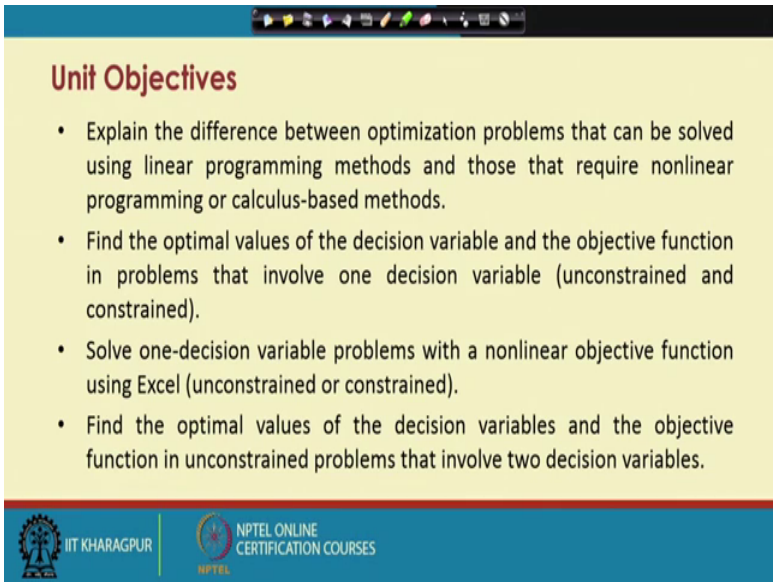

So, ultimately agains we like to discuss some of the concept like you know non-linear programming and, again under the non non-linear programming with a single objective, multiple objective, single constraints, multiple constraints so; that means, we have a different kind of you know structures depending upon the business problems.

So, we like to check you know which particular model will be fitted to a business problem and, then address the address the particular problems as per the particular you know requirement.

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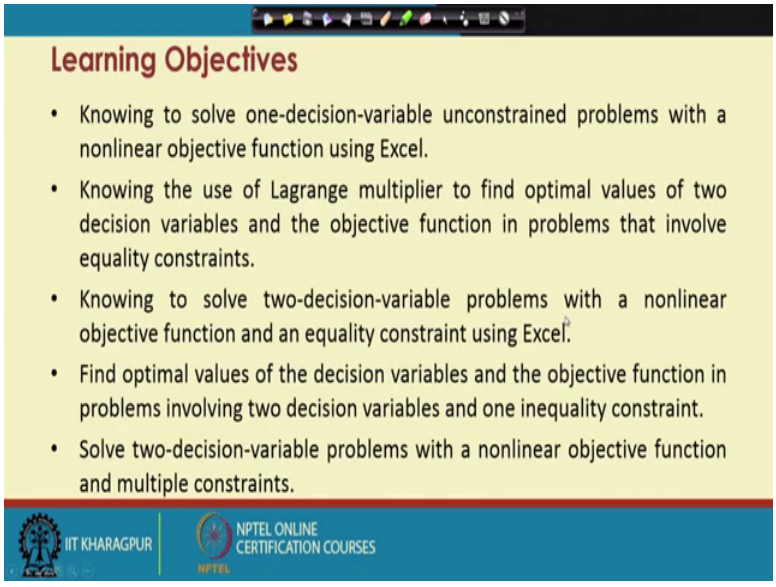

And in fact, you know we like to know various mechanisms through which you can solve also non-linear programming, with single constraint multiple constraint with 1 variables multiple variables. So, depending upon the objective and the kind of you know a business requirement.

But ultimately in the case of you know non-linear programming. So, it is with it is the game between you know objective function and constraints, and somewhere so, there is a non-linearity futures. And in that context how we have to address the business, business problems and come with a kind of you know solution, where you can address the or we can take the management decision more effectively.

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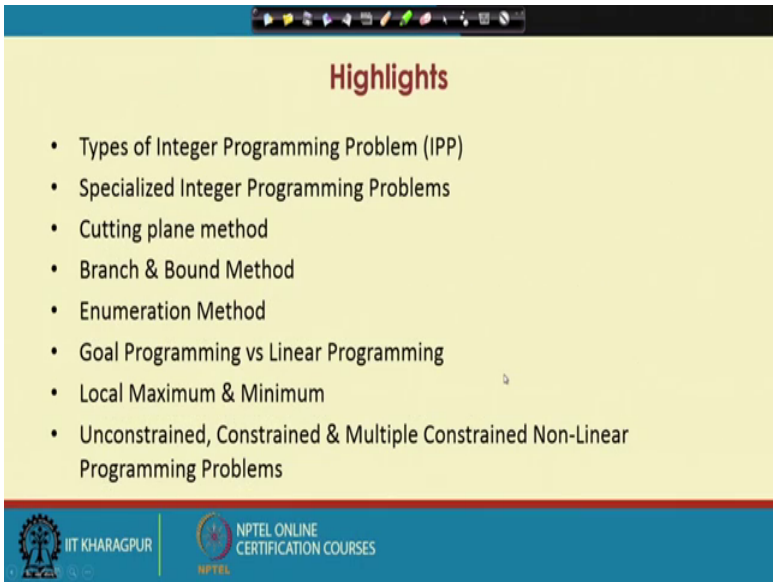

So, these are the following highlights which you like to you know address for this integer programming, goal programming and the kind of you know non-linear programming. So, or technically 3 things we are supposed to address in this unit. So, the first one is the integer programming, the second one is the goal programming and the third one is the a non-linear programming.

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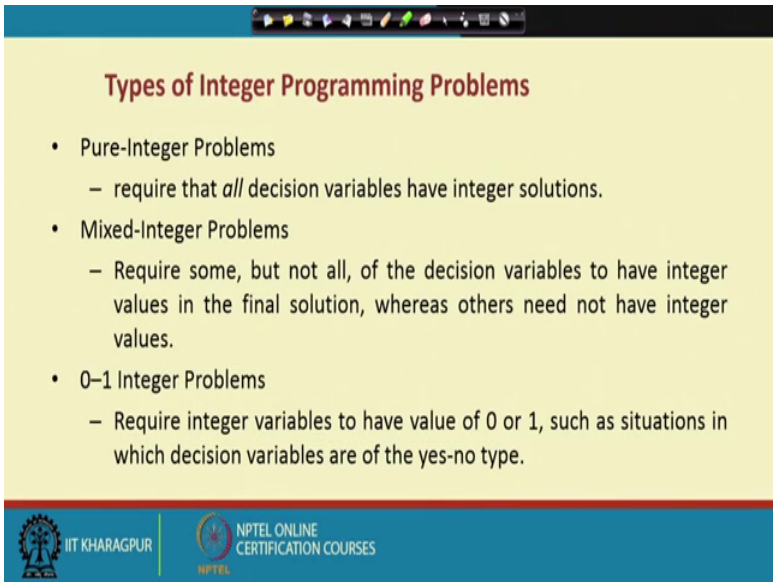

So, now, depending upon the particular requirements, we start with first integer programming. So, what I have already mentions it is a question of pure integer, it is a question of mixed integer and, it is the question of you know 0 1e integer programming. So, in the case of pure integers all the decision variables should be integer type and in the mixed integer. So, you know the requirement is a integer type not for you know all variables, for instance you know you know say you know labor is a one kind of you know decision variables and, then the you know or table is a kind of you know another distant variables.

So, now oh 1.2 labor can be you know can be meaning pools, but 1.2 you know table is not a meaningful so; that means, 1.2 labors mean see so, the kind of you know you know 1 labor and then he can work for you know half half dates. So, 1 labor means there is it kind of you know you know P you know it depends up in how many hours he or she will work in a particular in assignment. So, let us say 80 hours, then so the moment will say 1.5 labor or 1.2 labors depending upon the kind of you know situation.

So, we like to see how many hours ultimately he will work. So, this will be this will be you know more effective way to represent the situation and, then in this contest in the labor case. So, there is no require you know there is no you know typical requirement of you know integer type solution, but in the case of you know table there is a typical requirement of you know integer type solution. Otherwise we cannot actually you know

come with a kind of you know optimal decision and, that to effective decision through which you can address the business problem more accurately and, then finally the 0 1 integer programming.

So, the requirement is a integer variables to have hello 0 either 0 or 1 or you know sometimes you knows you know like you know the kind of you know situation a yes no type kind of you know situation or the kind of you know binary kind of you know situation.

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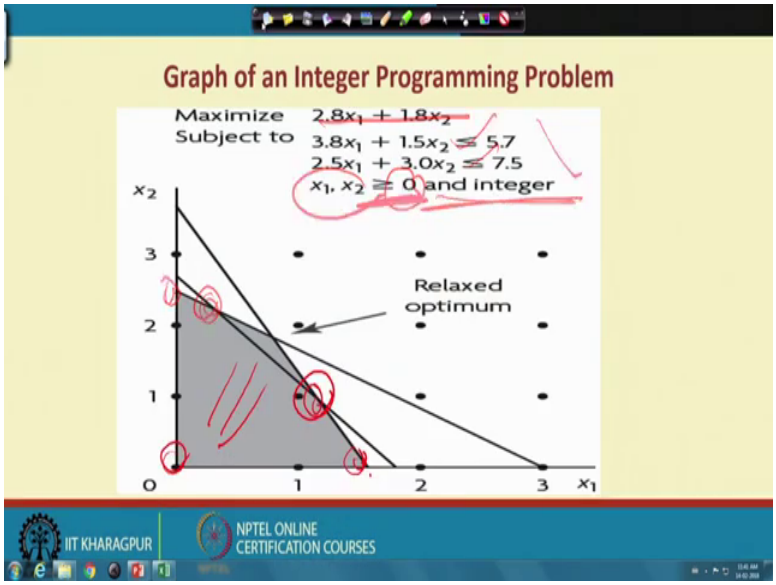

So, so this is what the a structuring and let us start with a simple kind of you know game here and, this is a digital actually linear programming problems. So, let us you know start with the kind of you know concept here. So, here is so this is a objective functions and the objective function is the maximization type and subject to certain constant here, and that that is actually less than type and as usual so, we look for this optimal solution and this is how the graphical structures.

So, so we have actually constants and, then we are you know applying the constants, then we look for the optimal solution. Since it is a less than type so; obviously, the feasible zone will be this side and; obviously, these are the corner point through which actually the optimal solution will be lines. So, there is high chance that you know in this case you know it is a 0 0, but in this case this is X 1 is having some value X 2 is 0, in this

case X 2 is having value and X 1 is 0, but here both X 1 and X 2 are there and here both X 1 and X 2 are there.

So, now there is a high chance that you know these values in this kind of you know line or in these points corner points may not have actually integer type, if it is integer type then we can stop or if not integer type, then again we have to you know or diversify the particular structure so; that means, corresponding to a particular problem. We first look for the optimal solution, if we you know you know without any kind of you know extra efforts, if you are getting the optimal solution which is your integer type we can stop there.

If not then you know we have to adapt another restriction, or you know for integer programming, we can actually redesign the structure the optimal structure by putting additional constant or the kind of you know you know restructuring through which again, we again go for the kind of you know iterative process, and then look for the optimal solution. So that means, a it is the combination of you know the you know the particular process the combination of what we have already discussed like you know primal structure, then simple you know the application of you know primal simplex mechanism, then the kind of you know duals dual simplex mechanism.

So, initially we start with the you know the particular you know problem, then look for the optimal solution up to optimal solution in the first stage. So, it is a kind of you know a primal simplex structure and, then since our requirement is the integer type of you know solution. So, then we are you know checking the kind of you know integer type solution, if it is the it is the case of you know integer solution, we will have actually final optimal solution, and that to no further you know if require means further processing is required.

And in fact, if it is not integer type, then you know we will add something you know additional constant or you know restructuring the problem, then again look for the optimal solution. So that means, it is kind of you know 3 stage process all together.

So, primal then look for the you know primal solution, and check the optimality and check the integer type you know whether the values of the variables, these are variables are integer type. If it is the case then you know we look for the a kind of you know optimal solution so; that means, see here X 1 and X 2 is a kind of you know greater than

positive signals that is the a condition and against there is the integer type of you know signal. So, now depending upon the particular you know requirement. So, you know we will look for the optimal solution and then if not integer type.

So, what I have mentioned that you know we have to create a new constant depending upon the optimal you know solution existing optimal solution and, then a revaluate the particular you know process. And the revaluate you know the particular process depends upon the second stage kind of you know requirement where, we will use dual simplex mechanism through which you can address the business problem and, then look the kind of you know optimal solution which is actually integer type. So, that is how the procedure through which usually you move, and will we like to see how is the kind of you know structure so, in this case. So, so this is what the kind of you know requirement.

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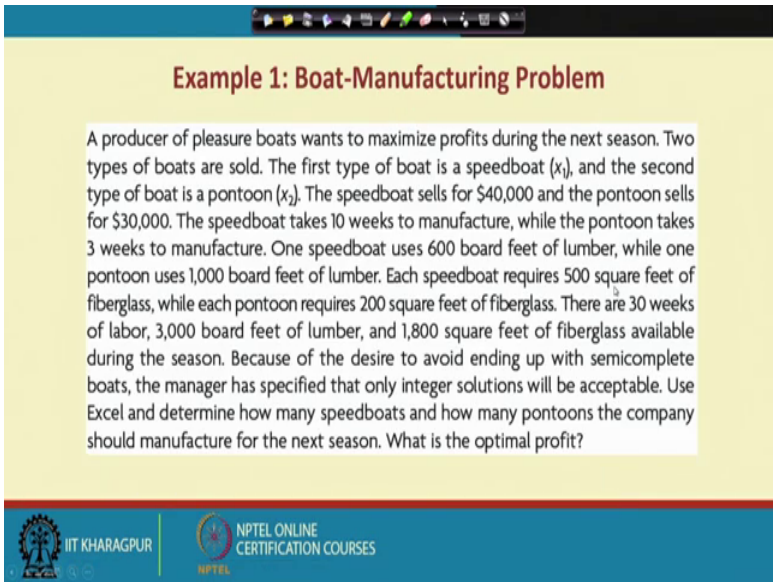

So, a another case this you know you know some kind of you know big problems. So, you know as usual actually a business issue. So, now the entire business issue will be transfer into a kind of you know models. So, here you know a speedboat and the kind of you know type of you know another boat that is called as you know X 2 so; that means, a it is the game of you know 2 variables. So, it is a 2 different boats X and they are represented as you know X 1 and x 2.

And since it is a kind of you know boat so; obviously, the values of the distant variable should be you know on numbers, and that to exclusively integer types like you know 10,

20, 30, 40 like this. So, it cannot be 10.5 10 point to like that. So, obviously, these are all constants and they having all these kind of you know structure.

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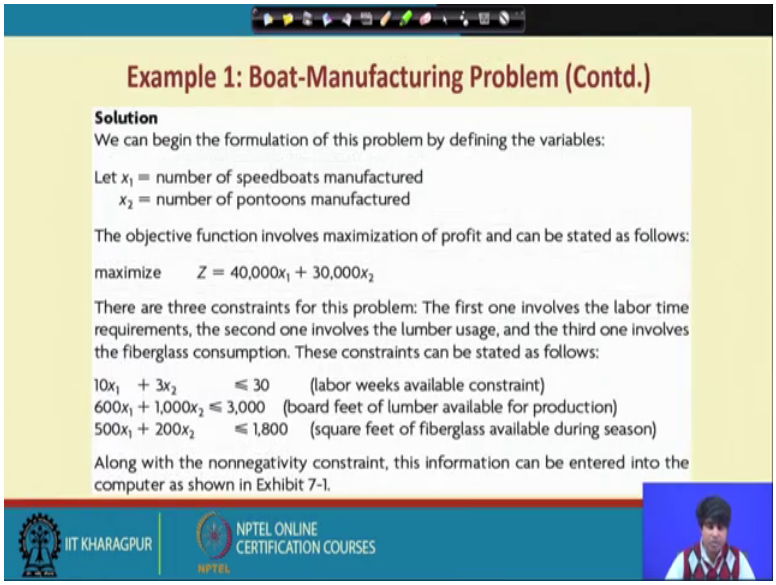

So, the first hand you know the requirement is the transfer the problem into model. And this is what the model. So, so X 1 and X 2 will be the, you know decision variable and the objective function will be a 40000 X 1s and 30000 X 2. So; that means, the particular you know business requirement is to set profit of you know 40000 for X 1 productions and 30000 for X 2 productions.

And subject to available constants that is the labor weeks you know constraints and, then the kind of you know are the other raw material constants that is 10 X that is 10 X 1 plus 3 X 2 less than 30, and 600 plus 1000 X 2 less than 3000, again 500 X 1 plus 200 X 2 less than a 1800 so; that means, you know having the kind of you know structures, then we look for the optimal solution.

And then; obviously, by default here the requirement is the you know non negativity constant or you know or the kind of you know, values of the decision variable which should be positive in nature obviously, it is a kind of you know production process and x one cannot be negative and X 2 cannot be negative. So, by default X 1 will be positive and X 2 will be positive. Either X 1 you know positive or 0 or X 2 positive or 0. So, now with the requirement of you know business or to address the business problem more effectively.

So, the values of the decision variables should be integer types. So, now if we do not fix the integer type of you know target then; obviously, it can be a you know 0 or you know some positive value, and that positive value may not be necessarily integer that can be the ratio type you know situation, but I here with respect to the a kind of you know problem. So, we are you know bound to have a solution optimal solution where the values of the decision variable should be integer type.

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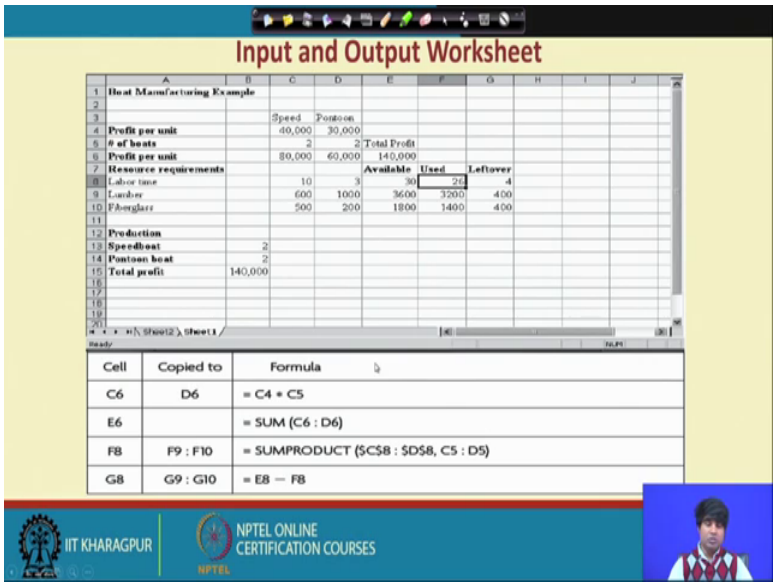

Otherwise we are not in a position to address the business problem more effectively and cannot come in a situation to get the effective management decision. This is the solution through solver packege. So, what fixing the target and then look for the kind of you know optimal solutions. So, then you know for that we can go to the objective functions and, then the constants and then look for the optimal solution.

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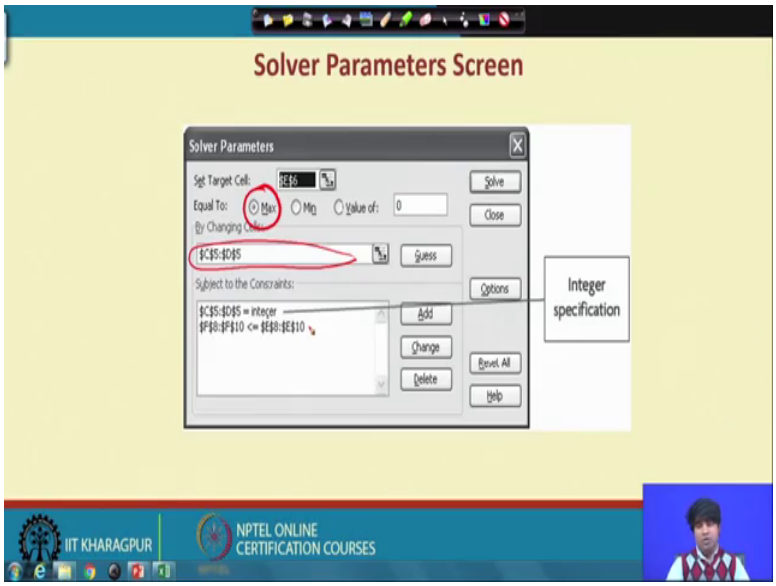

And the solvers if you know go to the solver. So, the first hand requirement is to put the inputs and then setting the kind of you know requirement.

So; that means, technically so here so this is the a you know objective function you know indications and then this is the objective function you know objective function structure that is actually a maximizing Z equal to C X; that means, C 1, X 1, C 2, X 2 since it is actually 2 variable case so obviously so it will be only 2 different combination and, then we have a constant. So, they pull the constant and you know look for the kind of you know solution.

So, now if there is no integer kind of you know specification, then you just add the constants and then ask the solver to give the solution so; obviously, corresponding to the objective a objective function and the constants. So, solver will give you the optimal solutions which is in which may not necessarily, the integer type but after getting the optimal solutions. So, will check whether you know the values are you know integer type, if not then you will put the integer type restriction, but by the way you know in the in the initial process, if you have already set up the mind or as per the business requirement, if you need actual integer type of solution, then you can actually set in the first levels the integer another constraint which will be the integer type.

Then by default so, the is the software will actually process the kind of you know problem, you know you know by more number of iterations. And then come with a kind

of you know solution, which will be having again optimal and the values of the decision variable will be a you know type of you know integer type only. So, means the kind of you know integer type. So, as a result so we can actually address the business problem effectively.

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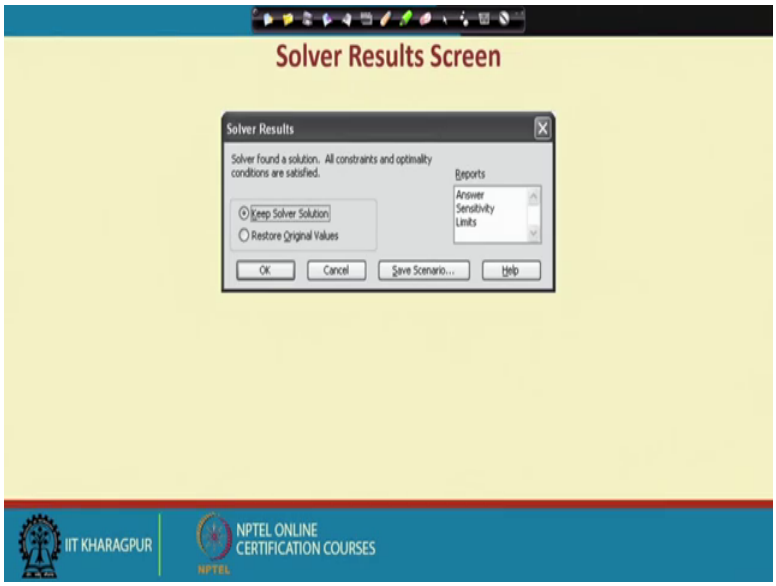

And, then come you know then we will come with your kind of you know solution through which you can you know take the strategic decisions. So, this is how the you know solver results.

So, you know we will take this problem and then will see how in the solver package the actual optimal solution and the kind of you know integer take and the kind of you know integer type of solutions; that means, when you look for the integer type of you know solutions we like to have some kind of you know compromise. That means, the original optimal value of the j means objective function and agains where the values of the decision variable is integer type the second optimal structures, then will find the typically you know difference here, the value of the objective function in the original framework will be definitely higher compared to the integer type of you know solution where, the value of the z will be little bit you know lower because we are putting additional restriction that to integer type of resource.

So, as a result while allocating the resource some compromise, we have to do as a result some productions you know you know I have to be interchanged so; that means, with a

given optimal structures. So, maybe somewhere it will be cut and, somewhere it will be added. So, then finally, we have a kind of you know optimal solution where the values of the decision variable will be integer and, then we can address the business problem effectively. So, with this will stop here is and.

Thank you very much.