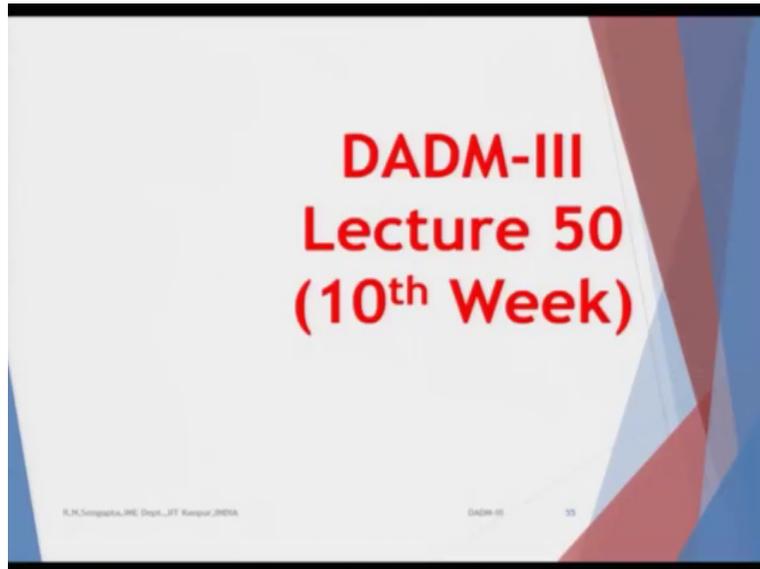


Data Analysis and Decision Making - III
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Lecture No. 50

Welcome back my dear friends and dear students, a very good morning, good afternoon, good evening to all of you wherever you are in this part of the globe and this is the DADM 3 which is Data Analysis and Decision Making 3 course under NPTEL MOOC series and this total course duration in the week number is 12 weeks and total contact hours is 30 hours which is spread over 60 lectures which means that each lecture is for half an hour.

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As you can see from the slide this is the last lecture of the 10th week which is the 50 lectures and after that you will take the 10th assignment before that you have already taken 9 assignments and you will complete it totality 12 assignments and then at the end of the lecture of 12 weeks being over you will take the final examination. And my good name is very is Raghu Nandan Sengupta from the IME department at IIT, Kanpur.

So if you remember in the 47th, 48th lecture we discussed a branch and bound for the integer programming again I am highlighting few points. It was purposefully taken as two dimensional case such that we can show the graph and solve the problem as you proceeded, but I did not show the branches which was very obvious I could have done but I did not show. In the next

problem which was little bit more more enhanced, it was a binary programming problem but the main issue was that it was an higher dimension such that drawing the diagrams would not be possible.

So I basically highlighted the overall branch and bound the steps and I also mentioned that I will give you the overall picture or the total solution but at each and every stage where we are and from where we have come in the branch and bound method I will highlight it using the different colors schemed on number 1 is from where we have come and what we have already traverse would be blue in color and where we are from the stage where we will go into the next branches would be highlighted by the green color.

So now when we left the 49th lecture last slide it was basically trying to highlight one of the arms which was basically the branches considering the actual binary variable, binary formulation was given and then we will basically now relax it, write the linear programming part and proceed accordingly.

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**Branch & Bound (B&B)
Example # 02**

► Branch # 03 ($x_2 = 0$ or $x_2 = 1$) and we consider $x_2 = 0$

LP relaxed problem

$$\text{Max } Z = 9 + 6x_3$$

s.t.:

$$5x_3 \leq 4$$
$$0 \leq x_3 \leq 1$$

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So relax problem when we solve which is branch number 3 is basically when we are taking, we have already taken as x_2 as 1, so x_2 could have been 0 and 1, so we take x_2 as 0 now and the LP relaxed problem formulation is like this, you maximize 9 plus $6x_3$ and here you have the object the problem formulation as $5x_3$ is less than equal to 4 and x_3 which was initially binary is now

were 2 were binary, 2 were non-binary, then we took this route and that we will highlight using (blue) green color and where we are I will again highlight using the green color.

So the z value as I mentioned is 13.8, the x_1 , x_2 , x_3 , x_4 values are corresponding and (x_5), x_4 values are correspondingly 1, 0, 0.8 and 0. Now here if you see that we would be taken the values if you remember I did highlight few minutes back or in the last lecture that the values here they were x_2 was non-binary, x_4 was non-binary, now we have taken the values in a way that now we have to branch depending on the x_3 value. So the x_3 values what we are going to take in the branches, one is x_3 is 0 and another is x_3 is 1 and we will basically try to find out which values it gives us the best answer based on which we will basically find out the solution for that. So we are in this green stage.

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Branch & Bound (B&B)
Example # 02

▶ **Branch # 04 ($x_3 = 0$ or $x_3 = 1$)**
and we consider $x_3 = 0$

The optimal solution is $Z=9$ and $x_1 = 1, x_2 = 0, x_3 = 0, x_4 = 0$

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**Branch & Bound (B&B)
Example # 02**

▶ With $x_2 = 1$ and considering the
LP relaxed problem

$Max Z = 14 + 6x_3 + 4x_4$

s.t. $9x_1 + 5x_2 + 6x_3 + 4x_4$

$5x_3 + 2x_4 \leq 1$

$x_3 + x_4 \leq 1$

$0 \leq x_3, x_4 \leq 1$

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Now x_2 is 1 here it is written, x_2 is 1 and the corresponding values which you are going to take are x_1 , x_3 and x_4 would be already highlighted and already being differentiated accordingly. So the problem formulation exactly the same, so the problem formulation is like this I will try and write the objective function, it is like this so we have been considering many steps at a time slowly 9 5 6 4, so x_2 is 1, so this is 5, x_1 was 1 which was 9, 9 plus 5 is 14, this is 14, this is 3 sorry and x_3 and x_4 remains so it is basically 14 plus $6x_3$ plus $4x_4$ so this equation what was original one and this one are the same, which is fine. Similarly, we will basically check the constraints, so what are the constraints? This is a slide 63 I will come back to that later, so 6 3 5 2.

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**Branch & Bound (B&B)
Example # 02**

LP relaxed problem

$$\text{Max } Z = 9x_1 + 5x_2 + 6x_3 + 4x_4$$

s.t.:

$$\begin{aligned} 6x_1 + 3x_2 + 5x_3 + 2x_4 &\leq 10 \\ x_3 + x_4 &\leq 1 \\ -x_1 + x_3 &\leq 0 \\ -x_2 + x_4 &\leq 0 \\ 0 &\leq x_1, x_2, x_3, x_4 \leq 1 \end{aligned}$$

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**Branch & Bound (B&B)
Example # 02**

▶ With $x_2 = 1$ and considering the

LP relaxed problem

$$\text{Max } Z = 14 + 6x_3 + 4x_4$$

s.t.:

$$\begin{aligned} 6x_1 + 3x_2 + 5x_3 + 2x_4 &\leq 10 \\ 5x_3 + 2x_4 &\leq 1 \\ x_3 + x_4 &\leq 1 \\ 0 &\leq x_3, x_4 \leq 1 \end{aligned}$$

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6 + 1 + 3 so which is $x_1 = 1$, $x_2 = 1$, 6 + 3 = 9 on the right hand side you have 10 so take 9 to the right hand side 10 minus 1, 9 is 1 which is right. So this equation x_3 and x_4 remains the same, those first constraints and the constraint which is written are right. So I will consider the second, third, fourth constraint. Second constraint is $x_3, x_4 \leq 1$, so x_1, x_2 are not there obviously $x_3, x_4 \leq 1$ which is fine, so this is also right. Now let us go to the third constraint, third constraint is minus x_1 plus $x_3 \leq 0$ because x_1, x_3 are 0, so obviously 0 is less than equal to 0 so obviously the third constraint vanishes, so let us check.

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**Branch & Bound (B&B)
Example # 02**

LP relaxed problem

$$\text{Max } Z = 9x_1 + 5x_2 + 6x_3 + 4x_4$$

s.t.:

$$\begin{aligned} 6x_1 + 3x_2 + 5x_3 + 2x_4 &\leq 10 \\ x_3 + x_4 &\leq 1 \\ -x_1 + x_3 &\leq 0 \\ -x_2 + x_4 &\leq 0 \\ 0 &\leq x_1, x_2, x_3, x_4 \leq 1 \end{aligned}$$

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**Branch & Bound (B&B)
Example # 02**

▶ With $x_2 = 1$ and considering the

LP relaxed problem

$$\text{Max } Z = 14 + 6x_3 + 4x_4$$

s.t.:

$$\begin{aligned} 5x_3 + 2x_4 &\leq 1 \\ x_3 + x_4 &\leq 1 \\ 0 &\leq x_3, x_4 \leq 1 \end{aligned}$$

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So the third constraint has vanished, come to the fourth constraint, the fourth constraint is x_2 and x_4 . So now for x_2 and x_4 , x_2 is 1 and x_4 is basically in this problem so you take 1 to the right hand side it becomes 0, so x_4 being less than equal to 0 is not feasible so obviously it will be out. So actually x_1, x_2 are not there, so in the case of binary problem x_3 is 0 and 1, x_4 is 0 and 1 when you relax it, but relaxed constraint which is of more important for us to highlight is this one. So you have a problem, LP problem formulation you solve it.

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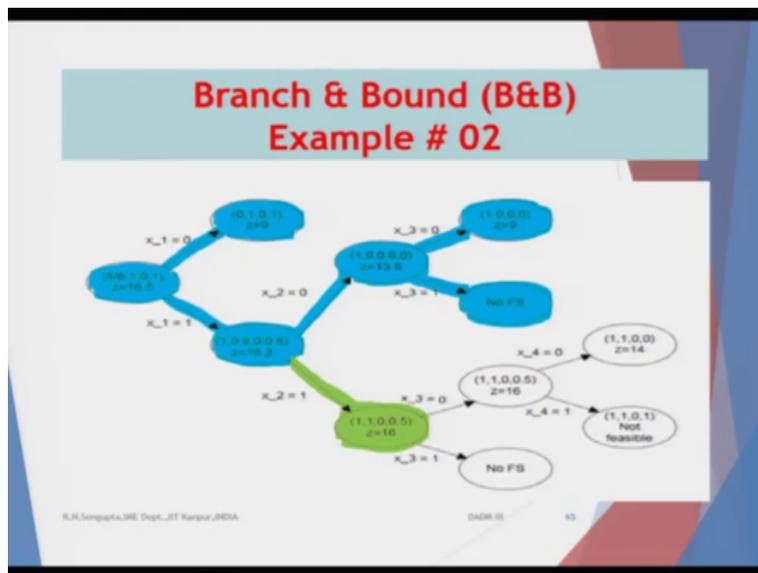
**Branch & Bound (B&B)
Example # 02**

▶ With $x_2 = 1$ and considering the *LP relaxed problem*
The optimal solution is $Z=16$ and $x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 0.5$

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So with x_2 as 1 and considering the LP relaxed problem you solve it and you get the answers accordingly, z is 16, x_1 is 1, x_2 is 1, x_3 is 0 and x_4 is 0.5 so x_1, x_2, x_3 are fine, x_4 which is 0.5 which is not binary we have to basically again branch it accordingly. So where we are in the macro and the macro stage or the total picture, in the branch and bound again I will highlight using blue and green color which I have been mentioning time and again in order to differentiate from where we have come, where we are.

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So we are here, which I will mark in is green, so I would not mark it immediately from where we have come will mark everything accordingly. Just 1 minute, x_2 is 1 right, so we have already traversed, traversed, traversed, traversed, traversed means already visited this place, visited traversed, traversed now are at these, took this route we are at this position. Now for x_3 is equal to 0, now if you remember okay by the way, why we are branching? Now the value of x_1 , x_2 , x_3 , x_4 are respective as 1, 1, 0.5 so x_3 can be either 0 and 1 so we will again branch it accordingly. So x_3 0, x_3 is 1 and other values of (x_1, x_2, x_3) x_1 , x_2 , x_4 sorry are highlighted accordingly, so they would be traversed in this following way.

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Branch & Bound (B&B)
Example # 02

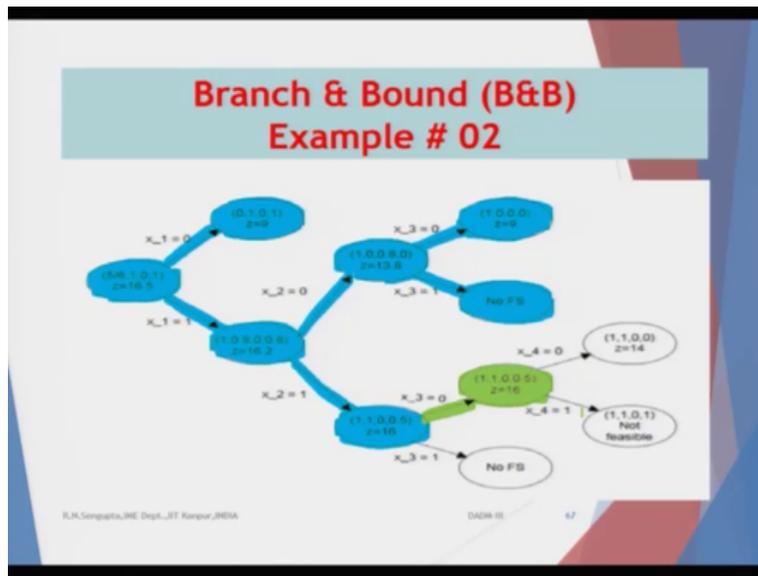
▶ For $x_3 = 0$, the previous optimal solution is $Z=16$ and $x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 0.5$ is still feasible and therefore still optimal

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So for x_3 , the previous optimum solution was 16 and the values of x_1 , x_2 , x_3 , x_4 were 1, 1, 0 and (0.4) 0.5 sorry it was 16 so is still feasible and there is still optimum solution can be obtained from there, feasible in the sense it is in the feasible region if you have able to draw the feasible region graph.

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So here I would just added it an extra so I will basically move in this direction so I basically I am here in this. So I will basically highlight this as green, so first I will mark the blue ones from where I have come already completed, already traversed I have already traversed this, I have already traversed travelled here and now here. So I use green color, so in order to highlight, so from here so I again come back and try to find out at x is equal to 3 is equal to 1 what is the situation? For x is equal to 4 okay by the way, here (x_4) x_1, x_2, x_3 are 1, 1, 0 and x_4 is 0.5 so it can be 0 and. So basically again I branch out this branch which I am going to highlight in the next slide so x_4 becomes 0, x_4 becomes 1. So let us check.

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Branch & Bound (B&B)
Example # 02

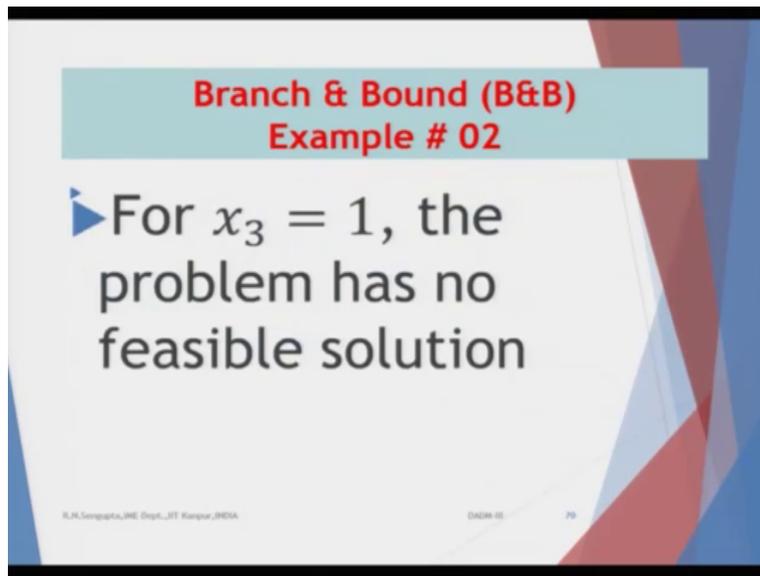
▶ For $x_4 = 0$, the optimal solution is $Z=14$ and $x_1 = 1, x_2 = 1, x_3 = 0, x_4 = 0$

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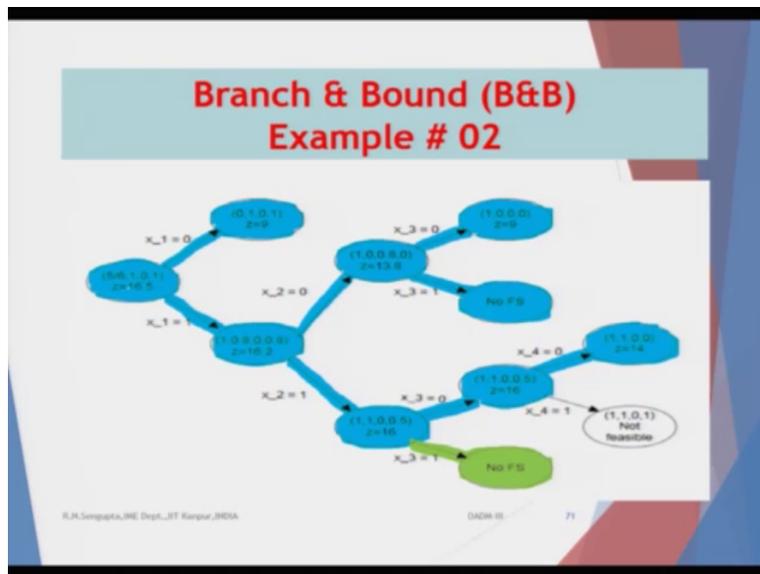
For x_4 is equal to 4, x_4 is 4 the optimum solution is 14 when you solve it, so this concept of trying to write the binary problem, linear relaxation, solved decayed they are repeated time and after time. So I am skipping it for the discussion, so when z is 14 x_1, x_2, x_3, x_4 are respective values of 1, 1, 0, 0 which looks interesting because you have to basically check it.

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For x_3 is equal to 3 equal to 1 the problem has no solution, if you solve it so x_3 is equal to 1 here this has no solution here so this infeasible region.

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So here we are, so now we are in this feasible region not being there, so dead end. So this would be green others are blue, this is also blue, this is also blue, this is also blue you have already visited these places, solved and found out the answers. We have already visited these place, this is the branches and the bound. Where we are, is here, so will be basically covering it accordingly.

The yellow ones will be highlighted which are actually feasible and which contenders, this is the contender, contender means contender for finding out your optimum solution and this is contender. So you will compare them which I told you when you compare, so these are not no more contender, they are not feasible not phantom, so they will re-route and out of the system. So you have started from the left hand side proceeded accordingly and reached the branch and the nodes where any further progress is not feasible and not possible.

So once I highlight this will be compared with amongst them, the best solution is when x_1 is 1, x_2 is 1, x_3 and x_4 are 0, 0 answer is 14. So with this I will end the 50th lecture which is the last lecture for the 10th week. We have considered two branch and bound problems in four consecutive lectures in details, so hopefully it is helpful for you and in the next class we will start, starting the 11th week we will start with the quadratic programming in some of the solutions, we did give some results but I will come to the details of the solution methods. Have a nice day and thank you very much.