Data Analysis and Decision Making – II
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Lecture – 04
Utility Analysis

Warm welcome to all of you; good morning, good afternoon, good evening to all my dear students and friends. This is the fourth lecture for DADM 2, which is Data Analysis and Decision Making. And as you know this course is for total duration is 12 weeks 30 hours; each week we have 5 lectures each lecture being for half an hour and after each week we have an assignment. And then after the 12th week there would be a final examination. And my name is Raghu Nandan Sengupta from IME department IIT Kanpur.

So, if you remember that, in the last lecture which you were third one we were discussing at the fag end and the last two slides about the concept of risk and how the properties of concept or risk could be analyzed. Risk means from the utility point of view. And we did mentioned and I did discussed one of them was the absolute risk aversion property, where the actual formula is given minus of U double prime divided by U prime. And as you know according to the two properties of utility, analysis in a very simplistic form I am not going to the detail mathematical content.

So, in the simplistic form that two properties were non-satiation with basically translated into the concept that the first derivative would be greater than 0 and the risk property would basically be divided into three categories. I love risk, I am indifferent to risk, I hate risk based on that you will basically have the second derivative would be greater than 0. If I am indifferent it will be equal to 0 second derivative and in the last case it would be less than 0 in the second derivative.

So, for the absolute utility function, we divided the decision into two sets one was W plus Z; Z was basically a fair gamble where the expected value of Z was 0 and the variance of Z was sigma square suffix n. And in other hand you have basically had a total investment of W suffix C and based on the concept of Taylor series expansion will basically derive the actual formula, which is given by minus U double prime divided by U prime.

Now, if the person is indifferent between the decisions on the choice of A and the choice of B then; obviously, may if he if he or she is indifferent then obviously, the expected value on for A or which is on table 1 and the expected loss of B which is on table 2 or whatever case 1 case 2 we say they would be equal.

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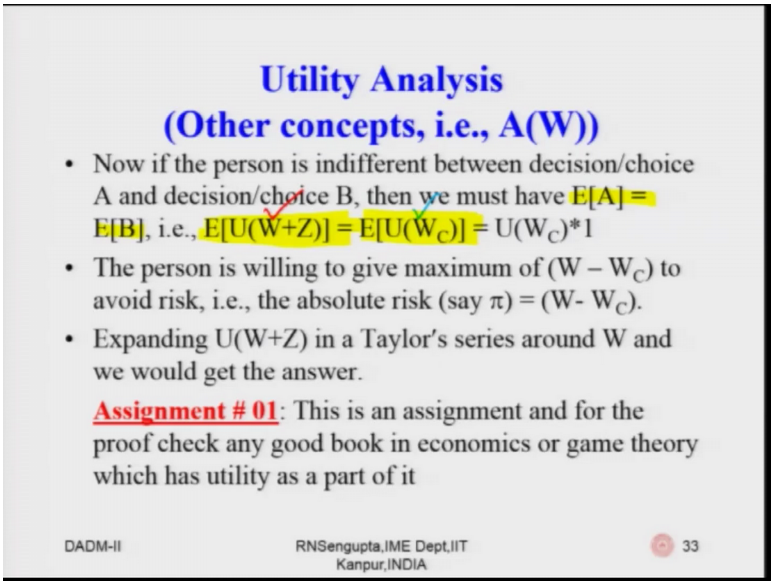

So, as it mentions the expected value I will just highlight it once. So, the expected value of A is equal to expected value B and if you remember for what was A? A was W plus Z and B what was B? B was W C. So, if you basically put them the actual formula comes out to be A is the left hand side which I am highlighting now and B is the right hand side which I am highlighting now and newly I will just put a tick with respect to colors. So, it would be much easier.

So, this is A and if I consider B this is B that would basically B. And if you remember the B was basically a decision which was certain. So, your W C only and the probability would be 1. So, you multiply them and we get U W C depending on the utility functions functional form. The person is willing to give maximum of the difference between W and W minus C to avoid the risks. So, because if he wants to take one decision with respect to the other whether he wants to take the risky one or non or the certainty one. So, the there has to be some counter payoff.

Now, if we expand W plus Z which is basically the risky one. W amount plus the fair gamble in their Taylor series expansion, we can basically get the under terms where from where we can basically find out that A prime is equal to minus U double prime divided by U prime.

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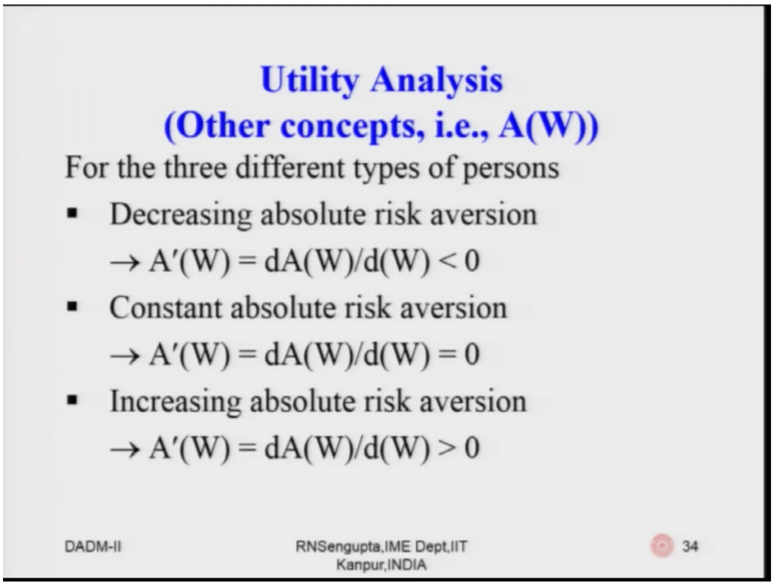

Now for the different and three different types of persons which we have already discussed; if we just write down the rule and we will come back to the using of the rules later on. So, decreasing absolute risk aversion property would mean that A prime or differentiation of A W with respect to W would be less than 0. For the constant absolute risk aversion property; obviously, A prime would be 0 and for increasing the absolute risk aversion property A prime would be greater than 0.

Now, what does this conceptually mean in a very in a simple practical sense? So, let us look at this table.

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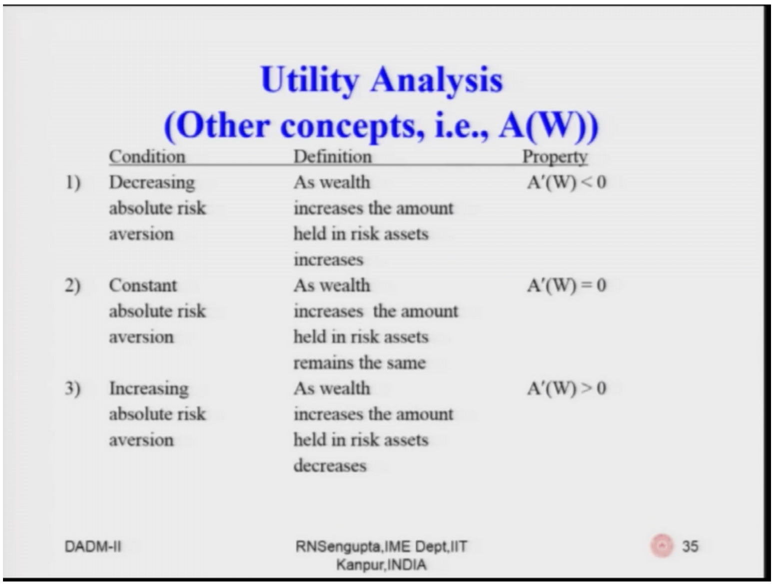

So, on the first column we have the conditions, what is the condition based on which we are trying to pass on the judgment? Second would be what is the actual practical definition based on which we are saying that, absolute risk aversion property or relative risk aversion property or the first derivative of absolute risk aversion property is greater than 0 equal to 0 less than 0.

So, that would basically come under the column which is the second one which is definition and mathematically when you try to utilize this for our calculation for the examples. So, because obviously, we will consider different type of utility functions try to analyze the utility functions what are the properties, whether the non satiation property is met, whether the risk property is met. And obviously, we will try to comment that what are those properties of the utility function based on the absolute risk aversion properties is derivative, relative risk aversion property is derivative.

Now, why it is important? I did not mention it, but I have been talking in the last class also today also mentioned that, they are important why they are important? The reason is that technically finding out the exact form or some functional form the utility function are knowing whatever utility function a person has is not possible. Because I may express my liking and disliking for a decision, but I may not be able to find out exactly whether it is a quadratic one or a logarithmic one or an exponential one or a power function one.

So, what we do is that, we use the properties of those utility functions for which we do not know the functional form. Use the concept of A prime A, R prime R R and then get some results based on which we can go backwards and say that yes this utility function is quadratic or else we say no this utility function is not quadratic, it is basically a logarithmic. So, we will come to that later on.

So, this is the basic main one of the main utilization of utility functions in our case. Now remember in if you go into the depth of utility function is a vast area, we are not going to apply all this concept we will keep it very simple in order to basically utilize; how MCDM and MATU which is Multi Criteria Decision Making in a Multi Attribute Utility Theory will be utilized for our non parametric decision making process in DADM 2.

So, coming back to the slide which is slide number 35; so, the property is based on which we will comment the mathematical properties are given in the third column. So, let us go one by one row wise. The first condition which states is that decreasing absolute risk aversion property is true. Now what does that mean in the practical sense? Which means that as the wealth of the person increases or as the total amount of so, called investment in the decision increases or whatever the decision variables are if the in keep increasing, then the amount held in the risky assets. I am using the word assets in a very general sense the risky decisions let us use the word risky decisions.

So, the amount held in risky decisions or assets also increases. So, if that is true then; obviously, it would mean that the A prime is less than 0 which will all recommended what it means with respect to loving risk, hating this or being indefinite producer. I will come to that once again when we do the problems please bear with me. Now if we can let us concentrate on the second row, it gives us constant absolute risk aversion property which means as wealth increases.

What I am reading now is basically the definition part in the second row. As wealth increases the amount held in risky decisions or risky assets remains the same and it means that the A prime would be 0. And finally, if we go to the third column it means that, increasing absolute risk aversion property is true which means as wealth increases the amount held in risky assets or risky decisions decreases and in that case A prime would definitely be greater than 0.

Now, you check the wordings in first column and the concept which is given in the definition in the second column. Now let us go again back to row 1. It means decreasing absolute risk aversion property; that means, my risk aversion property is now decreasing which means that I will be more tempted to invest in risky decisions and risk assets. If that is true then the definition part should also match with that and let us read it again as wealth increases in the amount held in this case it increases which is true; that means, my absolute risk aversion property is decreasing; that means, I am more willing to invest in the risky assets or decisions.

So, if you follow these principles in a very simplistic logical sense, in the wording sense this comes out to be true for the first row, second row and third row. And, we will again prove it that mathematically which is the last column would also be true as we try to basically solve the problems accordingly. Now, go let us go into the other property, which is known as a relative risk aversion property and this relative risk aversion property and absolute risk aversion property would be utilized as I said depending on their on their results, which we get for different type of utility functions and we can comment what type of utility function it is.

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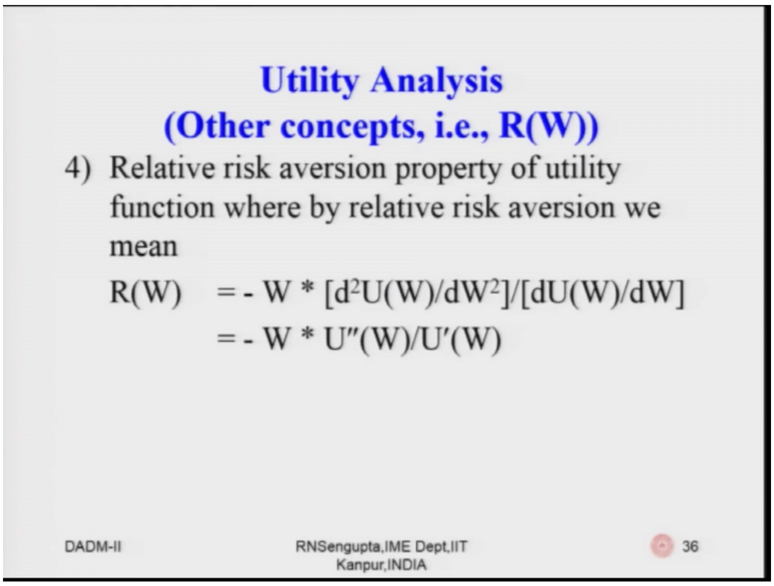

So, relative risk aversion property is given by the ratio, where we multiply the absolute risk aversion property multiplied by the wealth. So, the final formula comes out to be minus W into U double prime by W U prime. If you want to differentiate it, remember that we will always have that U prime is greater than 0 as per the non-satiation property. U double prime can be greater than 0, equal to 0 less than 0 depending on whether I am I want risk more risk which is greater than 0 about second derivative of U. If it is equal to 0 that means, I am indifferent if it is less than 0 that means, I am I want to run away from risk.

Now, the sign of the first derivative of R would depend only on U prime U double prime sorry because U prime is technically 0 was greater than 0 and W which is the wealth is also the amount of wealth which I have in my hands which is also positive. So, let us come back to the actual conceptual way, how you try to analyze and find out what is the expansion of R prime and what is the actual formula which will be utilizing it and how we derive R.

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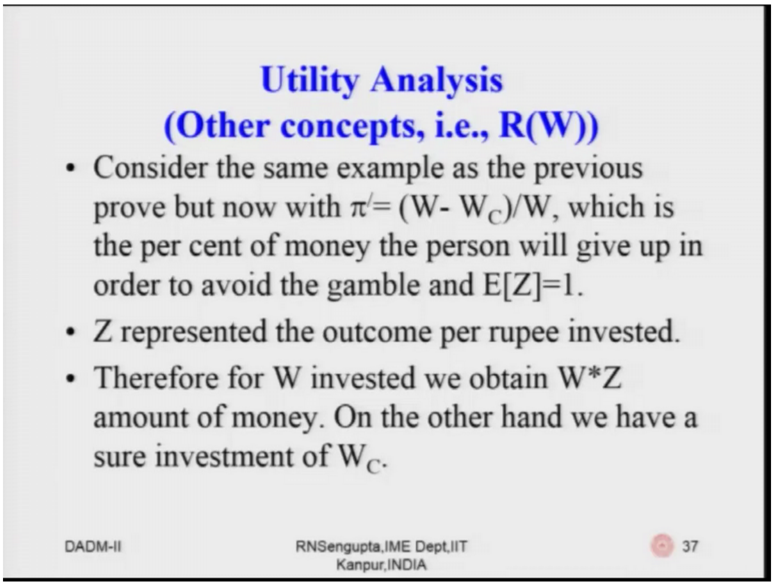

Consider the same example as we have done in the previous case when we are considering the absolute risk aversion property. But, now we consider that it is in the relative sense not in the absolute sense. Because in in the first example it was the absolute sense or difference of the wealth which was being given to us or we are considering that in order to prove that.

So, basically we have the absolute ratio given as W minus W C by W. Now remember that case 1 which is on the left hand side or on the table 1 and case 2 is on the right hand side which is on table 2. In case 1 we consider W as a wealth plus some amount which we are going to invest for which the expected value of Z was 0 it is a fair gamble and the variance of Z was given by sigma square suffix Z on the right hand side we had a total amount of his investment of W C.

So, based on that we are proceeding; so, the context based on which we are trying to draw the derivation is exactly the same example as we did for the first case for a. Now which is basically, if you stick into the ratio pi dash; so, this is W minus W C by W which is a percentage on the money the person will be will give up in order to avoid the fair gamble. Because the fair gamble you remember the it the expected value in that case was 0 now we are considering the expected value as 1.

So, I want to avoid the fair gamble; that means, I do not want to take a risk I want to go for the certainty event. So, per unit increase and decrease in my wealth what is the total percentage based on which I will take the decision. I will run away from the fair example or I will basically go for a certainty event. So, that is basically: what is the decision you want to take. So, Z depends the outcome per rupee investment or per dollar investment and this is if you remember expected value of Z is 1 in this case.

Therefore, for W investment we obtain an amount of W into Z and on the other hand we have a sure investment or W C which is given on table 2 or case 2 or on the right hand side if you remember we are discussing. So, left hand side right hand side we want to basically balance based on which you want to find out we have found out a now you are going to find out R also.

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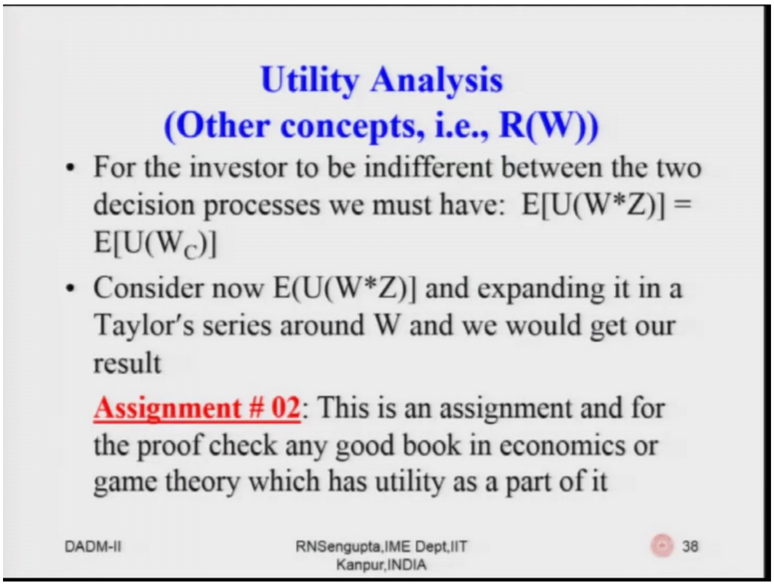

For the investment to be indifferent between the two decisions on table A table B. So, what we have is basically the expected value of the utility or W into Z and why it is multiplied? Because now it is basically ratio. So, we can find out by multiplying and finding out the total commitment on the amount of money which you want to invest and on the right hand side E of W C. So, that value should match the expected value of both hand size left hand side and right hand side should match.

Now, consider the function of utility of U of W into Z expanded using Taylor series expansion if we basically put the terms then the functional form R comes out to be minus W in the bracket U double prime divided by U U prime as we have already stated. So, this assignment word which is given is basically just if somebody is interested see; this is nothing to do with the course remember that it will carry no marks, it has no implication for the assignments 12 segment is a no implication for the examination, it is not counted. But, people who are interested then just can check that how these proofs are done in order to basically get themselves acquainted that how these proofs are very simply done.

Again I am mentioning this assignment one which was given for I forgot mentioning that before and when we are discussing absolute risk aversion property I am sorry for that. So, this assignment 1 which is given for A and assignment 2 which is given for R absolute risk and relative risk, they have nothing to do with the course they have nothing to do with the assignments they would not come into the in the assignment, they would not come into the in the final examination these are just inculcate or bring up the inquisitiveness in you if somebody is interested he or she can definitely check.

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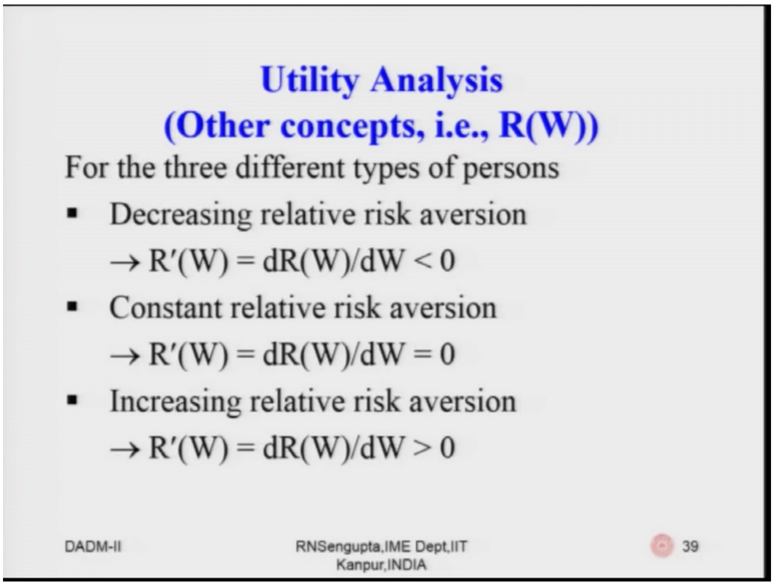

Now in the similar fashion, we will basically follow the same logic as we did for A for the three different per type of person which we have decreasing relative risk aversion property would mean that R prime which is d of R W by dW is less than 0, constant relative risk aversion property would be R prime is equal to 0 and increasing relative risk aversion property would basically be R prime is greater than 0.

So, now, mark the words. Decreasing relative risk aversion constant relative risk aversion and increasing relative risk aversion and these would be coming back again. And, I will try to explain the concept of relative risk aversion property increasing decreasing constant in the same context of the same type of explanation as I did for the absolute risk aversion property.

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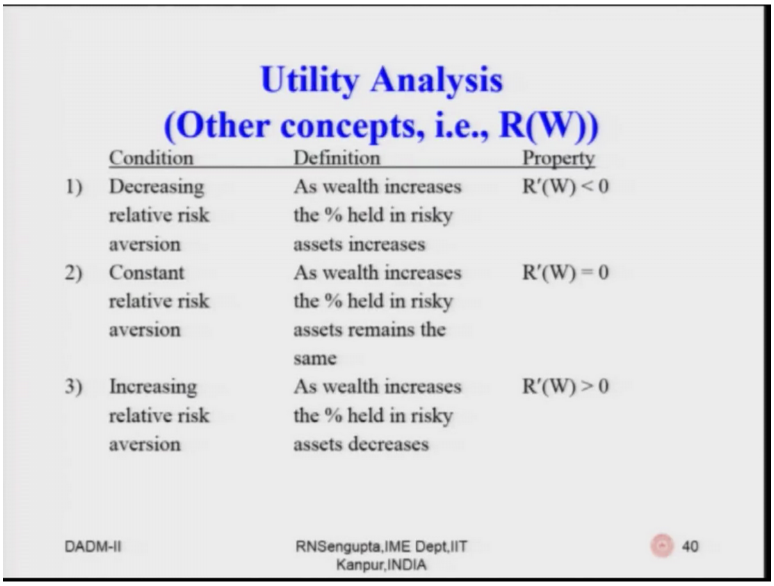

So, with this let us come back to the same type of table, which you have already discussed. So, in the table we have basically in the first column condition and the second column definition and the third column the mathematical property. Now, I will start again its this the context or the overall concept of condition definition properties are exactly the same as we have discussed in for the table for A which is absolute risk aversion property.

So, the conditions which basically are given let us go one by one in the row wise. So, decreasing relative risk aversion property means that my relative risk relative risk aversion property is decreasing; that means, I am more willing to go for this. It means that as wealth increases the percentage held in risk asset would increase, because my risk aversion property relatively is decreasing. And in that case R prime would be basically be less than 0. If it is constant relative risk aversion property it would mean that add wealth increases the percentage held in risky asset or risk decisions would remain the same.

So, R prime is equal to 0 and increasing relative risk aversion property would mean that the as wealth increases, the percentage held in risky assets basically would decrease. So; that means, I am basically being more attracted towards risk; that means, I have an increasing relative risk aversion property. In that case the concept of R prime is greater than 0 would be true.

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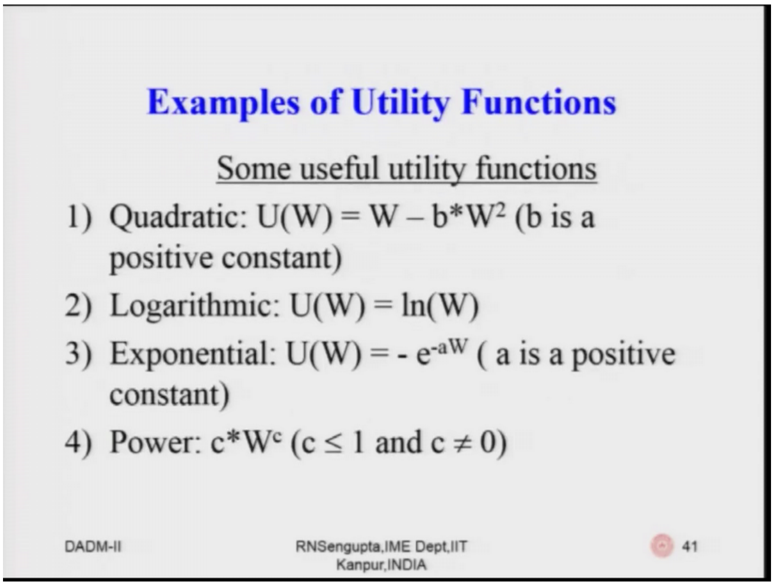

Now, let us come to the utility function where the actual applications of the utility properties of R and R prime and A and A prime would be important. And you will consider that why non satiation why U double prime greater than 0 U double prime equal to 0, U double prime less than 0 would be important as we consider the utility functions. So, we will very simply consider four utility functions, and I will appreciate if the students or the participants for this course pay attention to the first one, first utility function I will come back why it is important later.

So, these are basically the quadratic utility function, we where it is given by a quadratic form where U W is equal to W minus b W into W square where b is a positive quantity. Logarithm utility function would be given by log of W W remember all these four examples or whatever example, you are considering are the wealth or the decision variables based on which my overall value increases or decreases. The third one is the exponential utility property which is minus e to the power minus a W where a is a positive constant and power utility function property is equal to c into W to the power c and c is less than 1 and c is not equal to 0.

Now, I will come back to the concept of a quadratic utility function later on once again, but I will just briefly mention why quadratic utility function is important. Now if somebody considers and that I may not have discussed in much details in this course, but I have definitely done in in the other course which is DADM 1. Now whenever you are considering in estimation problem, what you have is basically trying to analyze the estimate from the sample and when we take the estimate of the sample we consider the square or the (Refer Time: 20:15) concept quadratic loss functions.

Now, quadrant loss functions are easy to handle, then they are theoretically proofs are very nice and they will give us very theoretical very good results. Another important fact is that if you try to minimize the expected value of the quadratic utility function of the quadratic loss function, it basically gives us the information that is has something to do with the variance. So, trying to basically minimize the expected value of the loss, would basically lead us to the fact that you are trying to basically minimize the variance, which is basically the main motivation based on which why it has been considered.

So, the quadratic utility function which will be utilized later on would be found out to be one of the best utility function based on which we will try to do many of our calculations in a very simplistic manner. And I will come back to these later on also. And I did not mentioned sorry before I start this slide, I did not mention about the loss functions I will try to bring up some of the loss function which I may have done in in DADM 1 again bring it back here. So, the many of the students who are doing this DADM 2 and not have done DADM 1, me they may not find it suddenly out of context that those concepts are just being mentioned without being discussed. So, I will try to bring those slides accordingly.

So, some examples of the utility functions which was the first one was the quadratic utility function it is U W is equal to W minus b into W square.

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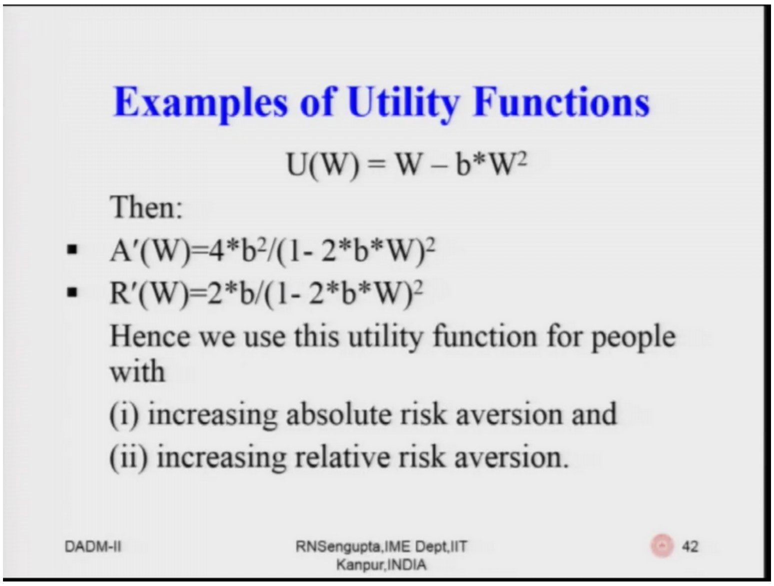

Now here is where I would like to again bring back the context of A and A prime and R and R prime. So, if you remember I have been discussing that and you may have been thinking what is the importance of R, R prime A and A prime. So, what we will do is that we will try to find out the R prime R, A A prime A for this utility function and those values are given. So, A prime is given by 4 into b square divided by 1 minus 2 b into b whole square and R prime is given by 2 into b divided by the same term as it is there in A prime.

Now, let us pause here for few seconds. If you look at A prime the value in the denominator which is a squared is definitely quadrant is positive and the value which is given in the numerator which is 4 b square is also positive because bs b becomes square. Now if that is the case it will immediately give us the information that A prime is greater than 0 and then immediately we know what type of property that utility function is. So, let us read it hence we use these utility functions for people with increasing absolute risk aversion property because that is therefore, for the bullet point 1 which is A prime being positive. And if I consider b as positive 1 negative we will basically find out that R prime would also be positive and negative accordingly because the denominator is quadratic.

So, we will say that is increasing relative is risk aversion property if b is positive and relative risk aversion property being negative. If you have decreasing one if you have basically b as negative, but in the problem we have already considered that b was positive so; obviously, it would be an increasing relative risk aversion property. Now, this is the actual information why those relative risk aversion property and absolute risk aversion properties are important because once you know what type of absolute risk and relative risk of other properties are true, we will just basically go back and excuse me and plug in into. Plug in the sense try to find out for which utility functions this is true and utilized that utility function for the case based on which we are trying to solve the problem.

So; that means, we may not have the functional form of the utility function, we may not know about the utility function, but we can find out what are the A A prime R R prime property is based on this we can comment that whatever you tell function it is. That is what the important actual application for R examples for A A prime R R prime would be coming out.

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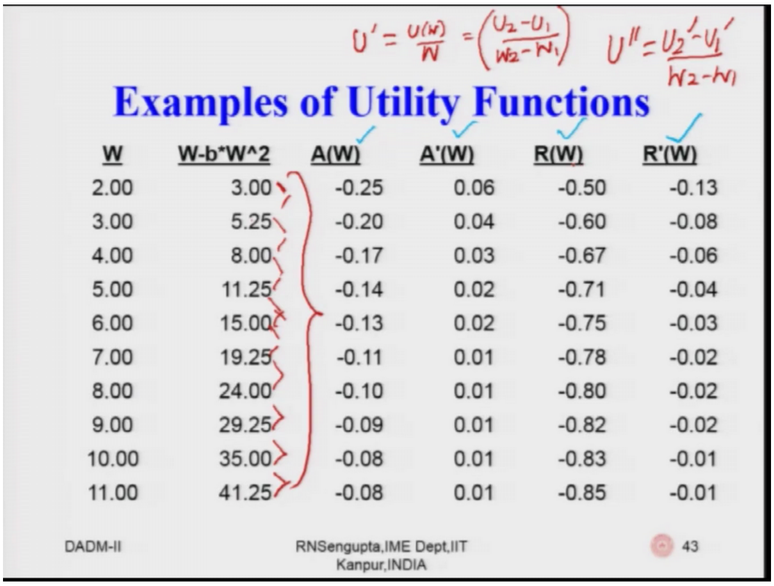

W	W-b*W^2	A(W)	A'(W)	R(W)	R'(W)
2.00	3.00	-0.25	0.06	-0.50	-0.13
3.00	5.25	-0.20	0.04	-0.60	-0.08
4.00	8.00	-0.17	0.03	-0.67	-0.06
5.00	11.25	-0.14	0.02	-0.71	-0.04
6.00	15.00	-0.13	0.02	-0.75	-0.03
7.00	19.25	-0.11	0.01	-0.78	-0.02
8.00	24.00	-0.10	0.01	-0.80	-0.02
9.00	29.25	-0.09	0.01	-0.82	-0.02
10.00	35.00	-0.08	0.01	-0.83	-0.01
11.00	41.25	-0.08	0.01	-0.85	-0.01

Now, let us consider an example let us consider an example for the utility function which is the quadratic one. So, you will basically have Ws the values of wealth on the first column at as it is given in the slide and because I have not put some values, which I will come to in the explanation. In the second column we have the utility function in the third column you have A, fourth column you have A prime, fifth column we have R and sixth column we have R prime. So, what are the missing columns I am going to come to that within few seconds.

So, the values I have written are the column 3 4 5 6 which is 3 4 5 6 I have put them accordingly. So, they have been calculated. So, how let us go one by one. So, first consider the value of W has taken. So, instead we basically if take a particular value starting from 2 to 11. So, the get can be any values it need not be integers also, but I have taking them integers. Based on that mean values of wealth and the values of b which we have we have considered value of b. So, we can b found out because it will be if we would is 2. So, it will be 2 minus b into 2 square 4. So, it will be basically 2 minus b into 4 which will give us 3.

So, from that we can find out what the value of b is, because in that case the second term minus b into W square would be 1. So, 2 minus in that 2 minus 1 wait it would be it is taken as positive. So, it is basically when we are taking negative values which may not be true, but it can be changed accordingly. So, based on this value we calculate the second column now comes the just simple calculations.

Now, what is U prime? So, U prime let me write it down. So, technically U prime is equal to U W by W. So, what you do is that find out the difference of U 2 and U this will be second value of U first value of U second value of W first value W based on that you find out the first derivative. So, find out the differences of Us. So, this is U 2 minus U 1 U 3 minus U 2 then U 4 minus U 3 U 5 minus U 4 U 6 minus U 5 U 7 minus U 6 so, and so, forth. So, find these values I have not written them they would be given here.

Now, you find out the similarly the difference of W 2 minus W 1, W 3minus W 2, W 4 minus W 3W 5 minus W 4 find it out and then divide it. So, once you divide it you will basically have U prime next you would not need to find out U double prime. So, U double prime; so, U double prime would basically be U 2 prime minus U 1 prime by W 2 minus W 1.

So, these values which you have calculated again find out the first difference divided by the first difference of W which have already got. Find out the ratios that U double prime by W put A, minus sign that will give you A, then find out the first difference of a and divide by first difference of W find out a prime similarly once you find out the values of A multiplied by W you get R and then again find out the first difference of R divided by first difference of W you get R prime so, based on that you can do the calculations

I will come to the graph later on in the next class. So, with this I will end this class and discuss these problems for all the examples for the utility function and you will find out why in the A A prime R R prime are important.

Thank you very much for your attention and have a nice day.