Advanced Green Manufacturing Systems
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Lecture – 07
Modeling with continuous variable – Part 1

Good afternoon students. Welcome to yet another lecture of the Advanced Green Manufacturing Systems course under the MOOC's program. And I am Dr. Deepu Philip from IIT Kanpur. And this course is aimed towards many of the our masters final year undergraduate students, practitioners and all who are interested in making or realizing sustainable green manufacturing systems and how can we realize those advanced systems and what are the tools and techniques that are required to do that or realize that is what is part of this course is about. And then I am Dr. Deepu Philip from IIT Kanpur.

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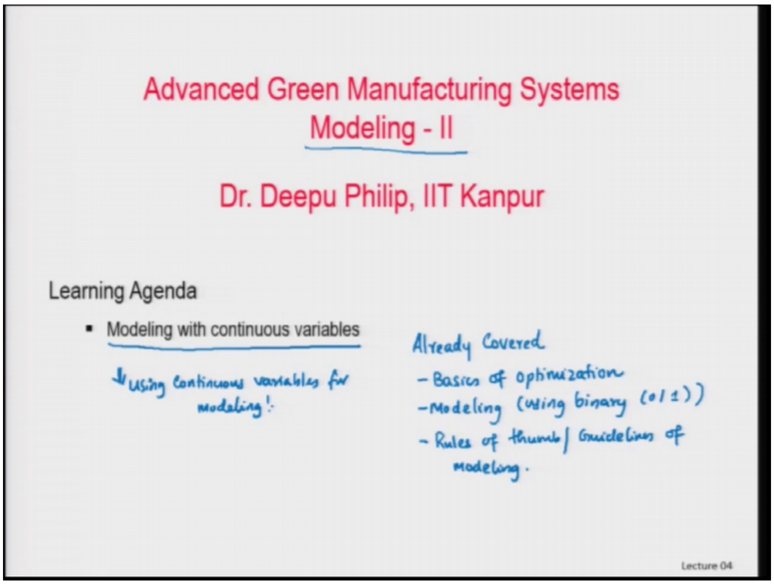

And also today's lecture we are getting into is the concept of what we call as modeling the second part of the modeling course. Earlier if you remember, we have already covered we have already covered basics of modeling, basics of optimization, and then we covered modeling using binary variable binary which is a 0, 1 system we already covered that.

Then we also looked at the rules of thumb or guidelines or guidelines of modeling ok. And in that guideline I have told about keep it continues right, I mean guideline for was about keep it continues. And today what we are going to do is how do we model with continuous variables ok. Today's lecture is all about using the continuous variables for modeling, how do we do that that is the fundamental question, and what we will do is we will work through an example.

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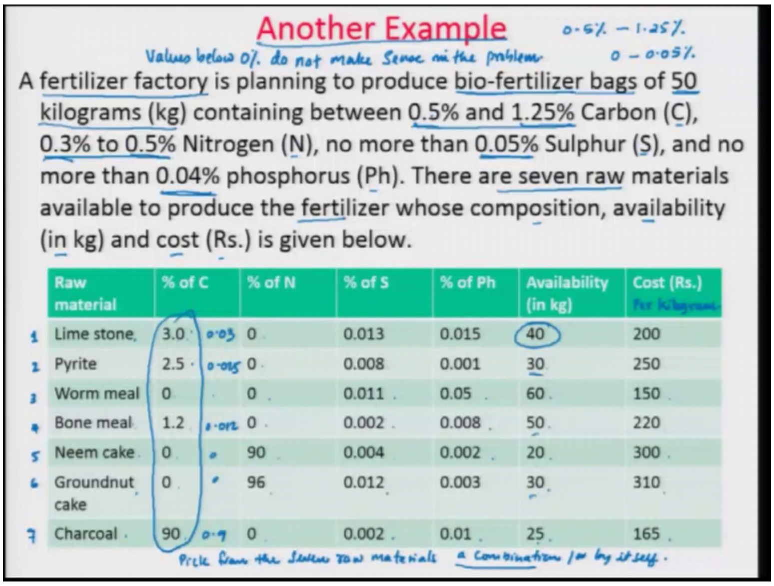

So, let us take another example today ok. And it is always better to work through an example, because once you do an example, it actually makes life easy. So, what we are going to discuss today is a fertilizer factory ok. Fertilizer factory is planning to produce bio-fertilizer ok.

The aim is to produced bio-fertilizer bags of 50 kilograms. So, the content of the bag is 50 kilograms 50 kilograms of bio-fertilizer containing between 0.5 and 1.25 percent of carbon. 0.5 percentage to 1.25 percent of carbon, carbon is denoted by the letter C. And 0.3 percent 2.5 percent nitrogen, nitrogen is denoted by letter N. And no more than 0.05 percent sulphur and sulphur is denoted by letter S. And no more than point 0.04 percent phosphorus ok, phosphorus is denoted by letter Ph.

So, when you say between 0.5 and 5 percent that means, it is 0.5 percentage to 1.25 percentage that is what we are talking about in this case this, this is an interval. When we say no more than point 0.5 percentage that means, it is 0 to 0.05 percentage, because the values below values below 0 percentage do not make sense in this problem ok.

So, what we are saying here is if we say 0 percent that means, sulphur is not at all present ok, so 0 does not mean that this present 0 means absolutely is the absence of one thing. And there are seven raw materials. So, ideally there are seven raw materials available to produce the fertilizer ok. So, there are seven raw materials available from which the bio fertilizer can be produced, whose composition availability in availabilities and kilogram and cost is in rupees is given below ok.

So, there are seven raw materials. So, the raw materials are given in this table. So, these are the seven raw materials 1, 2, 3, 4, 5, 6, and 7. So, the seven raw materials are limestone, pyrite, worm meal, bone meal, neem cake, groundnut cake and charcoal ok. And the limestone the first raw material limestone has 3 percent of carbon in it, 0 percent of nitrogen this means absence of nitrogen, 0.013 percentage of sulphur 0.015 percentage of phosphorus and 40 kilograms of limestone is available, and other cost of rupees 200 ok, so it is 200 per kilogram you can think about it that way, this is cost per kilogram ok.

So, then the pirate is the second raw material available which is 2.5 percentage of carbon, 0 percentage of nitrogen, the nitrogen is absent 0.008 percentage of sulphur, 0.001 percentage of phosphorus, 30 kilo grams is available at 250 rupees per kilogram like that. Then worm meal has no carbon 0 percent of carbon, 0 percent of nitrogen, 0.011 percent of sulphur, 0.05 percentage of phosphorus and 60 kilograms of it is available and it is at one 150 rupees per kilogram.

Same way then there is bone meal 1.2 percent of carbon, 0 percent nitrogen or if nitrogen is absent, 0.002 percentage sulphur, 0.008 percentage of phosphorus and 50 kilograms of air is available at 220 rupees per kilogram. Neem cake has 0 percent carbon, 90 percent of nitrogen, 0.004 percent of sulphur, 0.002 percent of phosphorus, 20 kilograms of it is available, and the cost is 300 rupees per kilogram.

Then groundnut cake changes 0 percent carbon which means absence of carbon, 96 percent of nitrogen, 0.012 percent of sulphur, 0.003 percent of phosphorus, 30 kilograms of it is available, 310 rupees per kilogram. And then final raw material is charcoal with 90 percent carbon, 0 percent nitrogen, 0.002 percent sulphur, 0.01 percent of phosphorus, 25 kilograms is available at 165 rupees a kilogram.

So, the aim is pick from the seven raw materials a combination or by itself, either use a option and so that you get it by itself or you use a scenario where a combination is being picked up ok.

So, the problem setting I hope you guys understand that we are going to produce bio-fertilizer bags of rupees of 50 kilograms weight containing 0.5 percent to 1.25 percent of carbon, 0.03 percent to point 0.5 0.5 percent of nitrogen, 0.05 percent no more than zero 0.05 percent of sulphur, and no more than 0.04 percent of phosphorus. Other seven raw materials using which you can produce a fertilizer and there composition availability and cost is given and we already discussed the details of it.

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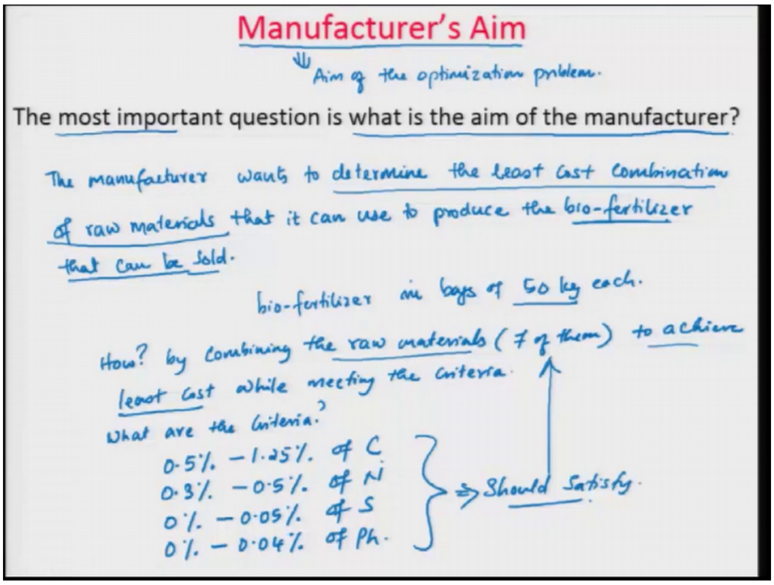

So, now the manufacturer's aim, what does the manufacturer want or what we are looking at is what is the aim of the problem or what is the aim of this optimization. This is the aim of the optimization problem. So, the most important question, we have is what is the aim of the manufacturer or what is the manufacturer want to do ok.

We can say that the manufacturer the manufacturer wants to determine wants to determine, so what does he want to determine? Determine the least cost combination the least cost combination of raw materials. So, you want to find the least cost combination of raw materials why, that it can use to produce to produce what the fertilizer, the bio-fertilizer that can be solved.

So, the manufacturer wants to produce bio fertilizer ok. So, bio fertilizer we are saying bio fertilizer in bags of 50 kilogram each ok. So, each bag is 50 kilogram and he wants to produce this bio-fertilizer that can be solved, it can be solved to the customer.

And how do the manufacture wants to produce this bio-fertilizer the how is by determining the least cost combination of raw materials or how he want to produce it is by combining the raw materials how many raw materials are there, 7 of them by combining the raw materials to achieve least cost while meeting the criteria ok, you want to do the least cost while meeting the criteria.

What are the criteria what are the criteria ok? The first criteria the criteria is already mentioned here, these are the criteria's the carbon and nitrogen and sulphur and phosphorus. So, if you think about it, the criteria's are 0.5 percentage to 1.25 percentage of carbon, then 0.3 percentage to 0.5 percentage of nitrogen ok, then no more than 0.05 percent sulphur which means 0 percentage to 0.05 percentage of sulphur and no more than 0.04 percent of phosphorus, so 0 percent to 0.04 percentage of Ph ok.

So, your bio-fertilizer should satisfy this criteria should satisfy ok. And while satisfying this criteria, you want to do this you combine them with the raw materials in one fashion or another combined raw materials to achieve least cost ok. You get the least cost, while meeting this particular criteria of carbon, nitrogen, sulphur and phosphorus. So, this is the aim of the manufacturer as far as we are concerned ok.

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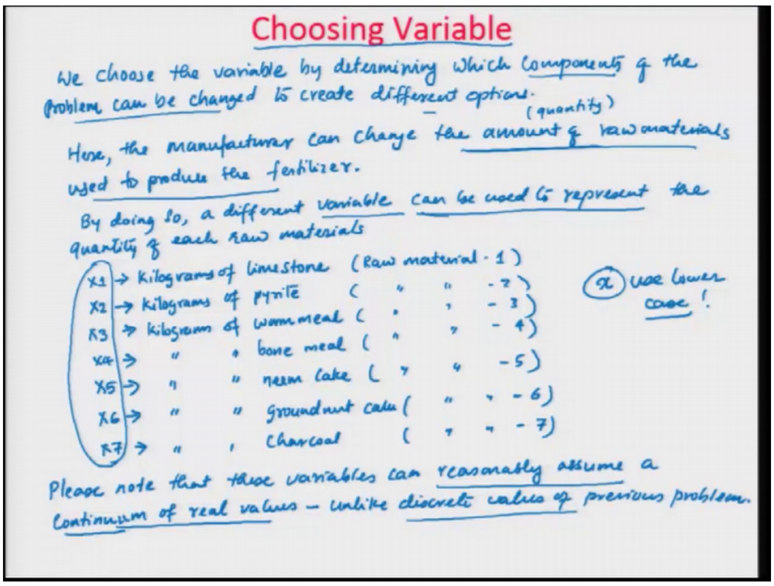

So, remember than the next major thing or not the next major thing, the first major thing for us is choosing the variable. In the stepwise process of what we called as the optimization. The first thing is about choosing the variable, so we say that we choose the variable by determining we choose the variable why would we choose the variable by determining which components of the problem which components of the problem can be changed of the problem can be changed to create different possibilities there are different options.

So, what we are trying to find out here is which are those components of the problem that can be changed or we can adjust those things, so that we get different options or different possibilities out of this ok. So, here manufacturer the manufacturer can change the raw materials manufacturer can change the amount of raw materials the amount of raw materials. And how many raw materials are there seven of them ok.

The raw materials used to produce the fertilizer; used to produce the fertilizer ok. The manufacturer can change the amount of raw materials, you can change the amount of raw materials there are seven of them to produce the fertilizer ok. And by doing so by doing so by doing the or by changing the amount of raw materials ok, different variable can be used to represent can be used to represent the amount of each raw material or the quantity of each raw material of each raw material.

So, since the or the manufacturer or he can do is he can change the command or quantity of raw material when we say amount, we are also thinking about quantity of the raw material, so that the fertilizer can be produced, we can use a variable to represent the quantity of the raw materials.

So, the variables will be let us say we define the first variable as X 1 ok, which denotes the kilograms of limestone ok. Your variable X 2 denotes the kilograms of pyrite ok, X 3 third variable denotes the kilograms of worm meal of worm meal, X 4 the kilograms of we will not bone meal this. So, this is your raw material-1 ok, this is your raw material-2, raw material-3, raw material-4 ok.

So, these are the raws of the table ideally speaking. X 5 will be your kilograms of neem cake raw material-5. X 6 kilograms of ground nut cake raw material-6, then X 7 is kilo grams of charcoal, your raw material-7 ok. When they are order like this, ideally the variables are to be little case little x ok. This x is equal to so use lowercase all right.

So, what we are saying that please note that please note that these variables can these variables can reasonably assume a continuum of real values; continuum of real values real values ok. So, unlike discrete values of previous problem previous problem.

So, the quantities can think that the values these X 1, X 2, X 3 all the way to X 7 can take our values between zero two, so we can say that so in the case of X 1 for the limestone ok, it can vary between different values we will write that in the next constraint ideally speaking. So, we can write that down ok.

But, ideally speaking that these values can take these variables can take continuum a continual values of real continuous set of real values or continuum of real values unlike the previous case where it was 0 and 1 kind of a scenario. So, all these variables these are our what we call as we are going to this is going to decide, how what will be the cost of the fertilizer or the bio-fertilizer that we are trying to manufacture at the end.

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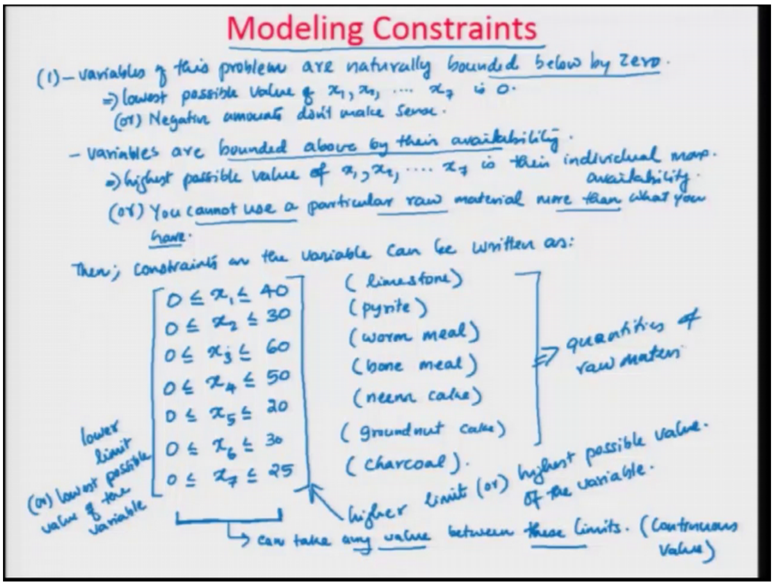

Now, we get into the second part of it, once we identified what the variable is we now get into what we call as modeling constraints ok. So, first thing is variables of this problem this problem are naturally bounded naturally bounded below by zero ok.

So, what are what we are saying is that which implies lowest possible value possible value of x 1, x 2, all the way to x 7 is 0 or we are saying is that negative amounts do not make sense, when we say minus 3 gram or minus 3 kilograms or something, it actually does not make any sense in this ok.

And variables second aspect ok. So, this is the first aspect. The variables are bounded above bounded above by their availability ok. So, how much is available, which means highest possible value e of x 1, x 2, etcetera, x 7 is their individual max availability.

So, how many kilograms of individually, they are available are the maximum value or you cannot use a particular raw material raw material more than what you have more than what you have. So, what does this mean this means that if you have 30 kilograms of something, then the max you can use is 30 kilo grams or you cannot use a particular raw material more than what you have ok. So, whatever is a quantity you have this upper limit of this.

So, we now have the lower bound or the bound the lowest possible value of the raw material of the variable and we also have the highest possible value of the variable. So, if that is the case then we can write the constraint as for x 1, then constraints on variable constraints on the variable can be written as ok. So, for variable x 1, it is 0 less than or equal to x 1 less than or equal to what is the upper value.

So, for that we go back to the table. And we find that for one the upper limit is 40. So, we go back and write 0 less than or equal to 40, so this is for the case of limestone ok. So, the minimum we can use is 0, the maximum you can use is 40, because 40 is the maximum kilograms of limestone that is available to you.

Similarly, for the second one 0 less than or equal to x 2 less than or equal to what is that value? We go back to the table, and that value turns out to be 30. So, we come back and say 30 and 30 is meant for the pyrite ok. And then the third variable x 3 is less than or equal to we think about it is 60 kilograms, so it is upper bounded by 60 ok.

So, I am just going to than that that is worm meal ok. So, you get the upper limit from the worm meal raw of the table. The fourth variable 0 less than or equal to x 4 less than or equal to that you go back again to the table and the fourth one bone meal you can see that the value possible is 50. So, we go back and we write it as 50 that is for bone meal right bone meal ok.

Then for the fifth one we write it as 0 less than or equal to x 5 less than or equal to because it is 0, because negative values this one or not make any sense, lower bound is 0 you cannot have negative values of this. So, we go back to the table again, and we find that the neem cake which is the fifth one is 20. So, we go back and we write it as 20 and we say it is neem cake ok.

Then the sixth variable x 6 the quantity of the next raw material, we go back to the table again and research groundnut cake limit is 30. So, we come back and say it is 30 for the ground nut cake. And the last variable 0 less than x 7 less than or equal to the quantity we go back again, and look at the table as charcoal its twenty 25. So, we come back and write it as 25, now it is charcoal ok.

So, now we know that the individual variables all individual variables what is the values. So, for limestone this is the lower bound, so this is the we call it as the lower limit or lowest possible value of the variable, we cannot take a value less than 0. And this is the higher limit or highest possible value of the variable of the variable. So, between this can take any value between these limits ok.

So, what we are saying here is that the each variable has a lower limit and the higher limit ok, so which means the highest possible value, it can take between these limits the variable can take any value between these limits, so that is why it is a continuous value can take for example like seven can take any value between 0 and 25, so it can be 0.325 or its 1.67 or 3 or 9.893 whatever it is you can put whatever values that that for any way any of these seven possible variables ok.

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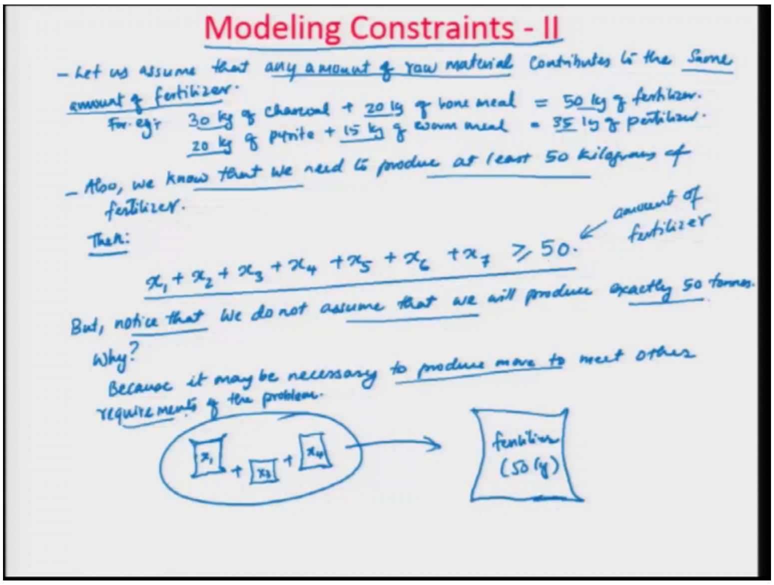

Then we talk about the second set of modeling constraints. So, the first constraint is we talked about is what about the how we can model the in the or how the individual variables are bounded in this. Now, we can say that now we say let us assume that any amount of raw material any amount of material contributes to the same amount of fertilizer same amount of fertilizer ok.

So, for example what we mean by this is 30 kilo gram of charcoal plus 20 kilogram of bone meal, if we do that we should be able to get 50 kilogram of fertilizer ok. If we take 20 kilogram of pyrite plus 15 kilogram of worm meal, then we will get is 35 kilogram of fertilizer ok.

Fertilizer means, it the composition of the other requirements might vary, but the quantity whatever you have the quantity, the same amount any amount of raw material will give you the same amount of fertilizer. So, if you use this many kilograms, it will give you exactly the same kilogram, these will match ok, this is what we assume ok.

Also we know that we need to produce, we also know that we need to produce at least 50 kilograms of fertilizer ok, we have to at least produce 50 kilograms of fertilizer to make it into a one bag of this ok. So, we also know that we know that we need to produce a at least 50 kilograms of the fertilizer.

Then if this is true, then what we have is we can say x 1 plus x 2 plus x 3 plus x 4 plus x 5 plus x 6 plus x 7 all of them together should be greater than or equal to 50. So, what are we saying here is in the previous cases x 1, x 2, x 3, x 4, x 5, up to x 7 where the quantities ok.

These are the quantities of the raw materials quantities of raw materials if that is true, and we have an assumption that the quantity any quantity of the raw materials that we take we summed up together same amount of fertilizer, we will also get if that is the case if you sum all the seven raw material quantities put together, we should be able to get 50 or more kilograms of bio-fertilizer ok.

So, but notice that we do not assume we do not assume that we will produce exactly 50 tons ok, you should notice that we do not assume that we will produce exactly 50 tons. This is not just equal to 0, it is greater than or equal to 0 why, why do not we assume that? Because, it may be necessary it may be necessary to produce more to meet other requirements of the problem.

So, let us say we only make exactly 50 kilograms ok. So, what let us say we let us think about is pictorially ok, so let here is a small bag ok. So, small bag of x 1 ok, there is another bag of x 3 ok, the third bag of x 4 like this ok, we combined all of these we should be able to get a bag of fertilizer would be 50 kg ok, this will be individual values ok. So, these plus, this plus, this these three together should give you 50 kg of this, but we what we are saying is let us say there is a scenario that if we make only exactly 50 kilograms.

And some 1 kilogram is lost that the time of packaging, then you will only end up with 49 kilograms. So, it is necessary for us to produce slightly more to meet other requirements of the problem, whenever they come up. So, usually that is the reason why we write this constraint in a fashion like this that will produce more equal to or more than 50 kilograms ok. Hopefully, that makes sense to you guys ok.

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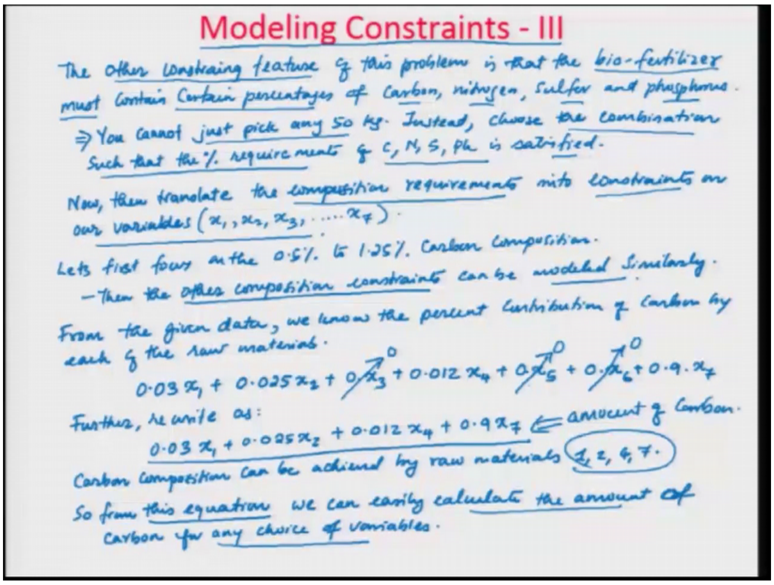

Now, if that is true, then we are getting into the third aspect of modeling the constraints of this problem. So, the other constraining feature the other constraining feature of this problem is that the bio-fertilizer the bio-fertilizer must contain must contain certain percentages of this of carbon, nitrogen, sulphur and phosphorus.

So, what we are saying here is we already talked about the constraints of the variables, and the total quantity constraint we have already put it in. But, other constraining factor another constraining aspect of this problem is that the end result bio-fertilizer must contain certain percentages of carbon, nitrogen, sulphur, and phosphorus. This is also a requirement of this, you cannot just mix and match ok.

So, this implies that you cannot just pick any 50 kg ok. Instead, choose the combination such that the percentage requirements of carbon, nitrogen, sulphur, and phosphorus is satisfied. So, you cannot just pick up any 50 ok, just do not pick up any 50. Instead, choose the combination such that the percentage requirements of these four are satisfied ok.

So, now then translate we have to translate the composition requirements the composition requirements, now we had to translate the composition requirements into constraints on our variable constraints on our variables, what are our variables x 1, x 2, x 3, x 4, x 5, etcetera ok. So, the variables are x 1, x 2, x 3, all the way to x 7, these are our seven variables.

So, the composition requirements that we have of carbon, nitrogen, sulphur, and phosphorus need to be put into constraints on the variables ok. So, what we do is let us first focus on the 0.5 percentage to 1.25 percentage carbon, composition ok. So, first we are going to focus on the carbon composition constraint.

Then the other composition constraints then the other composition constraints can be modeled similarly ok. So, once we attract the carbon constraint carbon composition constraint, then the other composition constraints we can be use we can be use the same methodology to model them.

From the given data the given data, we know the percent contribution of the percent contribution of carbon by each of the raw materials. So, we can say that if you go back to the previous table, we know the percentage composition of carbon ok. This gives you the percentage composition of carbon contribution from all the all these raw materials ok.

So, if I write it down, then I can write it as; I can write it as 0.03, so it is 3 percentage is translated to 0.03 times x 1 plus 0.025 x 2 right plus 0 times x 3 plus 0.012 times x 4 plus 0 x times x 5 plus 0 times x 6 plus 0.9 times x 7 ok, I can write it this way. So, where do I get this, this comes from the table. The 3 percent is equal to 0.03, this is 0.025, this is 0, 0.0120, 0 and 90 percent is 0.9 right, you have dividing it by 100.

So, if you follow that, then we can say that we can write a constraint like this. And we can see that this one 0 times this variable is 0, this is also 0, this is also 0. So, then we can further rewrite as 0.03 x 1 plus 0.025 x 2 plus 0.012 x 4 plus 0.9 x 7 ok. So, this will be the constraint, because these variables are all 0's, because there coefficient is also 0. So, you see x 1, x 2, x 4, and x 5.

So, to achieve the carbon or carbon composition can be achieved by raw materials 1, 2, 4, and 7 ok. So, what we are saying here is using this stuff, these of the any of these four raw materials, we can achieve the carbon composition right all right.

So, from this equation which is this equation this equation is this right from this equation ok, we can easily calculate; we can easily calculate the amount of carbon the amount of carbon; amount of carbon amount of carbon for any choice of variables any choice of variables ok. So, whatever the variables we choose from there we can calculate the amount of carbon, whatever be the choice of the variables ok.

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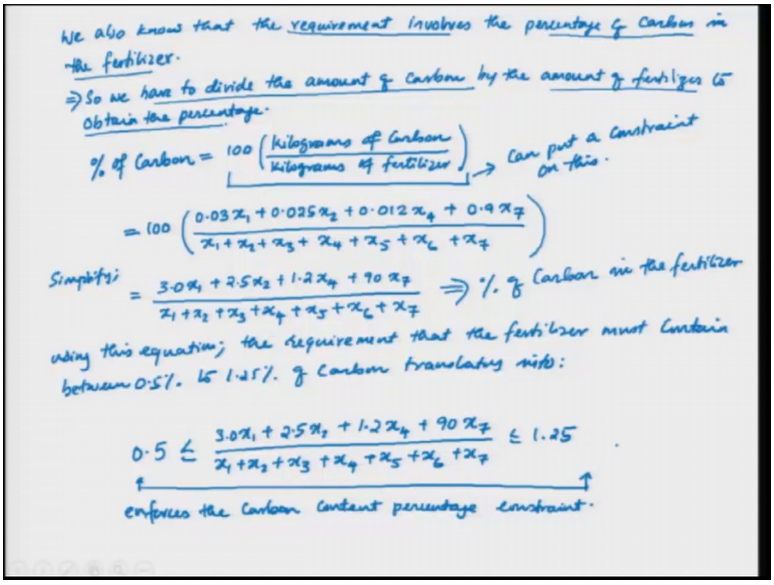

So, but it is not very simple, it is not completed here, because we also know that; we also know what we also know that the requirement involves; the requirement involves the percentage of carbon in the fertilizer the percentage of carbon in the fertilizer ok. So, requirement is on the percentage of carbon in the fertilizer, it is not about the quantity of the carbon, but it is the percentage of the carbon ok.

So, we have to we have to divide the amount of so we have to divide the amount of carbon by the amount of fertilizer to obtain the percentage. So, what we are saying here is that we have to divide the amount of carbon, whatever the amount of carbon where do we get the amount of carbon, we get the amount of carbon we obtained from here. This is the amount of carbon ok, this amount of carbon has to be divided by the amount of fertilizer to so the and multiplied by 100, so that we can obtain the percentage ok.

So, we can say percentage of carbon is equal to 100 multiplied by kilo grams of carbon divided by kilograms of fertilizers fertilizer ok. So, if we do this, then this ratio will give us the percentage. And we know that the percentage of carbon has to be between we know that this has to be between 0.5 0.5 percent to 1.25 percent.

So, now we can put a this can put a constraint on this, because we know what the constraint is ok. So, what does this implies to this is equivalent to saying it is multiplied by 100 multiplied by a previous equation, which is 0.03 x 1 plus 0.025 x 2 plus 0.012 x 4 plus 0.9 x 7. This is the amount of carbon divided by what we dividing it by x 1 plus x 2 plus x 3 plus x 4 plus x 5 plus x 6 plus x 7, this below is the total amount of fertilizer right.

So, this one is the amount of carbon this is the amount of carbon and as we said this is the amount of fertilizer right amount of fertilizer ok. So, if that is the case, what we can do is so now we know that this is a percentage and which can be further simplified by multiplying the 100 into the numerator ok. So, if you multiply 100 into the numerator, we simplify which will be equal to 3.0 x 1 plus 2.5 x 2 plus 1.2 x 4 plus 90 x 7 divided by x 1 plus x 2 plus x 3 plus x 4 plus x 5 plus x 6 plus x 7. So, this equation is the percentage of carbon in the fertilizer.

Now, we know that using this equation the requirement that the fertilizer; that the fertilizer must contain between 0.5 percentage to 1.25 percentage of carbon translates into so how can we write that we can write it as 0.5, this is the because this is also in percentage less than or equal to 3.0 x 1 plus 2.5 x 2 plus 1.2 x 4 plus 90 x 7 divided by x 1 plus x 2 plus x 3 plus x 4 plus x 5 plus x 6 plus x 7. So, this is the percentage of carbon less than or equal to 1.25 ok. So, these are all in percentages now ok.

So, this constraint this now enforces the carbon content percentage constraint ok. So, by looking into this; by looking into this approach, we can see now we have a constraint that tells you how the carbon content percentage can be easily modeled.

So, thank you very much for your patient listening, continue reading, and hope to see you soon in the coming aspects of this course.

Thank you very much.