Data Analysis and Decision Making 3
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Lecture 57

Welcome back my dear friends, very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe. And, this is the DADM 3 which is Data Analysis and Decision Making 3 course under NPTEL MOOC course series. And this total course duration as you know is for 12 weeks which when counted in the number of contact hours is 30 and when broken down into number of lecture is 60 in number considering each lecture is for half an hour.

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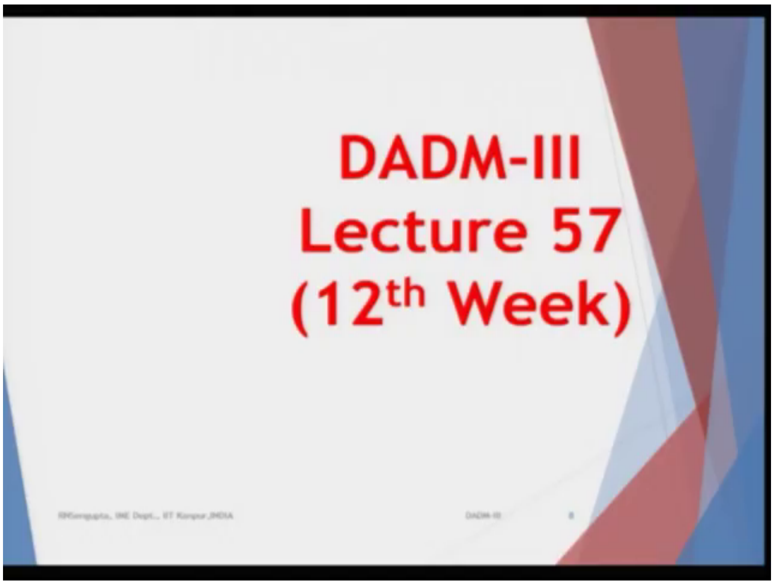

And as you can see from the slide this is the 57th lecture which is the second lecture in the 12th week. And after each week lectures which is 5 in number of half an hour each you take assignments. And then after the completion of this whole course that means after you have taken 12 assignments, you will be appearing for the final examination. And my good name is Raghu Nandan Sengupta from the IME department at IIT, Kanpur.

So, if you remember we were considering the concept of reliability with optimization and I was mentioning time and again. That you have the x plane in which the decision variables, the parameters which are stochastic or the parameters which are deterministic all are being calculated and measured. And in order to basically consider the concept of reliability based optimization or reliability analysis what you do is that you use Rosenblatt transformation.

Considering the fact if you remember I said that the main concept being the CDF values are between 0 and 1 for the corresponding distribution which when converted into the normalized standard normalized independent random variables used. You convert each of them of the axis into u1, u2 till un and also the probabilistic parameters. And then basically optimized that using the PME or the RIA method which is actually where you minimize the distance of U.

Subject to the constraint that g prime is equal to 0 or considering trying to basically consider minimize the g prime function or g function. Based on the fact that the norm of u is equal to beta, beta is basically the level of reliability. So when and when we are considering the level of reliability considering both of them are normal and have the same variants. They would be circles and the radius of the circle in the u space would be moving in such a way that it will increase or decrease depending on the level of reliability.

And the locai of the reliability point which is the best optimization solution based on level of reliability will have a locai. Now, when you come to the U space as I mention for the RIA or PME approach, your main concern is to find out the most probable point. Most probable point is basically the best point depending on the level of reliability which will give that optimum solution in the reliability space which when back transformed into the x space will give you the next stage of the iteration.

And we also mention one important thing the point from where we will start the iteration in the x space. Because, in the SORA method it sequential optimization is decoupled loop. You do the optimization, take the first step result and then go into the U space do the reliability analysis then map back into the x space and continue doing this till the level of accuracy is reached. Now, remember one thing in case if it is a non-normal distribution you will use the same concept, so there is no change.

Only the fact that I was mentioning time and again that in the normal distribution case, it is very easy to understand that how the calculations can be done. Considering the univariate case or the multivariate case. And I did mention the diagrams which are due will be repeated time and again in order to make things clear. So, after the 57th lecture most probably in the end of 58th or 59th and 60th will consider the robust optimization concept also.

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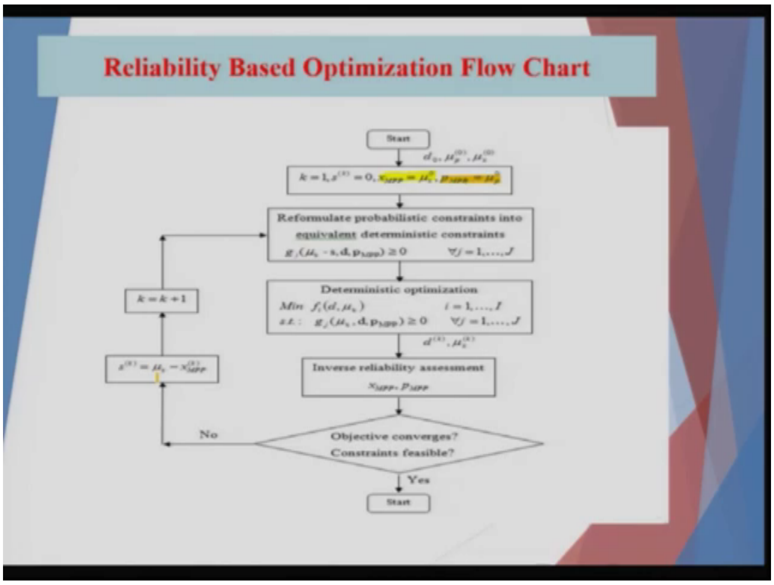

So, generally the reliability optimization flow chart works like this, so in the first step when the k is 1, so is the number of iterations you are doing. When k is 1 you take X MPP, X MPP is the most probable point in the x space that is equal to the mu suffix x knot, knot is the first iteration, so we will take the mean value. And similarly the, so I will just mark in order to make it clear. So, X MPP is the mu suffix x knot which is the first iteration, first value and the first value for P MPP which is the most probable point for the probabilistic parameters is mu suffix P.

So, also they are the mean values and obviously the knot sign which basically means the first iteration. So and you will basically be doing the iteration time and again so you will start with d knot, d knot remember is the deterministic parameter which would not be changing. So, d knot mu P for the parameter values which are stochastic or non-deterministic and mu X are for the decision variables which are non-deterministic.

So, you will basically reformulate the probabilistic constraints, so which are given and based on the equivalent deterministic constraints. So you just removed the probabilistic constraint have the deterministic constraints, solve the problem. So here they are but remember one thing, when you are replacing them you will be replacing them by the corresponding mean values of X and mean values of P which is the vectors.

And, this S when you start of the iteration is basically the amount of movement from the min value at each every step. So, the movement the rate of change I would not use the word rate of change, the change of the functional value is now no more relevant that means fx minus fx the previous value so fxt minus fxt minus 1 if that is less than equal to epsilon a very small value, then you will obviously terminate your iteration values. So, the constraints would be mu X minus S, S is the increment so which is 0 in the first step.

Because that is the decision variable which you are going to change to find out what is the optimum value and P MPP is basically the most probable point for P. So this greater than 0 less than 0 are immaterial they can be with respect to greater than b1, less than b1 whatever. And, you do it the optimization once you do the optimization the deterministic optimizations problem is like this you minimize or maximize whatever the problem is subject to the constraints of ga number of them.

Where you will replace again I am repeating replace x with the min values p with the min values which is the P MPP and do the optimization. Get the results once the results are obtained. So, here P are not changing only mu X or the X values are changing. When the mu X are changed you basically put them in reliability space or the reliability analysis coming. You do the inverse reliability assessment find out the next stage X MPP.

So, X MPP initially was mu knot suffix X, the next stage when you do the reverse reliability analysis, do the reliability analysis using the PME or RIA approach, map it back to x space it will be say for example X1 MPP that means X1 is the first iteration stage. Then you basically find out whether the objective function converges or not based on some set boundary values, if they do not then again you do the optimization in the X space.

Find out the values of X in the second iteration, map it back to the u space do the reliability analysis using the PME or the RIA approach, find out the new MPP points (reverse) map it back to x space find out x2 and continue doing this till you basically reach the optimum solution or the iterative value is basically reaches an optimum value in the sense the difference basically become the as small as possible.

So, you do the inverse reliability assessment find out X MPP and P MPP you ask the question whether, the objective conversions has been achieved were the constraints are feasible. Obviously, when you are doing the objective analysis to find out the conversions you will find whether all the constraints are applicable or at they are feasible. If this is achieved you stop the process, if not you basically shift that S amount the delta amount by each X values would be changing by the value of mu X which is the initial value minus the MPP points.

Then you will basically increment k by k plus 1 into next stage and then repeat the process till you basically achieve the optimality conditions. Not condition not mathematically but you basically reach the optimum values. So, generally the reliability stochastic optimization in very general sense I have discuss that but still repeat it is given by optimize a set of functions f1, f2, f3 and here f1, f2, f3 are all different type of functions which can be linear as well as nonlinear.

So, that means in the case we have the multi objective problem and x dp are the decision variables, d is the deterministic parameters, p is the probabilistic parameters and the constraints are like this gj which is which are probabilistic which will consider later on and we have already done that in the previous reliability analysis concept. Other case for the reliability analysis problem, we considered bj was 0, so it need not be 0.

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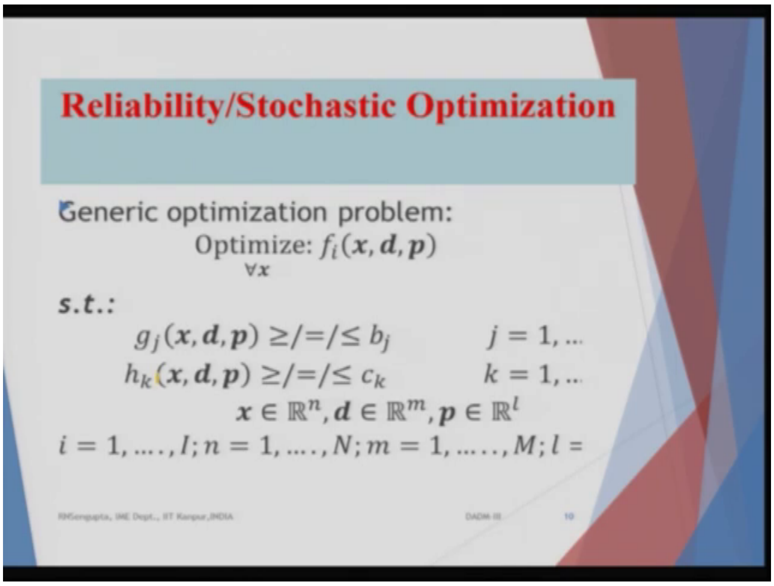

And, there are other sets K number of constraints K, small k is equal to 1 to capital K where the constraints can be considered as deterministic. So, we are trying to basically segregate into deterministic and non-deterministic part. So, if all of them are deterministic so it is a deterministic problem no stochasticity is there, no probability concepts are there, no reliability, no robustness.

And if all them are stochastic so obviously you have to consider their corresponding MPP points for each and every constraints accordingly. And X is in the n dimension, d is the m dimension and p is in the l dimension depending on number of decision variables deterministic parameters and probabilistic parameters which are there. So I is equal to 1 to n depending on number of such objective function which we have.

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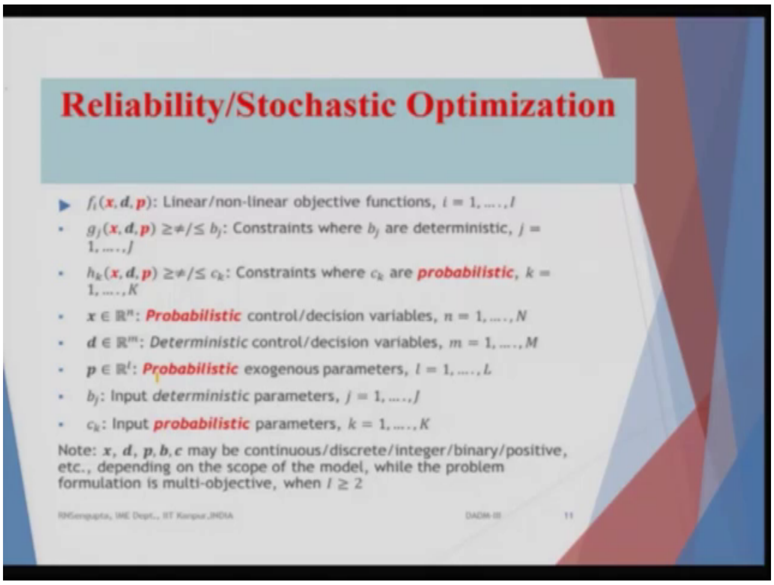

So here fx dp are linear or nonlinear objective function depending on how the problem formulation is done. And you see the red colors they actually signify which of them are stochastic, gj are the constraints were bj are the deterministic. We are not going to consider the right hand side as the non-deterministic, so they will be fixed only the concept of stochasticity would be coming in the parameter values p which are external to the system and the decision variables based on which you are going to take decision.

And, each case are the constraints were ck are probabilistic, so the right hand side here we have basically divided the right hand side into two different sets, sets means two different types of constraints one of them which are deterministic another sets are non-deterministic. X is the decision variable or the control variable which are probabilistic, d are the deterministic controlled variables or decision variables. And, p are the probabilistic exogenous parameters which basically detects how the overall effect is there on the constraints, bj are the input which deterministic parameters and ck are input which are probabilistic parameters.

So, here x dp b and c may continuous, discrete, integer, binary positive depending on how the problem formulation has been done. Depending on the scope of the problem and if it is a multi-objective problem obviously fi which is the objective function would be I would be more than 1. So, it can be as you as have seen in case of quadratic programing they were two parts. One was the quadratic part, one was the linear part. Quadratic was x qx, x are all vectors and q are is basically matrix and the linear part is c into x where c and x are both vectors. How they, how you denote them whether column vector or rho vector will depend on how the problem formulation has been done.

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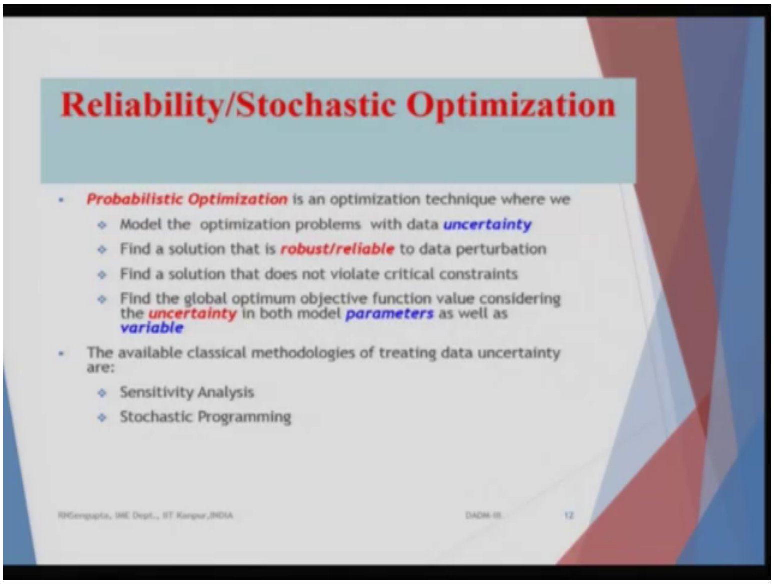

The probabilistic optimization, is an optimization techniques were we model the optimization problems with data uncertainty. You need find a solution that is robust and reliable to the data perturbation of the data fluctuations or data uncertainty which is there. We need to find a solution that does not violet the critical constraints so it will be feasible based on the feasibility get the optimum solution based on the conditions.

We need to find out the global optimum of the objective function value considering that uncertainties, in both the model parameters as well as the variables, model parameter are the P and the variables are the X. The available classical methodologies is sensitive analysis which we have considered though briefly in the linear programming, it can be done from the nonlinear part also. And, one is the stochastic programming or reliability programming or robust programming which are discussing in quite details.

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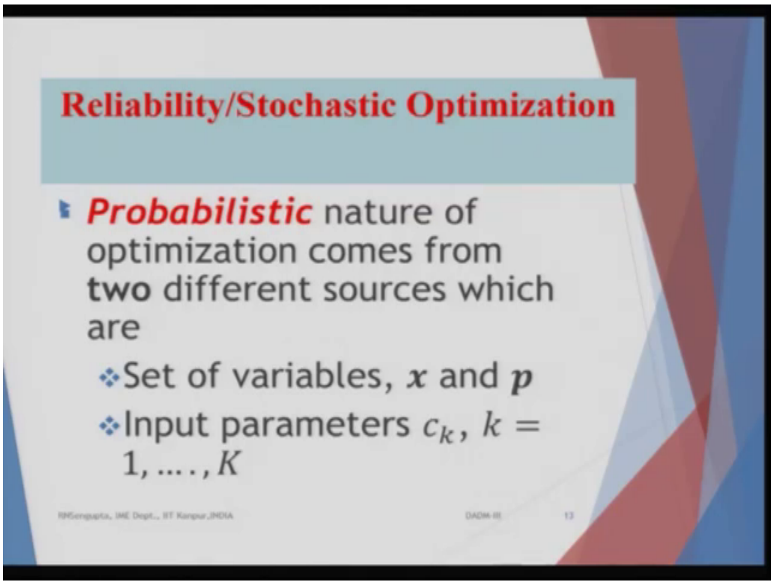

The probabilistic nature of the optimization comes from two different sources, which are set of the variables X and P and the input parameters which is ck which is on the right hand side. Because the bj are all deterministic, so there are 3 different sources, one of the decision variables which you want to take, one is the external set of parameters P and one is the right the constraints right hand side of the constraints. Few of them so the set k of them are non-deterministic because, c1 to ck are probabilistic in nature, they will have a distribution per say.

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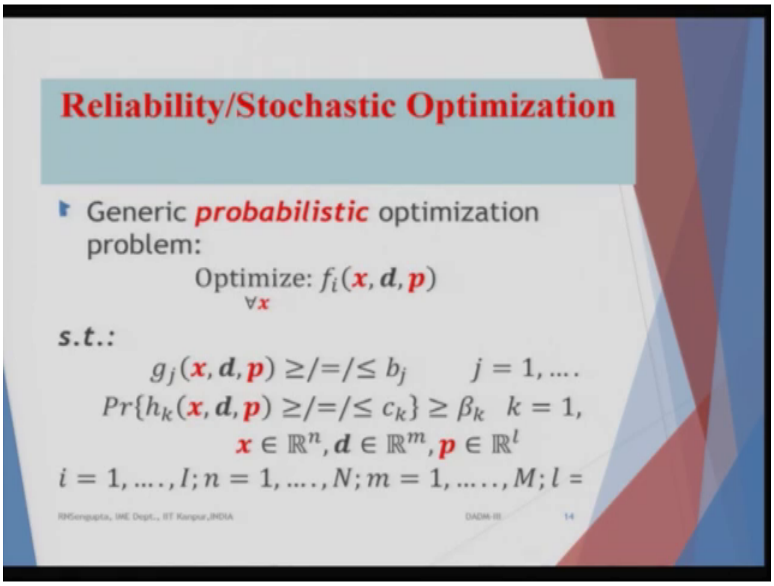

So, the generic probabilistic optimization would be optimize so here it is the same slide which we are consider but with the change, that the red colors are being marked for the case which are now no more deterministic but they are probabilistic. So, you optimize the functional form fi which is the function of xdp and xnp are probabilistic because, x is the decision variable p is the parameters which are external which are probabilistic. The constraints have been divided in to two parts, in one case it is this gj where the right hand side is deterministic so they remain as deterministic constraints.

And, in the case if it is right hand side is ck which are probabilistic will basically model that as a probabilistic constraint. Now, here if you remember I have been talking about alpha here we are replaced alpha by beta k. So, you have the probabilistic hk which is the function of xdp is greater than or equal to or equal to or less than equal to ck. And, that has a probability of bk, bk is means they will different values of betas and that will depend on the decision maker who is going to make the decision and an do the optimization. Again x is in the n dimension, d is m dimension and p is l dimension.

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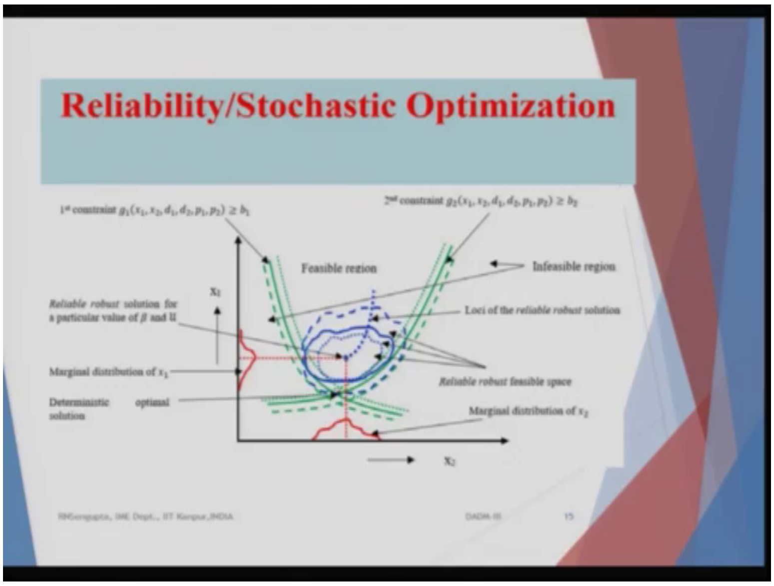

Now, this diagram I have already discuss but I still go through it, so here in the in the 2-dimension problem you have x1 along the x y axis x2 along the x axis. And, the actual boundary for the deterministic constraint is the bold green line. I do not want to draw too much here because or else it will get clattered. But, if camera is focused on this diagram I can continue talking, so rather than focusing on me if it is focused on this slide it will basically clear all the doubts.

So, the green bold lines are the constraints which are deterministic and the red point which is there, which I would not highlight again I will just hover the electronic pen. So, this is the deterministic optimum solution which can be solved using any of the deterministic tools.

Now, what we consider is that the constraints can be perturb as per the sensitivity analysis they can move like the right hand side changing plus or minus one.

So, the constraints for the green line, constraints basically fluctuate with some probability, so the small dotted one and the large dotted ones these are the range based on which the values we want to find out that how the constraint basically can fluctuate. And the feasible region which is inside it will shrink, the total number of such feasible points or feasible space will shrink depending on high or how low your reliability is. And, we consider that x1 and x2 are normal, in case if it is they are normal then you have a circle.

In case if it is 3-dimension you will have a sphere or like a football, in case if they are normal but one variant is higher than the other will have sort of baseball shaped overall ellipsoid which will give us the feasible optimum point thus the center of gravity of that will give the optimum point based on the level of reliability. In case if they are not normal then trying to find out the optimum space inside the feasible region would be difficult. So, there are different areas which have been shown depending on reliability.

So the blue one bold one is for the case when it is along the boundary and it is no more normal as I said and the small dotted ones and the large dotted ones or the last hashed ones are the level of the reliability space. And, the center of the gravity is the optimum solution depending on the level of reliability which you have. So, outside you have the (feasible) infeasible region and this dotted one which goes up is the locai of the reliable robust solution.

So there are different reliable robust feasible space which are given at the end remember one thing this red distributions which are given are the marginal distributions of x1 and x2 correspondingly. And, obliviously the joint distribution would be important for us.

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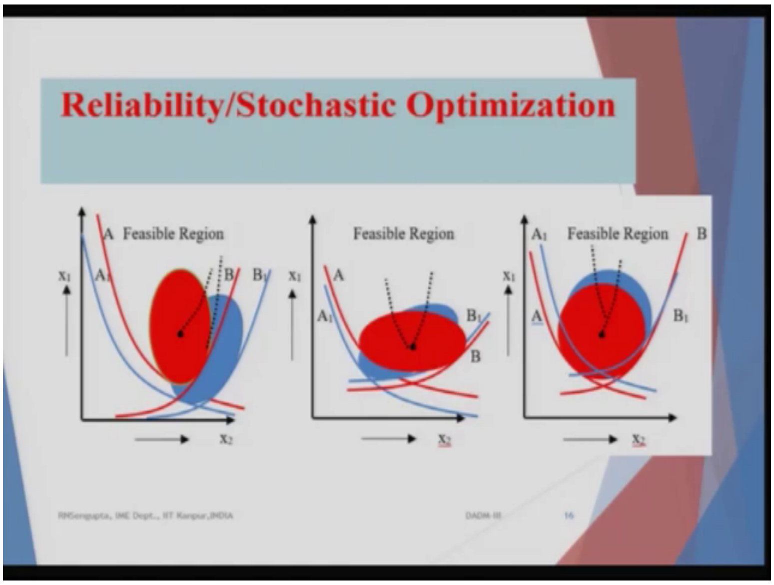

So, this is the fact if you have in the right most panel is the one where we consider both variances of x1 and x2 to be equal hence the common area or the feasible region are circles. And these circles would basically move inside the feasible region depending on the level of reliability being increasing or decreasing. In case if you have the second panel is in case if we have the variants of x2 is large so it will be a baseball or an ellipse or an ellipsoid kept where the major x is horizontal.

In the cases the variants of x1 is more in the initial case it was x2 was more in the first panel if the variance of x1 is more than it will be ellipse or ellipsoid where the vertical axis is the major axis. And, you can find out how the optimum solutions can be obtained so I have tried I give it but this I sort one nice diagram which you could be discussed and I have just made it in order to explain you in much details.

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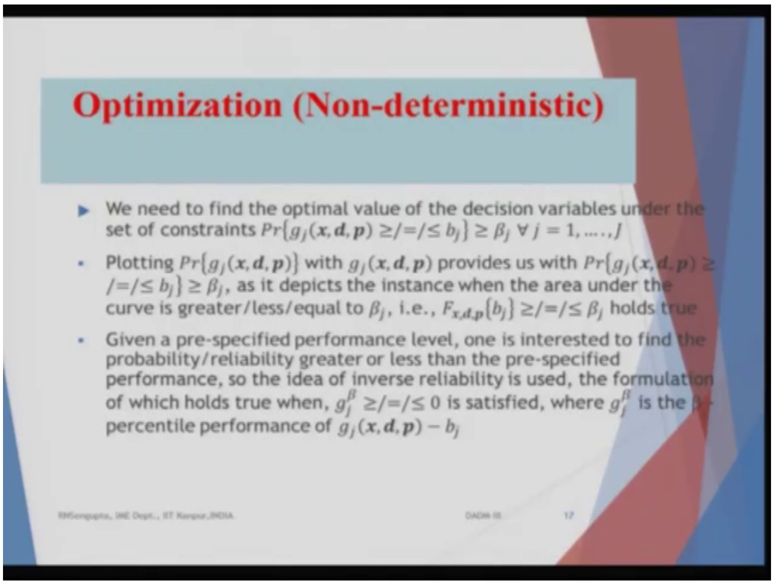

So, we need to find out the optimum value of the decision variables under the set of constraints which is the probability of Gj or hh whatever it is where you define is greater than equal to beta and plotting the properties with which gj provide us with the probability as it depicts the instances when the area under the curve is greater or less then equal to beta. If you remember the graph which you just said on the right hand side so alpha area was there.

So we want to find out the probability that it is equal to of bj, so bj are the values based on which will basically find out what is true or which is feasible. So you can find out greater than equal to, less than equal to and less than equal to beta j which holds 2 and we basically find the solution. Given a predefine performance level one is interested to find the probability or reliability greater than or less than the predefine performance. So, the idea of inverse probability would be utilized and it is formulated by trying to find out.

So if you remember what we did, if you have capital f of x is equal to value of alpha you find out the inverse probability and find out the small x value or the small z values. So this inverse probability will be utilized to find out the values of depending on the values of beta you want to find out the values of x which is the decision variables. And obviously they would be utilized in order to find out the X MPP points and similarly do the reliability analysis.

So, this would be satisfied when g to beta is basically beta percentile performance would be given by the difference of the functional form of the constraints gj minus bj. So, we will consider few very simple models and I discuss about that.

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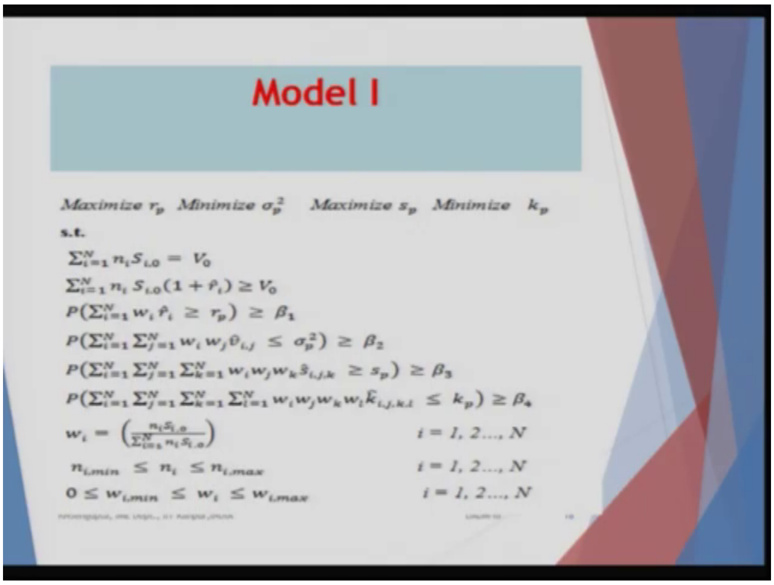

So, in the first model it is multi objective where f1, f2, f3, f4 are given. The 1, 2, 3, 4 values are corresponding to the case where the first value for I is equal to 1 is the expected value hence you are trying maximize rp. Next I is equal to 2 basically you want to find out the minimum variance hence it is minimization of the variance. The third one is basically you want to find out the maximum value of skewness, so it is given by maximization of value of ss with a suffix p is basically for the portfolio.

And, the last one is minimization of the Courtesies, so we basically have two maximization problem and two minimization problem. And I want to basically find out the multi-objective solution for this. What are the constraints? In the simple case the constraints are given like this, 2 are deterministic and 4 are probabilistic, what are the deterministic part? Deterministic part is the initial wealth which is the number of stocks multiplied by the corresponding prizes is equal to v knot which is the money which I have.

Second one is that the prizes of each and every stock would be increasing in decreasing we do not know but, in overall sense we should find that the some of the multiplication values of the stock numbers multiplied by the corresponding prizes. If they are added up it would always be greater than V knot, or else it does not suffices that why we are investing it, if we cannot increase our wealth obviously it does not suffices. The next 4 constraints are the probabilistic one which are like this.

And with level of reliability of beta values is given by beta 1 to beta 4 depending on the level how we perceive the level of risk for that be, risk means the level of reliability to be. The first constraint is that the portfolio returns considering that you are taking a sample should be greater than the rp value which you want to basically maximizes. Maximize means if you are considering from your side you will try to push it more on to the right returns being higher-higher since it goes away from 0 on the right hand side.

And that has the probability of beta 1, the second one is basically the overall combination of the portfolio variants, which is double formation wi wj into multiplied by sigma aj is less than or equal to sigma square p which is where sigma square p is the set variance, which you want to minimizes. That means push it want to left-left more away from the 0 but on the left hand side and the probability is beta 2. Similarly, the probability is beta 3 for the third constraints which give us some idea that what should be this skewness values of the portfolio.

Such that it is greater than sp or which is which you want to maximize so push it or more on to right-right away from 0 to the right hand side. The fourth constraints has a probabilities of beta value as beta 4, so beta 1, beta 2, beta 3, beta 4 are as I mentioned they are dependent on the decision maker they can be equal also.

Hence, coming back again to the forth constraints it is the Courtesies value or the overall portfolio would be less than equal to kp, kp is set value which we have taken in a optimization problem and you want to minimize it like push it on to the left more away from 0 and one should remember the constraints have been formulated other sets are. That ni values which is the number of stocks which I want to buy is between a minimum or maximum value for each and every stock.

Will also consider the wights which can be found out using the ni values is also bounded between 0 and 1 or between the values of minimum and maximum so it can be depend how you able to formulate the problem. And the weights wi are given by for each stock I am basically finding out it total investment which you are doing in that particular stock which is the multiplied value of number of s is multiply it by the stock prizes divide by the total amount of money which have been in my pocket.

Now, remember in the initial case a summation of ni into si should be exact equal to v knot which you have already done as one of the constraints. So the weights can be found out accordingly that means total number of investment which I have done for that particular stock divided by that total investment I am doing in totality.

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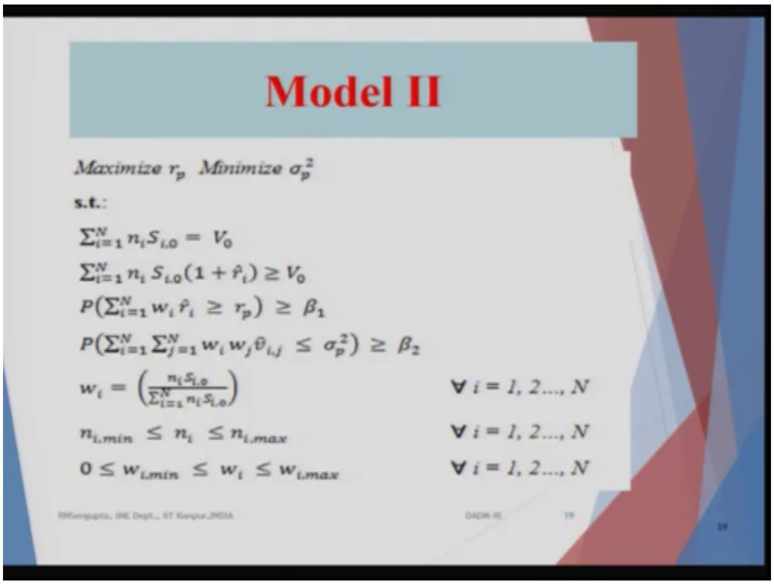

The second problem is also a multi objective problem but now we have 2 objectives, so fi is i equal to 1 to 2 and the without repetition I will just go through it. So, the first come constraints remains the same and which is the fact that the sum of your portfolios that is the ni into si is exactly into v knot the money which you have in a pocket. The second one is where the initial amount of money which you had in a pocket should increase with the interest of ri returns was for each and every stock which you have.

And that the overall summation of the number of stocks which you want to by multiplied by the corresponding increase in the prize it should be greater than or equal to V knot. The second and third are corresponding to the first movement and second movement which is basically related to the return of the portfolio and the variants of the portfolio and the beta values of beta 1 and beta2. And again wi is exactly equal to the same where I invest total amount I have in my pocket is V knot.

So, each investment which I do in each and every stock that value divided by V will give me wn and obviously the sum of wi should be 1 which I am not written here because I that can definitely has to be consider. The ni values is basically bonded between the minimum and maximum which is the number of stock which I can invest and wi is obviously is bonded between minimum and maximum where minimum can has to be 0 maximum need not be 1.

Because, based on the case this maximum value need not be 1 which I am saying that in that case the negative value should also be true for wi if short selling is there which I am not going to discuss now.

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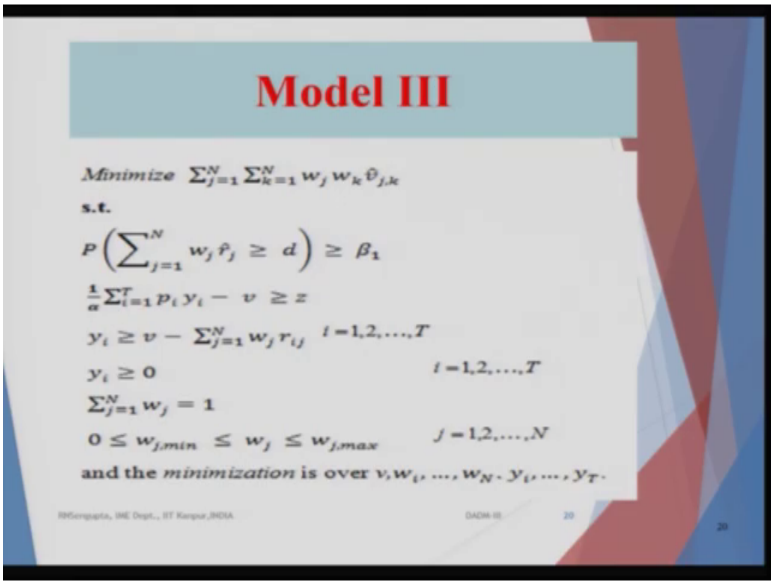

Model 3 is basically considering the minimization of the variance subject to the concept that we are basically trying to find out the returns greater than a value of d with a probability of beta 1.Similarly, we will consider the concept of the risk where the conditional value at risk or value at risk can be consider and the model can be done accordingly. Again, we will consider the minimization is there for all the values are wi, the wi value are between minimum or maximum and the sum of the (())(29:23) is equal to 1.

So, with this end this I will end the 37th lecture, so the 38th, 39th and this 57th lecture so with the 58th, 59th, 60th I will basically complete the course but obviously I will lay stress on the robust part. And in one of the last lectures I will extend it with for about 15-20 minutes I will rap up few important thing is which I think would be important for you, how you analyze the optimization problem. So, as I end the lecture I have already thank you all of you but I want to wish all of you a very happy Durga Pooja, Dashera, Diwali.

So, obviously you will be it hearing all this lectures in the dew course of time may be during the just before the Dashera but I want to wish all of you a very prosperous and a nice festival and I am sure with Gods blessing all of you will do well in life. And, as I said I will also wrap which I am just repeating it I will wrap up few extra minutes in the last lecture to basically high light the important points which are there for this course have a nice day and thank you very much.