E Business
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Lecture-59
Winner Determination Problem

Welcome, today we are going to talk about another interesting phenomenon in auction. In fact this is about computing the auction problem in fact, in an online environment this is a more important problem. Last class what we discussed, we discussed how to design auction rule, this is a strategic level decision-making situation, but once that rule of the game is decided about the how to conduct the auction, how to determine the winner, et cetera, now determining the winner will be the winning bidder is the next problem. So here in this lecture we are going to learn about winner determination Problem under various settings.

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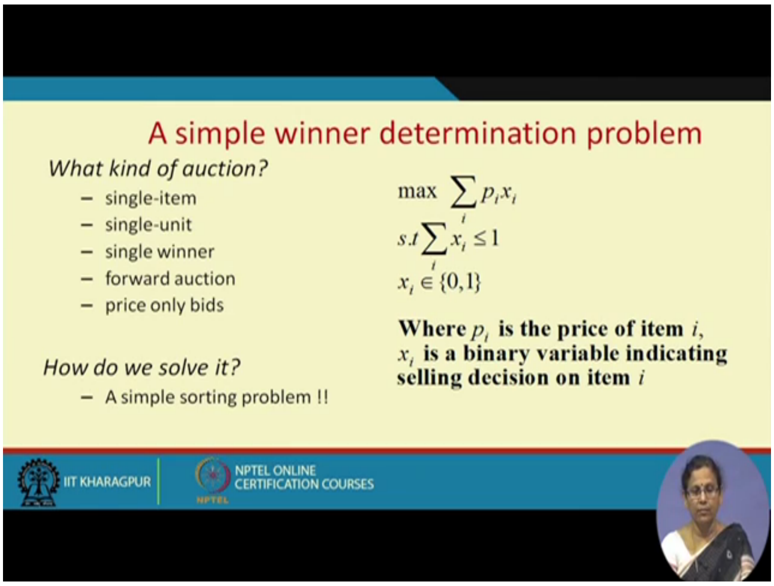

To start with, consider a very simple scenario in which you have a single item with single unit under selling and you have naturally the item will be going to the only one winner and it is forward auction and the bids are only Price bids okay. Now the situation goes like this, people who have little bit idea about linear programming, literature programming, et cetera they will be able to understand this but others can also understand because it is fairly written in the fairly simplistic manner. In fact, when we go for complex auction format I will not be talking about it but at least to give you a feel there is a mathematical angle to it I am explaining this one.

Let us say, what is my problem? My problem is to discover who is the winner who pays me the highest value. Naturally it is a sorting problem, I will be sorting all the Price bids and whosever gives me the highest I will be choosing that but there is a mathematical angle to it as well. So what is that winner determination Problem? The winner determination Problem is I maximise I try to maximise my sales revenue where by multiplying, I associate a variable x i with each of the bid, let us say I have 10 different bidders participating and all of them simultaneously submitting the bid. So first person's bid I represent by x 1, second person by x 2 and all of them are binary variables.

By binary variable we mean if the person wins, x i becomes 1, if the person loses x i become 0. Now the point is I have to determine which one of this x i should be 1, so what do what do I do? I maximise P I into x i so if fifth person is the winner so x 5 p 5 that that will give me my revenue and sum of this should be less than or equal to 1. So though the mathematical problem appears to be an integer programming problem which is little bit complex, otherwise if you solve using traditional integer programming solution approach but here it turns out to be a sorting problem and I solve it by simply sorting all the bid values, but all the auction formats will not be that simple.

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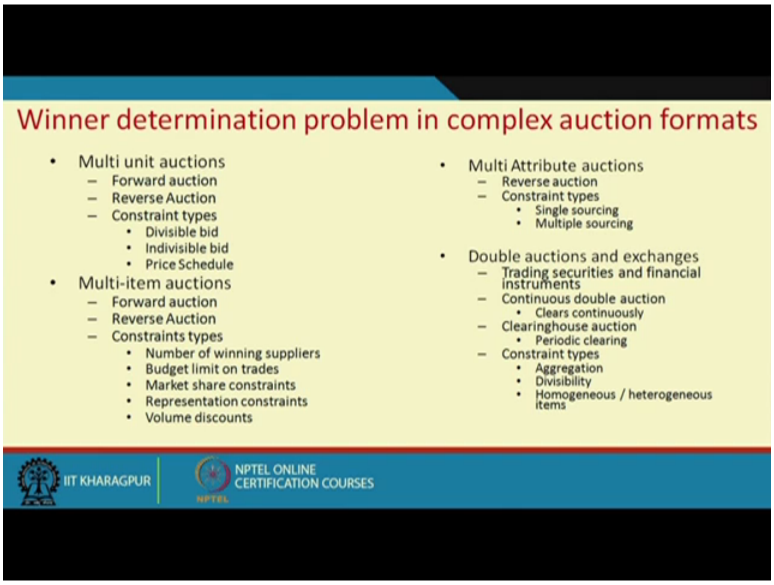

There can be multiunit auctions which can be forward auction or a reverse auction, it can have many constraints like bid the bid can be divisible, I mean the by divisibility remain in fact we are going to discuss more about it. Similarly I can have multiunit auctions, multi-attribute auctions, double auctions, in fact about all this while talking about how to how to distinguish between various auction types how to classify them we have discussed about all of them little bit now we are going to see about more of this. First one is start let us start with multiunit auction.

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In case of a forward auction it is a it is maximisation of selling price, in case of reverse auction it is minimisation of the procurement cost.

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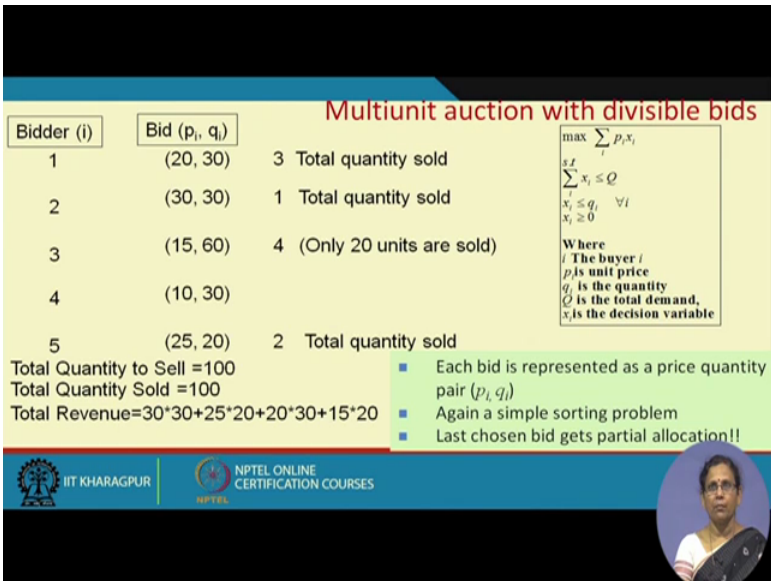

Look a situation in which a multiunit auction is happening with divisible bids, my divisible bids we mean. Now look, here if you look at this you can even if you connect it with the earlier lecture of how to classify the auction, once we said that we can classify based on the nature of the bid. Look in the first case it was price bid only, now we have a bid which is a pair of price and quantity so bid represents a price quantity pair. So here the first person's price is 20, quantity is 30 and so on. In case I go for divisibility of the bid, suppose my I am trying to sell some hundred units and these are the units asked by various people, so if I allow divisibility of bids so all my buyers are willing to willing to purchase less than what they specify then my auction will be like this.

I will be look here, see the point that I am going to, you must be thinking why I am showing this mathematical formulation. The point that I am going to make is from the very beginning I have told you we are going to discuss about decision-making situations mostly related to online business activity. Now while discussing about decision support system I told you decision support system can be either model based, model driven or it can be data driven. About data driven, few examples about your customer behaviour modelling, about your recommender system we have seen. But in case of model-based so far we have not seen any example situation, here is the example situation this is called a model.

So much kind of behind every model based decision support system there will be a model of this type, my sole purpose of showing you this mathematics is making you understand that when you talk about a decision support system and talk about a model driven decision support system, we have such a mathematical model in the background. So but right now you do not worry because basically when I offer this course to our students, they have a background on this so decision makes relevant.

But here anyway for simplistic problems I will try to explain little bit and for difficult problems and if you have taken courses on optimisation, decision modelling, linear programming, et cetera, it is anyway becomes very obvious to you. So here the idea is to sell some 100 units to whom? Because nobody wants exactly 100 units, they want in fractions, first person was 30 units, second 30 units and so on. So what do I do? Now this is a forward auction and I am trying to maximise my profit so if I maximise my profit then I some over P i into x i where p i is the price and x i is the unit that is being sold to a particular person. And each x i let us say this is the ith bidder, the amount what he will be getting should be less than the quantity he has demanded okay.

And all these x i you see x i are here really variables, they are no more integer variables. And total quantity I sold some of all these x i should be less than or equal to q, this is again even if in spite of this complex making mathematical formulation it is a very simple problem again, how do you solve it? You sort all the bids based on their price okay and the person who is offering highest, in this case this is the highest bidder you give him the total quantity. So out of 100 units now 70 units are left. Now he is the second-highest bidder, you satisfy his demand so 50 units you are selling, he is the third bidder, satisfy his demand 30 units so 80 units you are selling. Now this is the fourth bidder, his price is some Rs 15 or 15 dollars or something and his quantity is 60 units.

So now I have already sold 80 units, I have already decided to sold 80 units so next 20 units I will be selling okay. So this particular problem again can be solved as a in case of the bidders agree to take consider this divisibility, they agree to even purchase less even if they are asking for more than such a situation arise.

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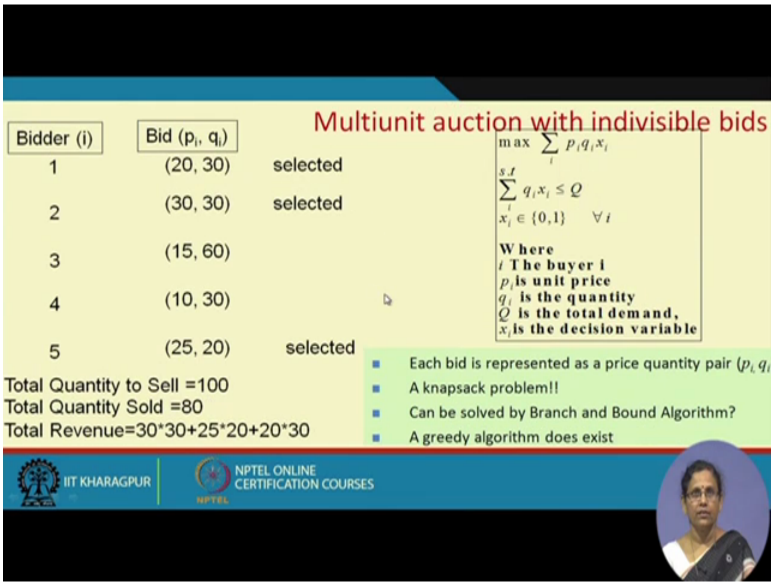

Now situation becomes little different, if they are not willing to do so. So in that case what do I do? I will be solving this problem in which I have to select which all suppliers are within my quantity and still they provide the highest price to me. So this is a typical Knapsack problem situation and knapsack problem you need not think about what a knapsack problem is if you do not know people with optimisation background will be knowing and their typical algorithm to solve this is branch and bound. But here also you can solve it very easily, How? Sort it and after sorting the highest bidder gets the full amount, second-highest bidder gets the full amount, third highest bidder gets the full amount, the moment you encounter a bidder where you are not able to fulfil the total demand you stop, that can be greedy algorithm of for solving this problem okay.

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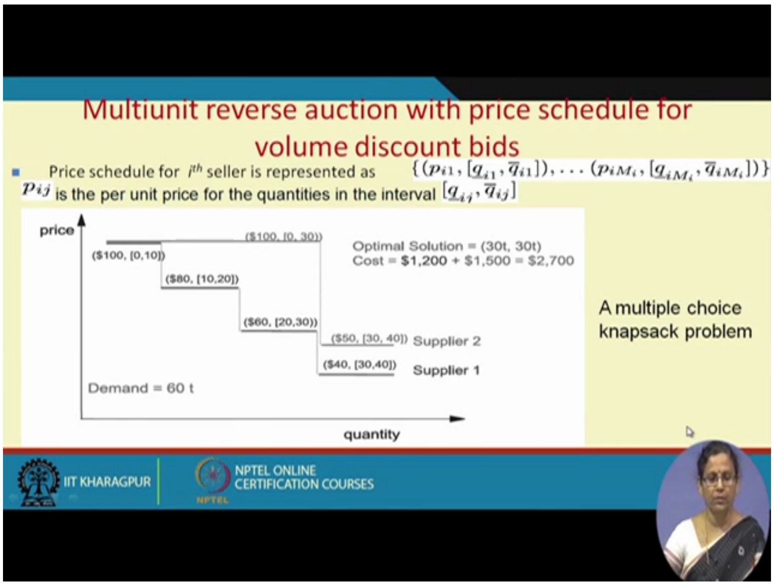

So now situation even becomes complex, the bit type is now complex, you have a multiunit reverse auction with price schedule for volume discount bit. Here the situation goes like this, everybody provides a bid of this type, they provide a price and a range within which this price is valid. So if let us say this is a situation of 2 bidders where the first bidder offers 100 dollars for first 30 units and 50 dollars for between 30 to 40 units. So as I buy more the price drops so this is the price schedule provided by the second supplier. So you will be giving one value such valuation for the first range, another such valuation for the second range and so on. Now the situation in case of the first supplier is bit different, first 10 units 100 dollar, second 10 units 80 dollar, next 10 unit 60 dollar and finally next 10 units 40 dollars.

Now I can have many constant here as well, I should be buying some quantity from supplier 1 and some quantity from the supplier 2, so how much to buy from supplier 1 and how much to buy from the supplier 2. My total demand is some 60 units so there there will not be very straightforward formulation to solve this, so what will happen? There will be a formulation which I have not mentioned here and formulation will look quite complex and it is very difficult to explain to everybody without proper background in in optimisation so therefore I just show you the optimal solution in this case. Considering all the combinations of the price ranges, the optimal solution turns out to be 30 units from each bidder I will be buying.

You can try yourself, if I decide to buy something else some other combination, let us say my demand is 60, I buy 20 from here and 40 from here, so how much I end up paying and how much I end up paying in this particular case, okay. So this is this particular problem is a multiple-choice knapsack problem, so so the bidding system that you are implementing will be solving such complex optimisation problems with appropriate algorithms.

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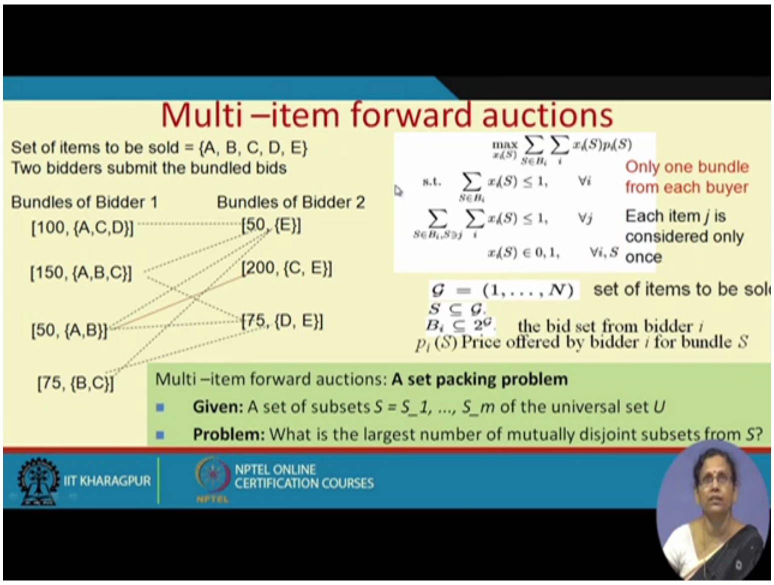

This is again another situation, this is a example of a multi-item forward auction . Again do not worry about this formulation, let us try to simply understand the winner determination problem. Suppose it is again a selling kind of auction where my aim is to maximise my profit so the set of items to be sold are A, B, C, D and E. And 2 bidders submit the bundle bids, so the first bidder submits 4 bundles, for A, C, D if he is buying together he is offering 100, for A, B, C together he is offering 150, for A, B together he is offering 50, for B, C together he is offering 75 and so also the second bidder.

Now I have only one unit of each, now look we are considering a very simple situation in case of this combinatorial auction, this is a combinatorial auction very simple situation where each unit there is a single unit of each item, but situation can be more complex and you can have more units of each item, but here in this one unit is this so while selling this item I should be very very careful, in what way? First of all I have to I have one objective I have to maximise my profit but I have certain constraint as well, what is the constraint here? Constant is, while selling the item I should not be selling one item more than once because I have only one piece of the item.

Similarly, while deciding the bundles from each bidder because one bidder is after all providing all the bidders, I should be only giving only considering one bid from the bidder, he is providing many auction s to me but I should be considering only one. So only one bundle from each buyer and only one item is to be and each item has to be considered only once. Forget about this formulation part, now to find out the solution you have to solve this combinatorial problem which is quite complex and you have to for this model which is there in your model base of your decision support system you have to actually write certain algorithms, but anyway we are not going to discuss about that but let us try to find out what are very possible combinations considering these 2 bundles.

In fact this is your assignment, find out all the combinations for this particular problem and values of each combination for example, if I take the combinations A, C, D and E from here, this is available combination because each item is getting sold once and one bid from each so my price is what I am getting is 150. Now if I go for this this bundle A, B, C and C, E, it is not possible, so all the possible bundles are now connected out of which this solid line which connects these 2 A, B and C, E, here I am getting 200 rupees, here I am getting 50 rupees so total 1 item is not getting sold that is D but that is fine with me but I am now getting more price.

Look if I consider this one; A, B, C and D, E, all my items will be sold but I will be getting much lesser price around 25 rupees less. Here I was getting 250, here I will be getting Rs 225 so actually even if one of the items is not sold I will be going for this okay. Now this particular problem if you can if you know have little background, it is a set packing problem but anyway you do not have to concern about it and such kind of optimisation related things like whether it is a set packing problem, whether it is a knapsack problem, no questions will be asked so do not worry.

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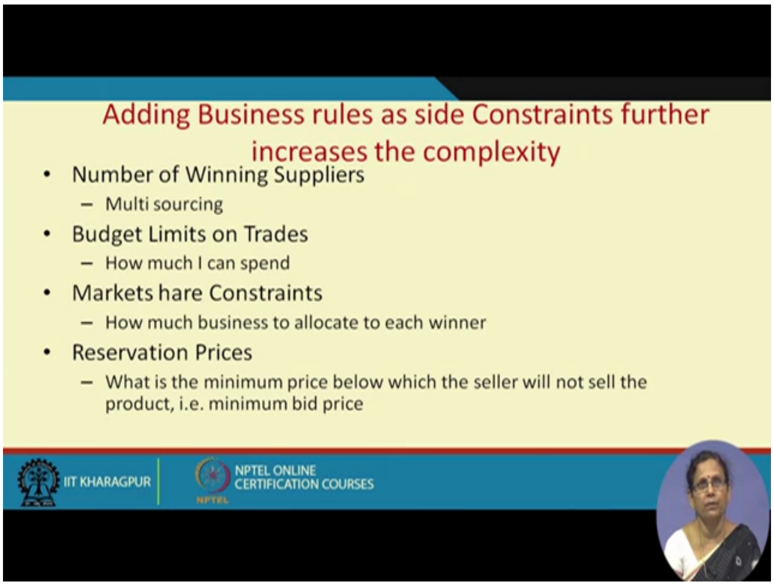

Now along with this, this setting many business rules can also be added, this business rule will appear in the form of constraint in the optimisation problem and will make the problem even more difficult. For example, I can introduce a number of winning suppliers suppose I am going for multiple sourcing, to minimise procurementries I am going for multiple sourcing so I will be choosing 2 or more suppliers, in that case I have to add it as one of the constraints in the optimisation problem. Then budget limit on trade, how much I can spend in case of reverse auction this is a very valid situation. Then market share constraint, in case of market share how much business is to be allocated to each winner.

So in case of for example in case of reverse auction I can decide how much units I will be buying from each buyer, I can decide I will buy from 2 buyers that is my number of winning supplier constraint then I can have one more constraint that I will be buying let us say maximum 600 units from its supplier that is my market share constraint. Then I can also said resolution price, now this is I can decide what is the minimum price below which the seller will not sell or what is the maximum price above which the buyer will not buy in case of a reverse auction.

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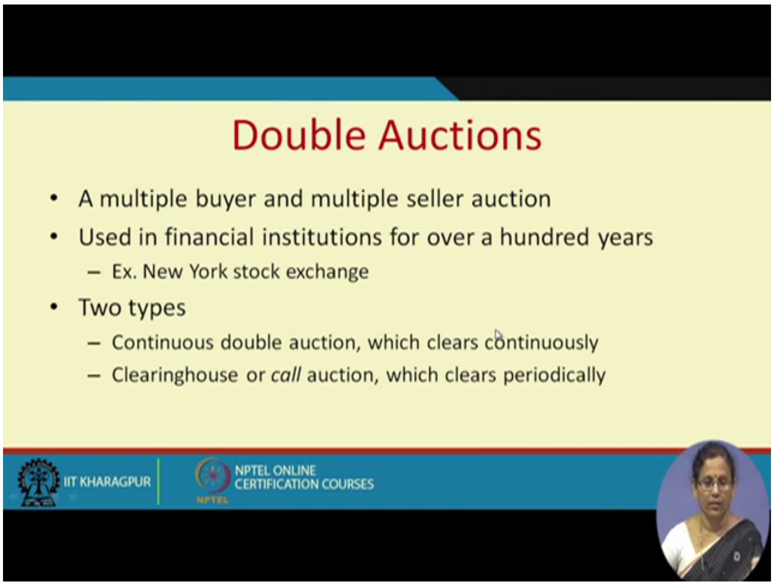

Then another kind of auction is your double auction, in double auction you have a situation in which you have multiple buyers and multiple sellers. Usually such auctions happen while exchanging financial security. In fact, many financial institutions use such kind of auctions for many years; example is your New York Stock Exchange. But with introduction of this online auction these things are happening in a more convenient manner, in a more automated manner, the activities are now automated.

So there are basically 2 types of double auction s, one is continuous double auction which clears continuously and second one is your clearinghouse or call auction which clears periodically. By continuous which mean the transaction period is very short maybe every 5 minutes, 5 minutes, 5 minutes will be getting clear, but in case of clearinghouse or call auction may be in a day or so it will be getting cleared once.

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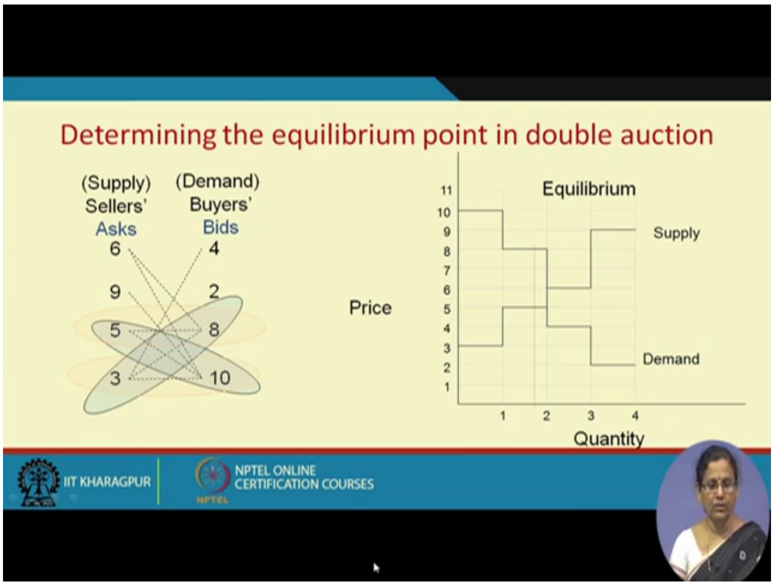

So this is the situation I was talking about, you have some seller one seller, you have 4 sellers and you have 4 buyers okay. The seller asks and only one item is under one single unit items are under consideration, so how many items are then in the market? Every buyer is trying to sell only one item so there are 4 items in the market. Now similarly, sellers are asking for 4 items so there is a demand of 4, there is a supply of 4 items there is a demand of 4 items as well. Now let us see whether all items will be getting sold or not, now what are the asked prices of the seller? First person is saying I will be paying 6 rupees if I get one unit, second person is saying 9 rupees, third is 5 and so on. Similarly the bidders also have their expectations who are trying to buy.

So the sellers are trying to sell at these prices and buyers are trying to buy at these prices. To find out this market equilibrium price let us look at this situation, this is my supply curve, at price 3 rupees that is this buyer, only one item is under sale ,with 5 rupees 2 items are under sale that is this person is willing to sell, the person who is asking for 3 rupees he is also willing to sell, he will be getting more price why will he see, he stop he will also sell. Then at Rs 6, 3 items are under sale, at Rs 9 all the 4 items are under sale, now look at my demand curve, at price 10 if the market price is 10, this is your buyer's bid okay, so if the market price is 10, only 1 item will be bought, if the market price is 8, two items will be bought because the first person who is willing to buy at 10 will be willing to buy at lesser price as well.

So then the third person the price is for 4 rupees so now 3 units will be sold at 2 rupees all the 10 all the 4 items will be sold. So when this demand curve and supply curve meet together, that should be my market price so my market price is basically this much so my market price is, how many items will be sold? 2 units will be sold and at a price market price 5 at market price 5, 2 units will be sold. So with this market price 5 how many transactions will be possible? Now these 2 sellers are asking 6 rupees and 9 rupees so naturally they will not be able to sell because their price they are asking is more. And which 2 because only 2 items are now under can be sold which person will be buying it, which will maximise my market surplus so either this I mean only the highest bidders will be getting the items.

These 2 items the highest bidders will be getting so that this person I mean both the persons both the parties are benefited, they will be getting the item at a lower price and this will be, these 2 fellows will be getting prices at as high as possible, now market price is 5 so the selling only 2 items are getting sold.

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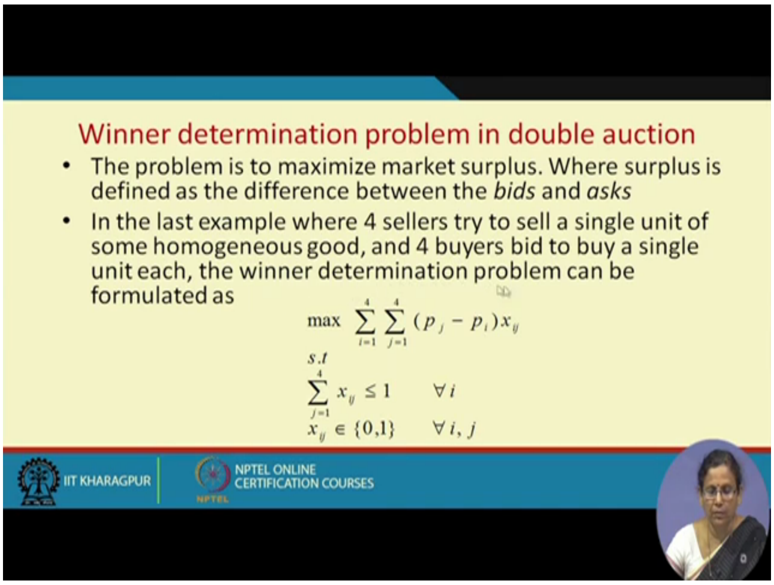

So this corresponding winner determination problem with that particular setting can be written in this manner and the problem is to maximise the market surplus, this is a market surplus the difference between the ask price and the bid price, sorry the bid price and ask price. And where the surplus is defined as the difference between the bids and asks. The last example where 4 sellers try to sell a single unit of some homogeneous good, and 4 buyers bid to buy the single unit of each, the winner determination problem looks like this and you have to devise corresponding algorithm for that.

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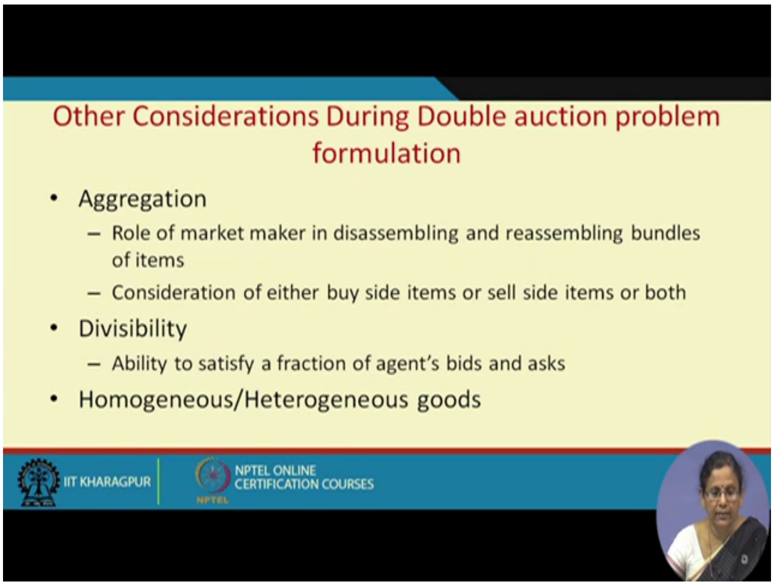

Now besides this simple setting there can be other considerations during double auction problem formulation, it is you can go for market aggregation where the role of the market maker in this assembling and reassembling the bundle of items. So which means it is no more a single item situation, multiple items are getting sold, these multiple items can be homogeneous or heterogeneous. Then consideration of either buy side item or sell side item or both so disassembling can happen either whether with the buy side item or at the sell side item or both sides.

Divisibility; then in case again multi-item situation, see these are mostly again for financial securities and shares, many shares of the, many unit shares of the same company or from the multiple companies so different items multiple units you can now relate possibly and homogeneous and heterogeneous goods okay. So with this definition our discussion on winner determination problem in various auction settings and these problems are to be solved online and looking at their complexity you can realise to make them happen in an online environment you need faster algorithms, so this is so we end our discussion on winner determination problem, thank you very much.