Data Analysis and Decision Making – 3
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Lecture 14

Welcome back my dear friends. A very good morning, good afternoon, good evening wherever you are in this part of this globe.

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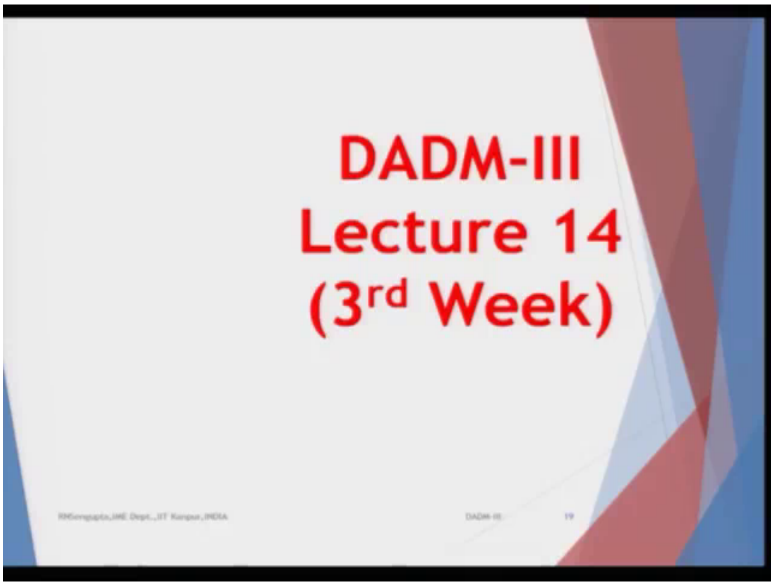

And this is the DADM 3which is Data Analysis and Decision Making-3 course on the NPTEL mock series and as you know this course total duration is for 12 weeks. Hence if you consider the contact hours it goes into 30 hours and the total number of lecture as each of them are half an hour, each is half an hour, the total numbers of lecture is 60 and this course after each week where there are 5 lectures each for half an hour you have an assignment.

So we have already wrapped up 2 assignments. We are in the third week and you can see form the slides, we are on 14 lectures, so 14 and 15 should finish of your third week and after that you will have the third assignment. Likewise, you will have till 12th week, so 12 assignment and after the end of this course you will have the final examination, and my good name is Raghu Nandan Sengupta from the IME department at IIT Kanpur.

So if you remember we were discussing the problem that given some constraints and I was talking from the 2 dimensional point of view only. So we will stick to that and then later on try to give you how the overall simplex method or the algorithm works. So in the 2 dimensional method all you actually needed is you have a overall feasible region.

And this feasible region and technically has infinite number of points, where you would basically you have some value for the objective function. Why I am saying some value of the objective function? Because it will depend on the type of the objective function you have, it can be either a maximization problem or the minimization problem. Now this constraints can be of 3 types, would be for greater than a type or less than a type.

For equality obviously it would be and if the constraint by itself is equality you would basically have only one straight line. So the overall area to find out the feasible region would not be actually possible if all the constraints are equal to, equality sign is there. So if they meet at you only one point you will basically have one point solution that means the optimal solution is only one point.

Now whenever you are solving this, this problem the concept if you remember I mentioned though very fleetingly is basically try to utilize the simple Gauss-Jordan elimination method. That means the concept of matrix solution technique is used to solve a set of simultaneous equation but before that we will try to basically understand what is the logic based on which we proceed.

The, this part of the lecture and the later part, which is the 14 and 15, would be a little bit more algebraic in concept. The concept, which we all know we have studied in class 10 or below that, the matrix we may have studied later but the concept of the finding the solution of simultaneous equation is very simple, we all have done in before class 10. So those would be utilized with some added on concept that how you will be able to get the feasible solution whether the feasible solution is possible, whether the constraints are applicable, whether the constraints are actually do have a feasible region, whether they are unconstraint solution for that, whether the unbounded solutions for that?

So all we will basically simply consider with simple diagrams and then obviously come to the concept of that how solved using the Gauss-Jordan method. So consider the first part is that exceptional examples, which are not a part and parcel of the solution but you, need to know that how they look like. So what is this solution?

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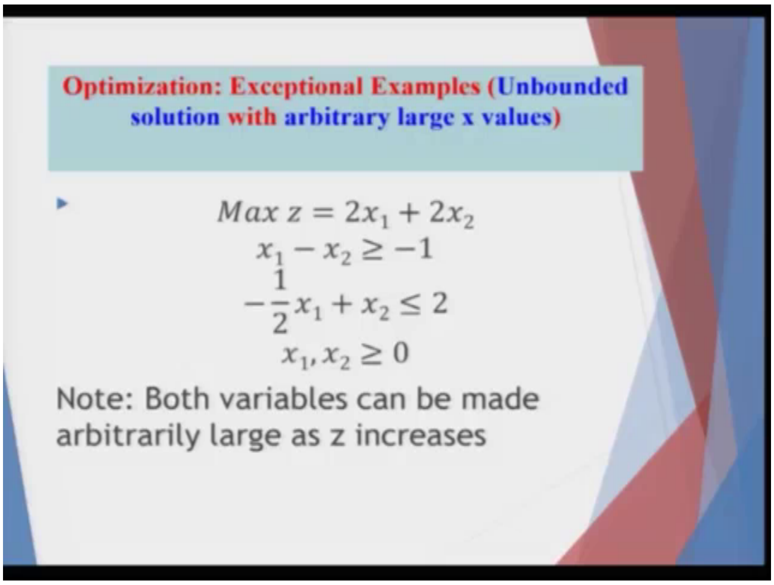

So I will tell you before the end result then tell you basically through other equation and the diagrams how it looks like. So you will have unbounded solution here that means the value of Z is unbounded which is the objective function can be either minimization or maximization, maximization obviously will consider because we are considering inherently that x1 and x2 are greater than 0, so they would be always be in the first quadrant.

So obviously if x1 and x2 are unbounded they can be negative also, in that case you can have unbounded minimization problem so because it will slowly go into the area which is basically the quadrant which is applicable for that. Considering the objective function is you are trying to minimize so obviously x1 and x2 can go as far as negative as possible.

So and that is point 1, point number 2 is that as you can see from the heading it has got arbitrary large x values, x means basically the decision variables, they we have got large x value. So let us see the problem. The problem is you have maximization of Z which 2 x1 plus x2 and the constraints are 2 number. The third and fourth are very, very I will not say intuitive they are very simple to understand, because as per the assumptions in linear programming will be considering all the variables to be greater than 0.

Hence the last set of constraints would be almost inherently considered (whenever) whether you write it or not. So the first constraint is x1 minus x2 is greater than minus1 and the second constraint is minus 0.5 or half of x1 plus x2 is less than equal to 2. So greater than minus 1 and less than equal to 2, the other variables which I said the constraints are x1 is greater than 0 and x2 is greater than 0. Now the concept which I am going to discuss need not be applicable for the 2 dimension only.

It can go into higher dimension but the reason I am trying to basically give this example is that easy for us to visualize how they look like when you solve the problem. So here the actual answer is when we see the solution we will understand, actual answer is I am giving you beforehand. Both the variables can be made arbitrarily large infinite value, as Z value increases that means x1 and x2 can be in infinitely large so would be the values of Z which is the objective function. So let us see the overall feasible region and how the objective function looks like and how it moves.

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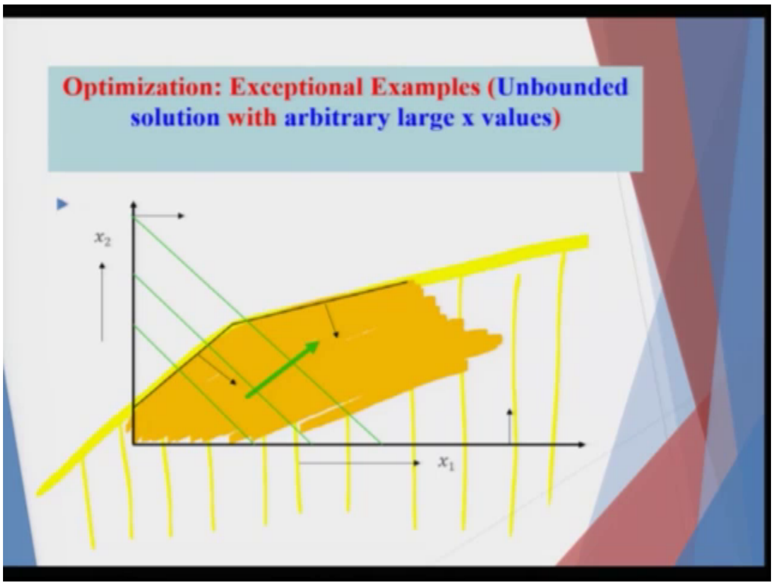

I am only drawing the first quadrant because that is where it is applicable. So if you consider the first two this constraints the overall boundary region I will basically mark it utilizing the this coloured pen. So I basically mark it using the yellow one. So I am only considering the value of the first and the second combined together. So If I consider x1 and x2 are not bounded technically they would be here. So all the region, so this basically the whole area below that which will definitely consider the fourth quadrant some part of the definitely the first quadrant and the third quadrant.

Now you add up x1 is greater than 0 that means it goes on to above the X-axis and x2 is greater than 0 it goes on to the right of Y-axis. So the moment you consider that the overall feasible region now becomes again use a different colour. So this colouring I did not do it in the diagram because I thought I will mark it as we solve the problem, so all this area is there.

Now consider the maximization problem. So the maximization problem if you place the objective function and where it cuts I did not draw it but if you use a simple excel sheet you will be able to do that. So if you have the objective function, the green lines of the objective function and as it is a maximization problem it will start moving away from 0 more away from the 0 into the first quadrant.

So what it happens? So as I said the boundary condition, the corner point would basically give you the objective function with maximum. So the corner points are actually going towards infinity, hence z value also goes to infinity and how it is moving the objective function moving is basically the bold green line which you see. So you here you have an unbounded solution where the x value be infinite and arbitrarily infinite values of x which is x1 and x2 is applicable here.

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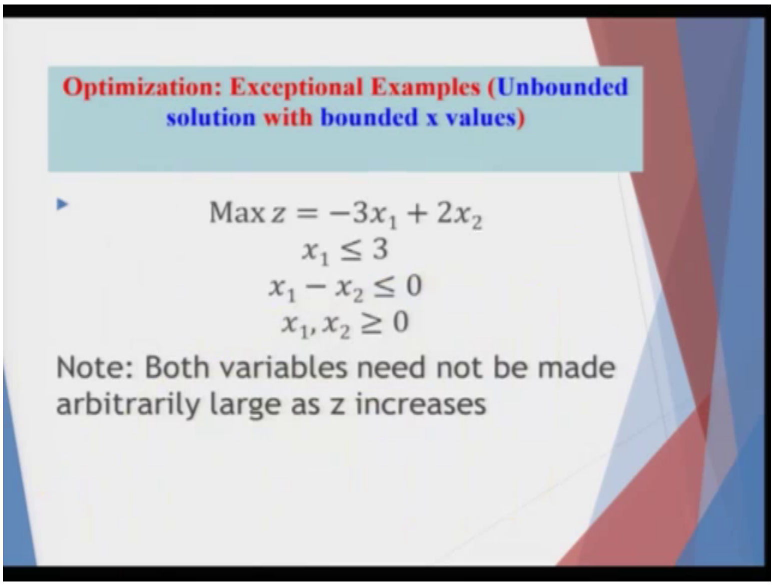

Next time, first let us read what is written here, it gives you unbounded solution but bounded x values. That means one of this, one or more of this x's are such that they are not infinite but the actual values of z is actually infinite. So it may be possible that if there are two variables one x is bounded has a fixed value or not infinite value but the actual value of the other x is infinite hence the actual objective function which you are getting z is infinite. So let us see what the optimization problem is. So with respect to the first one, there also z was unbounded, here also z is unbounded there all the x's were unbounded and infinite, here some of the x's are bounded.

So in this case as there are two variables, one is bounded, one is unbounded. Problem again maximization, again I am repeating why maximization because we are only considering in the first quadrant, quadrant considering the assumption being that x1 is positive x2 is positive. So maximization of z, which is equal to minus 3 x1 plus 2 x2 and how you draw the objective function would immediately come out from the, how the objective function looks like.

X1 is less than 3, x1 minus x2 is less than 0 and as usual x1 is greater than 0, x2 is greater than 0. So here note, both variables need not to be made arbitrarily large as z increases to infinity.

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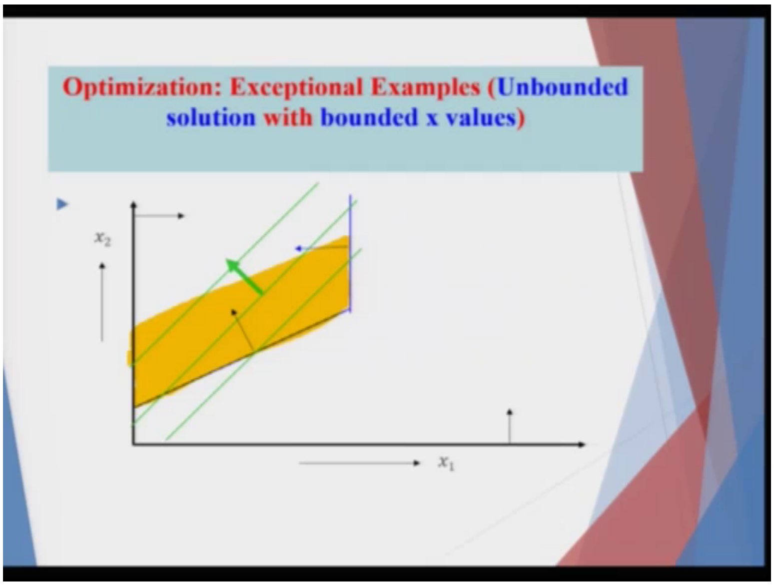

So let us draw the boundary region of the feasible space, take the first two constraints, now in this case you have x2 is greater than 0, x2 is also less than 3. So x2 greater than 0 would basically mean anything on to the right of Y-axis (x3), x2 (greater than) less than 3 means anything if you see my hand or if I point this pointer anything onto the left of this and the other constraint if you consider it is the upper portion which is here .

So technically if I basically have the whole region, the feasible region I will just mark it as it goes. So whole region will go along and there in the first quadrant. Now objective function you mark which is the green line they are moving parallel. So you want to increase the objective function value because in maximization problem it keeps moving but the corner point obviously you would always reach at infinite point, but remember one thing x1 will keep increasing to infinity, x2 is fixed because it is less than equal to 3.

So obviously the maximum value of x3 can be 3 which is a bounded solution, solution means bounded value but the z value if it keeps increasing it will go higher and higher up. So as x1 increase x2, x1 increases the value of z also becomes infinite. So here you have unbounded solution with bounded x values.

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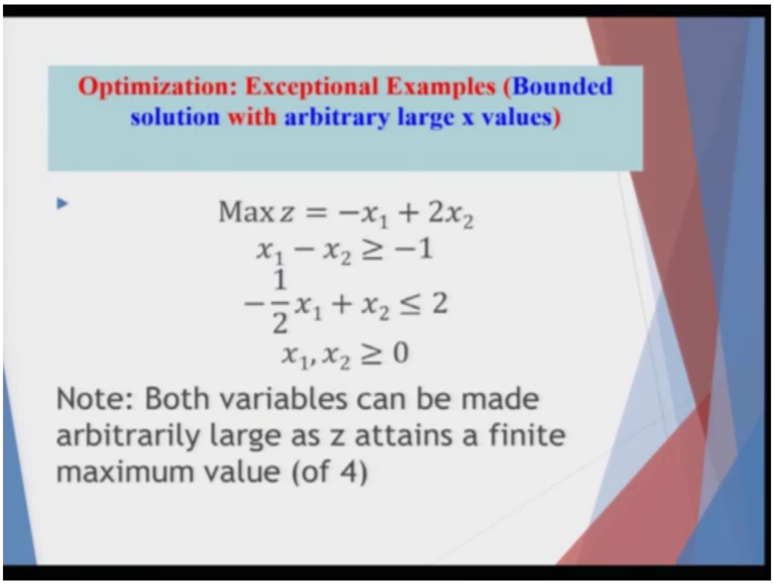

Now let us consider the case of a third example, when you have a bounded solution but with arbitrary large x values. So that means the z values is bounded it will have a maximum value, but the actual, actual x values can be infinite. So what is the solution? What is the objective function? Again the maximization problem, because you are considering only the feasible region which is applicably in the first quadrant, because due to the fact that x1 and x2 are both greater than 0 as per the assumption.

So the maximum value, maximization value z is minus x1 plus 2 x2 and the constraints are x1 minus x2 is greater than minus 1 and minus half of x1 plus x2 is less than 2. So these equation which are drawing please-please use excel and draw it and double check what I am showing is exactly matches. Because I am more interested to show you that how the feasible region looks like, how the x values are unbounded or bounded and how the z value can basically infinite or it can be bounded.

So here both variables can be made arbitrary large as z attains a finite maximum value where the value of z is 4. So let us see how it is, so have another look at constraints, constraints are x1minus x2 is greater than minus 1 and minus 0.5 of x1 plus x2 is less than equal to 2 as the constraints obviously other two being x1 is greater than 0, x2 is greater than 0 and you have the maximization problem is minus x1 plus 2 x2 you have to maximize.

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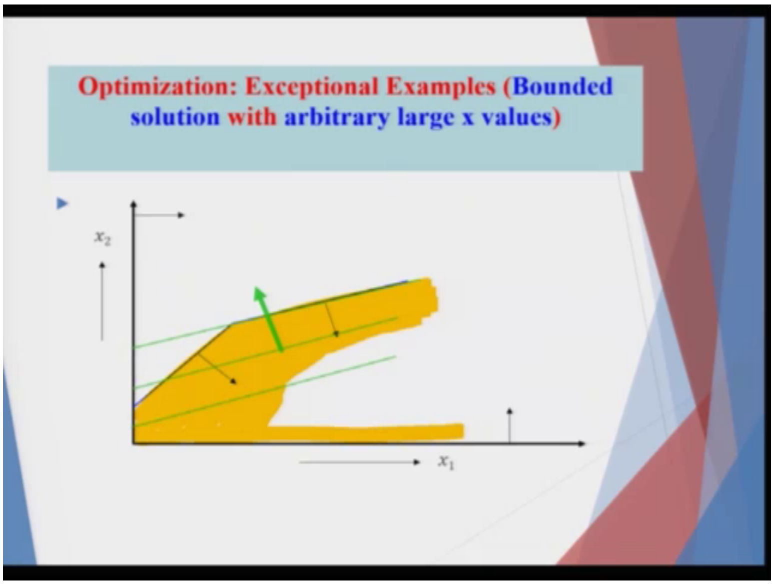

Now comes the constraints so when you basically marks the constraints, so you have the region x1 is positive that is obviously in the first and the second quadrant but obviously this second quadrant will be ruled out because x2 is also positive. So obviously any values onto the right of Y-axis, any value onto the top of the X-axis and the common region which is there between the constraints would give you the feasible region. The feasible region is this one, so I am trying to just mark it as clearly as possible it will go onto there.

Now interesting fact is that when you draw the objective function it is parallel to one of the constraints. So if you see the green line and the bold arrow of the green line basically says that in order to attain maxima it will should move up. So where it crosses the feasible region is basically parallel. So technically the z value is fixed but the number of such combination of x1 and x2 which will give you that value which is the fixed value of 4 can be technically infinite.

And if I consider the x1 and x2 value which will give me the same objective function which is 4 can basically go into infinity with because any higher values of x1, x2 as I moving my right hand more away from the 0 I am going to the first quadrant far away from 0 would basically give you x1 and x2 being arbitrarily large and the z value is basically fixed at 4.

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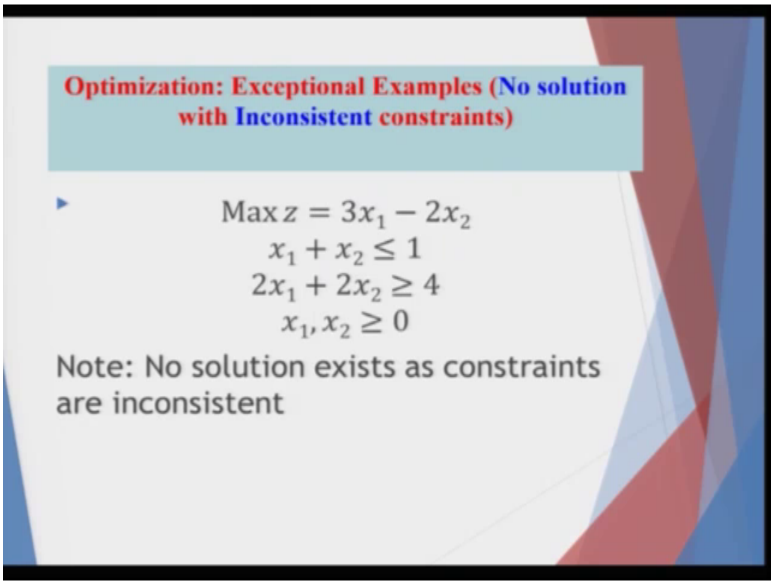

The (third) fourth type which is an exceptional examples is, no solution with inconsistent constraints. So what is the problem? Problem is what is the objective function? You maximize 3 x1 minus 2 x2 all these are maximization problem I am repeating it time and again because you are only considering the problems where x1 is greater than 0, x2greater than 0 as per the assumptions. So the first constraint is x1 plus x2 is less than equal to 4 and the second constrain is 2 x1 plus 2 x2 is greater than equal to 4.

So if you divide the second constraint by 2 you have x1 plus x2 is greater than 2. So obviously the region which you have to satisfy it is x1 and x2 is below 1, less than 1 and x1 plus x2 is greater than 2. So here (what) and obviously you have x1 is greater than 0, x2 is greater than 0 as per the norm. So here you have no solution exist as constraints are inconsistent. So let us see that and let us draw the constraints and then see the concept that no solution is possible.

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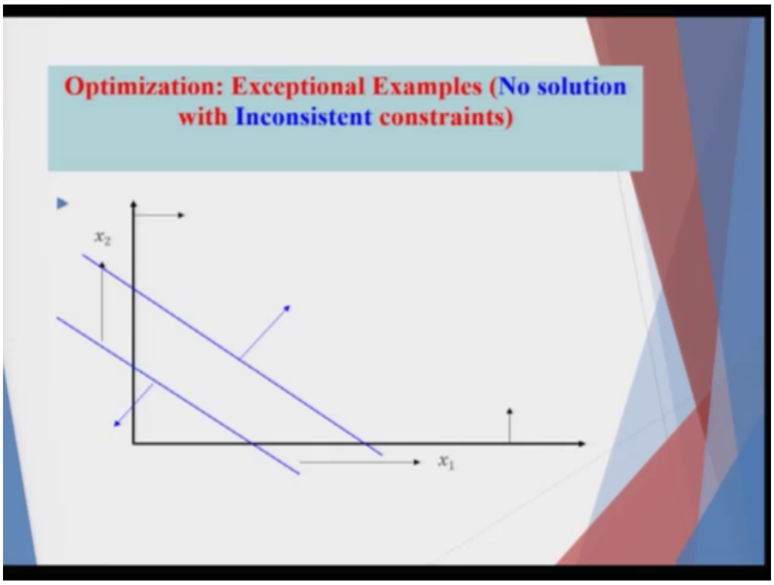

Again the overall region is given here the first constraint which you have is this one, I am not going to mark it I am just marking hovering my electronic pen here. So it is less than 1 so obviously any area on towards 0 with basically we with the area applicable for the first constraint, for the second constraint is greater than, so obviously it will be away from the 0 on to the right hand sides so this is there but if I consider the common region between these two constraints, there is no common region.

So obviously the constraints are looks plausible (possible) but the overall feasible region is not possible, so the overall feasible region is 0. So trying to find out any objective function which will whatever the objective function is there but trying to find us solution is not possible because the constraint are inconsistent and they do not give you any answer and obviously it means that in this case also x2 is greater than 0 means onto the right hand side of the Y-axis, x1 is greater than 0 means above the x-axis but the common area obviously in the first quadrant technically and obviously you will combine them with the first constraint and the second constraint.

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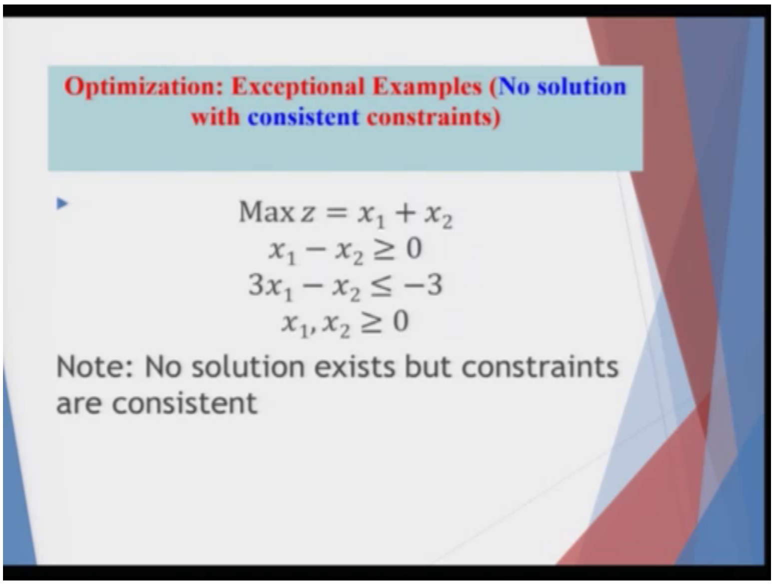

The fifth example, here no solutions with consistent constraints, so the maximization problem is again max of x1 plus x2 again the maximization because we are only considering simple examples where x1 is greater than 0, x2 is greater than 0, x1 minus x2 is greater than 0 and 3 x1 minus x2 is less than equal to minus 3 and as usual you have x1 is greater than 0, x2 is greater than 0. So here the end result is which I will show is that no solution exist but constraints are consistent as per the concept which is stated here. So let us draw the constraint and see and why we will see that no solution exist.

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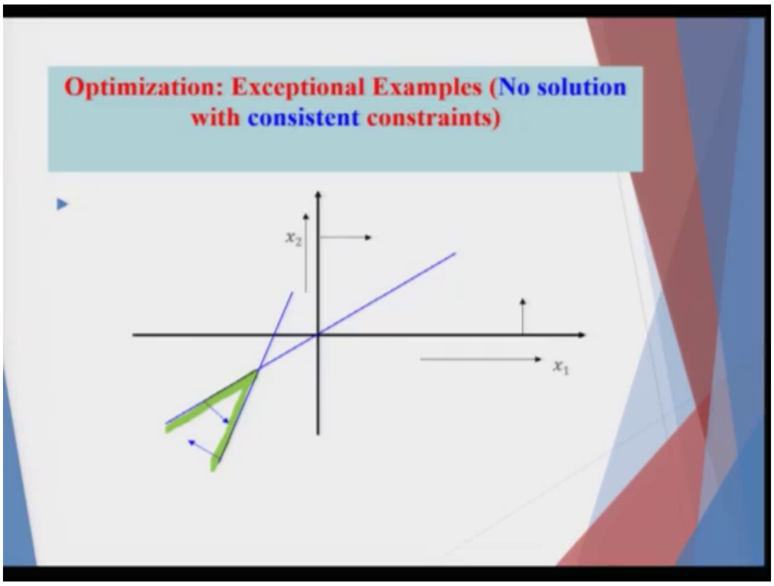

Now when I draw the constraints the first two, so the overall region even though that is not the actual feasible region because in the feasible region every the constraint should satisfy that and in this case the first last and the second last constraint which is x1 is greater than 0, x2 is greater than 0 are not applicable because all of them x1 is greater than 0 and x2 greater than 0 basically points to the first quadrant. But if you see the overall area common to the first constraints only, this I will use a different colour because the feasible region I was using a light orange.

So I will use the colour say for example light green. So in this case of green has nothing to do with say for example the objective function I am drawing. So the common area between the 2 constraints is this one but obviously the second and the third and the fourth, basically do not match and give a feasible region. Hence the constraints are applicable, are consistent but the overall solution is 0 here that means you do not have any solution.

So that means solution with even if the constraints are consistent. So you have considered on the 5 different examples, in the first case the x values can be arbitrary large similarly z values is arbitrary large and the second case, some of the x values are arbitrary large but some of the x value are not arbitrary large still you basically have arbitrary large value of M z.

In the second, third case we considered that the constraints what basically giving you a solution where the z value, the x values were infinite but the z value was finite which was 4. In the fourth case you had a case problem or problem or the type of error which you face, the constraints were inconsistent hence there were no solution and in the final case which is in front of you, you saw that.

Constraints were consistent but the feasible region obviously was not possible. Hence there was no solution. So obviously we may (count) encounter of such type of concepts in the higher dimension. So it was easy for me to show that in the 2 dimension hence I showed that with an example. In the problems which you are going to solve the technically if the problems I is the practical one which is solvable and the constraints and the all the objective function are actually as it should be.

Then obviously the overall feasible region and the concept which we saw this 5 one may not definitely be applicable to the case but I wanted to show you that these type of problems can occur where the x and z would definite give you different sets of information in the practical sense. Now I will discuss before I go to, before we go into the solution technique of the simplex method by Danzig.

I will basically consider very simple algebra, it is nothing serious, very simple some of the definitions may be little bit involved but I will try and there is definitely no proof here I am not going to consider any proof here, I will only explain it conceptually in order to make you understand.

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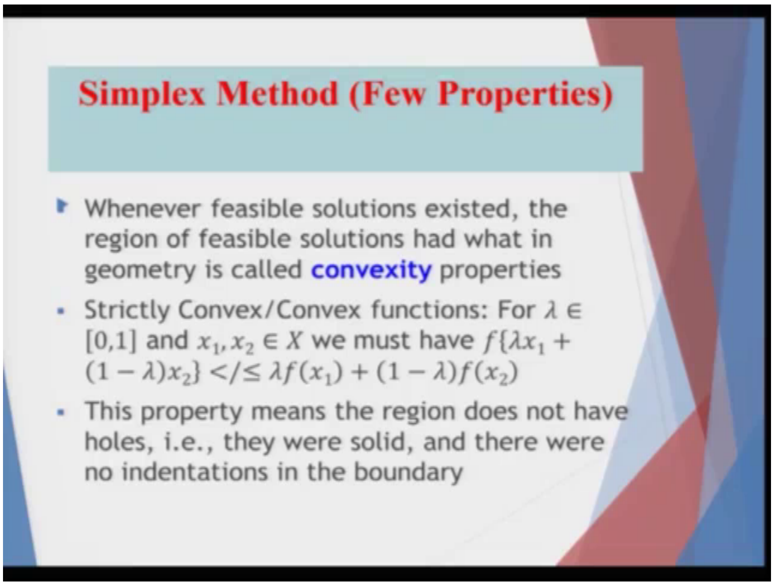

So few are the properties, wherever the feasible solutions existed the region of the feasible solution had what is in geometry we call the convexity properties. So hence the convex, convexity property should hold true in order to for us to find out the application of the linear programming is in the simplest method such that you will have a feasible region, set of corner points and the corner points would basically lead us to the case whether the objective function was either when you are trying to maximise or minimise would give you some feasible optimum solution.

Now what we mean by convexity? We all know that if a function is convex you will basically have the and if you consider lambda any of taking convex combination of the function lambda is the value between 0 and 1 inclusive. Hence lambda into F x1 plus 1 minus lambda into F x2 should be either the greater than equal to or greater than equal to, the greater than functional form of lambda x1 plus 1 minus lambda x2 in that case and obviously for concavity it will be the different one.

So when the greater than equal to sign holds it is strictly convex and for just convex function it will be only greater than for the case sorry, for the case when it is convex function it is greater than equal to and for strictly convex it is only greater than sign it would hold true. So this property means the region does not have any holes such that when you are joining two points in that convex region those points are set of points all the straight line which is joining them considering the convex region would always fall inside that region. So I will try to basically give you a diagrammatic representation for that.

So again I am mentioning strictly convex and convex function or concave strictly concave function are from the conceptual point of view from simple mathematics but how they look like considering the word is which as mentioned. This property means that the region does not have any holes that they are solid and there were no indentation in the boundary would also be made clear. So let we make, take a blank slide and explain it there.

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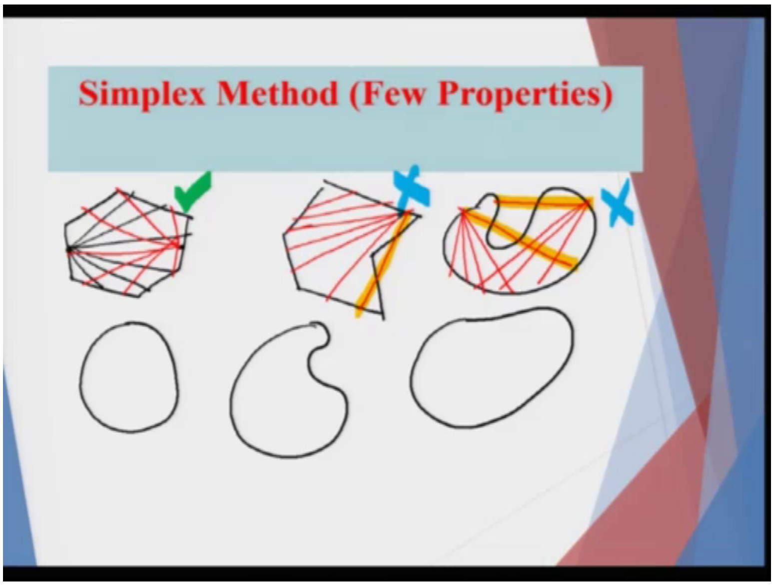

So here it is I will draw it. So I will use the black pen. So consider this area in 2 dimension and consider this area, consider this area, consider this area and consider this area and consider this area. So let us go by one by one, this if you join any 2 points, I am use another colour join any 2 points this is always inside the overall region so it is convex. Let me put a tick mark use the red colour it we will be easy.

Till now it is fine, the moment it comes here this goes out of the area so obviously it is not applicable so I will mark it. So this is not done, so hence I put a blue colour no. Consider this area again, this is done, done, done, done, not done, this is done, done, done, not done. So again I use the highlighter so this is not done, this is not done if I use whether is applicable, no it is not applicable.

So I will consider again this 3 diagrams in the corresponding 15th lecture so I will end it here and continue the discussion of what is feasible region and how it can be done. Have a nice day and thank you very much.