Total Quality Management - II
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Lecture - 33
Confounding in the 3^k Factorial Design – II

Welcome back my dear friends, very good morning, good afternoon, good evening to all the students for the TQM II subject lecture under the NPTEL MOOC series and this is as you can see this is the 33rd lecture and I am Raghunandan Sengupta from IME department IIT Kanpur.

So, if you remember we are basically deciding on the higher factor models, higher means more than 2 levels of dependence right the factors can be k, but the different levels of important different levels of effect different levels of combinations between the variables of that attributes because, they can be quantitative as well as qualitative can be 3 can be 4 can be 5. So, fractional factorial model should be 3 to the power k, 4 to the power k, 5 to the power k and so on and so forth.

And then later on and part of the 32nd lecture we consider the blocks such that you can basically differentiate the overall effects in different combinations of, say for example, a b c are the variables whether attributes are variables are not the important question. So, how you define divide into blocks and basically continue analysis such that, your main task is that if you please forgive me I am repeating it time and again, I did mention that the issue is basically to find out the different variables.

So, in the chart if you look the need different type of variables, in different type of combinations whether they combined effect, individual effect then try to find out the concept of how the total sum of the errors would be there for each of them considering then they third column would be degrees of freedom. Then the f value and based on the f value you pass a decision whether you want to accept or reject that idea or hypothesis based on which you will basically say, that which factors are important which factors are not important at a certain level of confidence.

So, that will also becoming from the p value, in between we consider the folding effect we considered different type of regression models regression model for basically first order, it will be of second order, third order depending on different combinations of variables you have and then you were required to find out the coefficient of the regression models and then we had the different assumption for the errors. So, all this things kept arising time and again during our discussion. So, will continue the same analysis, considering different concept, but the general properties general assumptions always remain the same.

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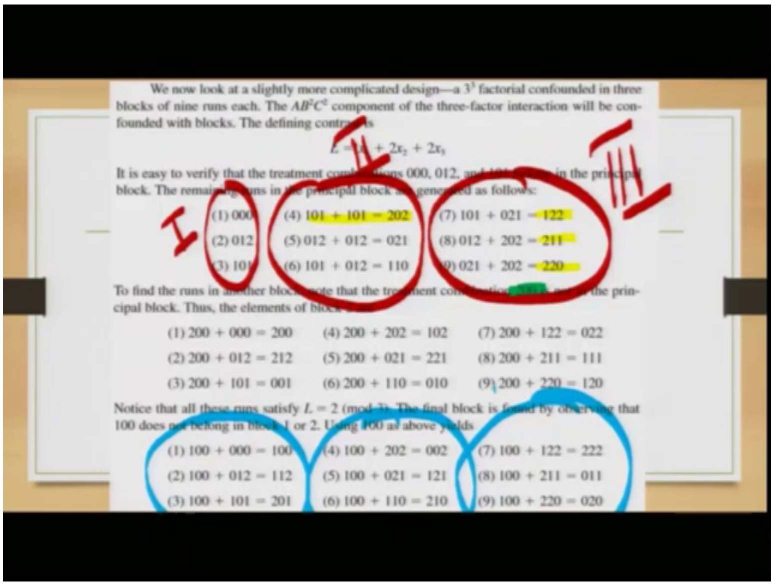

So, we now look at the slightly more complicated 3 to the power 3 factorial model so; obviously, there are technically there are 3 factors at 3 levels. So, again they can be minus 1 0 plus 1 1 2 3 whatever and their confounded in 3 blocks. So, you will basically have the blocks those vertical silos in inside which the effects among themselves will be combined in such a way that the interrelationship between the blocks would be.

So, called minimized, but; obviously, you will try to maximize the overall dependency structure when you considering this ANOVA models. So, there are 3 blocks on 9 runs each the a b square c square component of the 3 factor interactions would basically be given as, as the contrasting model with the contrasting model will basically have x 1 plus 2 x 2 plus 2 x 3 depending on the powers of b and c which you have. So, if it was basically a, b square c square d square it would be x 1 plus 2 x 2 plus 3 x 3 plus 2 x, sorry my mistake 2 x 3 plus 2 x 4 and so on and so forth it is easy to verify that the treatment combinations.

So, now we are considering the levels as 012. So, the at the level when it is 000 which means x 1 is lowest level, x 2 is lowest level, x 3 is lowest level, x 1, x 2, x 3 are basic a b c. So, they can other combination can be 012; that means, low medium high I am talking respectively of a b c, it can be 101 which is medium low high m m medium low medium for a, b, c. So, belonging to this principal block the remaining runs in the principal block are generated accordingly.

So, you can find for the first one it will be 000 and first block you have 012, 101 and the combinations which will have for different blocks that see if see if you consider the second block the first element, even though the word first element may not be right, but I am trying to basically highlight the fact what is of interest to us.

So, in this block you will basically have a at both the levels at medium and then basically the combinations of b would be at low, c would be at medium. So, it is basically 202 then you have basically 021, then you have basically 110 in the third block you have 122. So, the levels levels means a as medium, b and c at high high then 211 with basically means a at high, b and c at medium medium and the third one is 220 which is a and b at high and c at low. To find the runs in each block note that the treatment combinations of 200 in the is not in the principal block so; obviously, you have to take decisions according. So, these are what we are talking about I will try to use another colour.

So, the blocks 1, blocks block 2, block 3. So, if we consider this 200 effect or the combinations which we have which we have. So, this 1 200 is not in the principal block principal block means where I hovering my cursor. So, these are basically block 1, block 2, block 3.

So, let me continue reading it to find the runs in another block note that the treatment combination of 200 is not in the principal block thus the elements of the block 2 would basically now cor correspond to the value of 200, 212. So, I am reading the numbers basically to denote the level are importance of a b c 200, 212, 001 102, 221 or 221, 010, 021,111 and 120 or 120 notice that all these runs basically for mod 3 the hence the final block is formed by observing that 100 does not belong to this block 1 or 2; so using that we find out the combinations now as corresponding to. So, this would be 1, this would be 2, and this would be 3. So, technically we can have the blocks accordingly defined.

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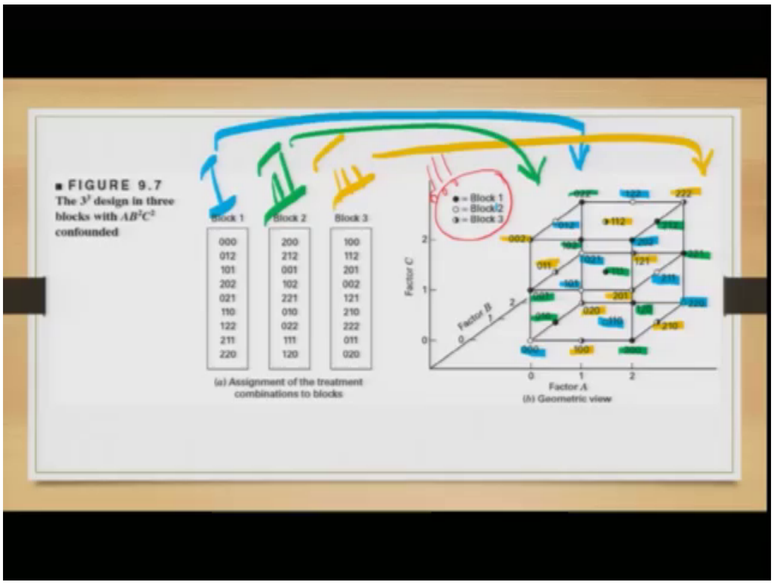

So, once we have the block. So, in the 3 3 model. So, the blocks are given as 1, it is mentioned, but I am just writing this is 2 and this is 3. Now, remember that you have basically 3 blocks with their level of importance level of important means I am not talking from the word using the word from the from the English language sense.

But basically gives you the level at which each of these variables a b c because this is a combination of 3 3 with there are 3 parameter or 3 variables or 3 attributes whatever the 3 levels of significance. So, if we combine them and we try to plot them you; obviously, will have a 3 d diagram and the 3 d diagram again I will repeat it.

So, along the x axis which is my right is which is my left term, which is vertical right term is basically horizontal and the z axis which will be either going from the common (Refer Time: 08:26) in the point where both the right term and the left term meet. Either towards you or away from you and they are all orthogonal so I basically is considered in a room. So, and the corner point you have the axis going up towards the floor, another 2 going orthogonal along the towards the roof and other to going towards the floor.

So, the blocks combinations would be if you basically denote block one as denoted as 000, 012, 101, 202, 2 021, 110, 122, 211, 220. So, this blocks which will have would be and mark them there are this, this, this, this, this, this, this, this, this. So, you have basically 1, 2, 3, 4, 5, 6, 7, 8, 9, 1, 2, 3, 4, 5, 6, 7, 8, 9. So, if I use the other combinations, why I am drawing it with the different colour that will understand the overall layout of the sequence of how the blocks 1 2 3 have been done if. Obviously, can be more than 3 blocks depending on what is the fractional factorial models which you have, if we say for example, n to the power k so; obviously, n would be the level of significance for each of the k factors which you have.

So, I will use for block 2 the green colour. So, the blue one which I have is basically for the blue then I will erase this so in order to make all simple for understanding. So, this you have the so this is block 2. So, block 2 would be the with 1 2, then you have 3, you have 4, you have 5, you have 6, you have 7, whereas 1 2 3 4 5 6 7 and where are the rest 1 2 3 4 5 6 7 8 9. So, it would be on the let me check let me check let me check.

So, you have let me check it. So, it is easier for. So, 200 is, into 200 would go for block 2 so; obviously, this is marked something wrong because they should not have been block one wait let me go for (Refer Time: 11:25) which is green colour. So, this I will, so this is 3 you have 100. So, 100 would come here you have 112 which will come here you have 201, 201 will come here this is 302, 002, 0 it has been they will have 0 they will have 0002 then you have 121, 1 then 121.

So, you will go 1 here, 2 here, 1 here this one, then you have 210, 210, then you have 222, 222 you have 011 011 and finally, 00 200 20. So, you have how many yellow 1, 2, 3, 4, 5, 6, 7, 8, 9 if I check the block 2, let me go through. So, I am taking time please and try to understand that my apologies for that. So, this is for the yellow one if I do the green one, so 2. So, let me first erase because or else it will get confusing ok.

So, now I go to the highlighted green, green 200, if we have 200 this would be. So, this should be highlighted with 200 green 1 212, 212. 212 this is also my mistake, this should be this is. So, let me one and the diagrammatic one there is a problem. So, I am trying to rectify that please, bear with me 001, 001 again an error 001, 102, 102 1102 this one, they should also be colour should change then you have 221, 221 yes this is fine, then you have basically 010, 010 010.

So, this should also be removed then 022, 022, 022 this one you will have 022 then you have 011 you will 011 this one. So, this should 022, 111 sorry my mistake, my mistake, my mistake. So, it is 120. So, the last one would be 120. So, this is also, so I think this should be done.

So, you have 022, let me mark it 022 is here, I am going through this then 212, 212 is here then 102, 102 is here then 221, 221 is here 111 is here then 001 is here, then 010 is 010 is here then 120 is here and 200 is here. So, you have 1, 2, 3, 4, 5, 6, 7, 8, 9. So, these are for the green one for block 2. So, block 2s are given. So, the colour scheme here there is a problem, which will basically highlight the problem basically is here the way of or denoting and if I go to block 1.

So, block 1 would basically 20 202 block one would blue in colour. So, it basically go to the blue colour scheme, 000 means here, 012 01 012, 012 is here 101 is here then 20, 202, 202 is already done, 021 021 is here, 110 here, 122 here, 211 here, 220 here. So, technically you will have 1 2 3 4 5 6 7 8 9, 1 2 3 4 5 6 7 8 9, 1 2 3 4 5 6 7 8 9 ok. So, this is done.

So, the only nomenclature there was a problem in the in the this circles. So, one was basically dark circle, one was absolutely unfilled circle and one was half filled circle. So, the nomenclature was anyway I am sure you have got it. So, it can be (Refer Time: 18:13) expand it from third dimension to 4 dimension and we can make it more complicated with, more number of factors more number of variables. So, you have to just analyze accordingly and divide the blocks accordingly.

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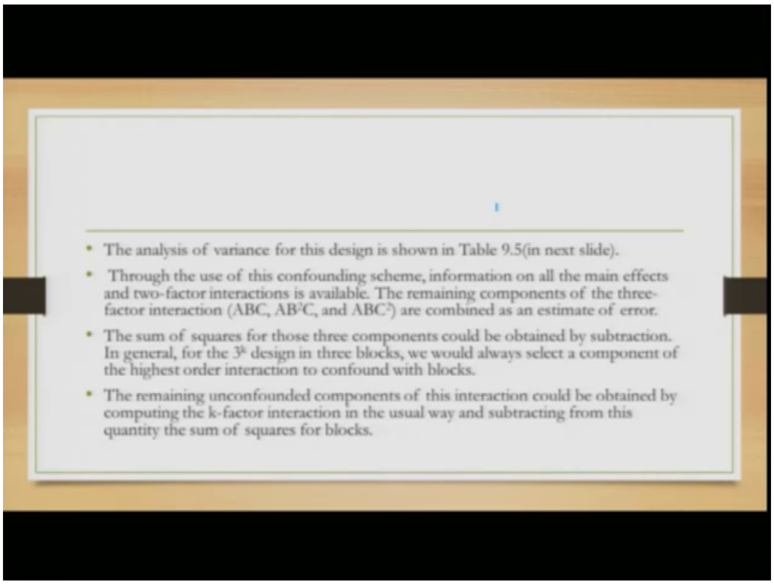

So, ANOVA of the analysis variance for the design is shown in table 9.5. So, through the use of this compounding scheme the information on all the main factors and 2 factors interactions are available or is available, the remaining component of the 3 factor interaction which is basically ABC.

So, at each level of importance being of first order a BC is, AB square C and ABC square compounded as an estimate of the error based on that you again do the same table. I am repeating it first the factors on different combinations then the sum of the squares corresponding to those factors and then the degrees of freedom and then the f have value and the p value based on which will take a decision.

The sum of squares for those 3 components could be obtained, obtained by subtracting in general. So, basically what you do that, the sum of the totals errors should be the total sum of the squares including the errors; obviously, you want to minimize the errors, in general for the 3 k design in 3 blocks you have to always select component the highest order interaction to confounded with blocks. The remaining unconfounded component of this interaction could obtained by compute in the k factor interaction usual way and subtracting from this the quantity the sum of the square blocks and so on so forth and basically to the combinations accordingly.

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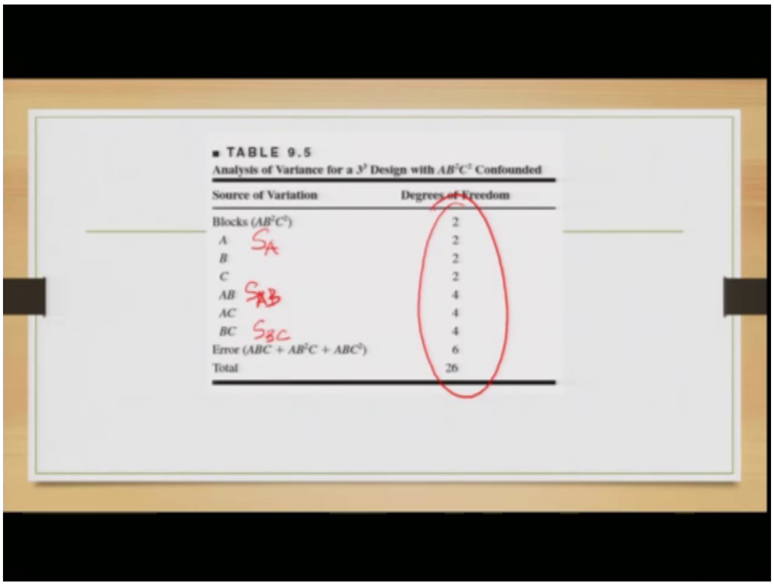

So, now I basically ANOVA and the analysis of variance for the 3 3 design and with AB square C square confirmed it; that means, you are trying to basically find on the maximum effects from there. So, the blocks are for AB square C square, the degrees of freedom is 2 I am just reading the first column and the second column. For ABC the corresponding degrees are 222 and for AB, AC and BC the degrees are 4 each and the errors; obviously, would be the total sum minus the degrees of freedom for others combined.

So, it is 6 you have total number of degrees of freedom would be 26. So, 26 is basically you are trying to find out the sums and based on that and degrees of freedom would also be important because once you find out the total sum of the squares for each of these so; obviously, you will denoted as s a I should use a much more pointed writing, so easier for me yes. So, this sum of the square of the would be s a and this would be say, for I am just giving examples so; obviously, the second one would be s suffix b, the third would be s suffix C, fourth I would did as a suffix AB and the just before the errors the suffix would be s suffix BC and correspondingly degrees of freedom and basically given.

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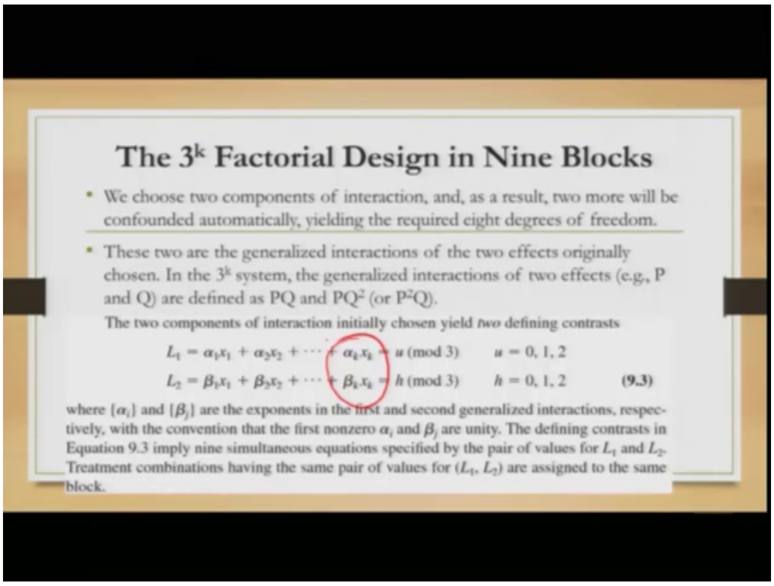

The 3 k factorial design in 9 blocks, so we choose 2 components of interaction and as a result 2 more will be confounded automatically, yielding the required 8 degrees of freedom corresponding to that we can do it, these 2 are the generalized interactions of the 2 effects originally chosen; so in the 3 k system generalized interaction of the 2 effects.

So, example you have been defining the p and q, they would be PQ or PQ square or P square Q and corresponding to that because why it is basically 3 k because the total sum of the powers for ABC or PQ would basically take as 3. So, if it is P square Q. So, it will be 2 plus 1, it is PQ square it will be 1 plus 2 in case if you are say for example, the 4 effects so; obviously, it could be given as P square Q square because that would be 2 plus 2 is 4, the 2 components of interaction would be given.

So, now, for mod 3 and different way of mentioning you would basically have for the effects given as alpha 1 x 1 plus alpha 2 x 2 now; obviously, it will be alpha k x k and in the other way of defining, it will be beta 1 x 1 plus beta 2 x 2 plus dot dot dot beta beta k x k. Now, the reason why they have been given a different parameter is x and x 1 in both the cases or you could have been given has y 1 in first case and x 1 in the second case the suffix 1, I am basically I mean that it would be in the first case it would be y 1, y 2, y 3, y 4 till y k and the next case it could be basically x 1, x 2, x 3 till x k.

So, what you are trying to do is that you are trying to basically break them into for into orthogonals. So, the minimum spanning would basically be given in 2 different directions, those sets of orthogonals you are trying to find out would give you the best possible introduction of say for example, the first and second best possible combination of the orthogonals to give you the total effect.

So, what you are trying to do is that you are trying to basically break down the overall effects in different orthogonal direction, because the interaction between orthogonal should be 0 because 90 degrees and try to find out the effects of the orthogonal sum them up depending on the concept of square error loss, error loss. If you remember I did mention the example was basically the linear exponential loss function that was nice practically, but theoretically square error loss was always utilize because you want to if analyze both over estimation and estimation in the equal proportion and square them up.

Now, if you come to the this orthogonal which you are talking about this orthogonals either in the combinations of y 1, y 2, y 3 till y k with factors being alpha 1 till alpha k and in another direction between basically it will be x 1 to x k with factors a beta 1 to beta k would be done in such a way that you try to basically find out the minimum set of these orthogonals to basically predict the maximum amount of the variations. So, here where alpha I and beta I of exponents of the first and second generalized interaction models respectively, with the convention that the first the non zeros alpha once and beta j is basically you they add up to 1.

So, technically you will try to find out even though it has been mentioned I will just highlight it even though it has been mentioned as k need not be k in the sense the set of xs and set of ys. So, that the corresponding the sets of alpha I s and beta j s may be different where I would say for example, b form 1 to k and beta j can be say for example, 1 to l whatever the combinations are.

So, this through this you will find out the best effect in different orthogonals.

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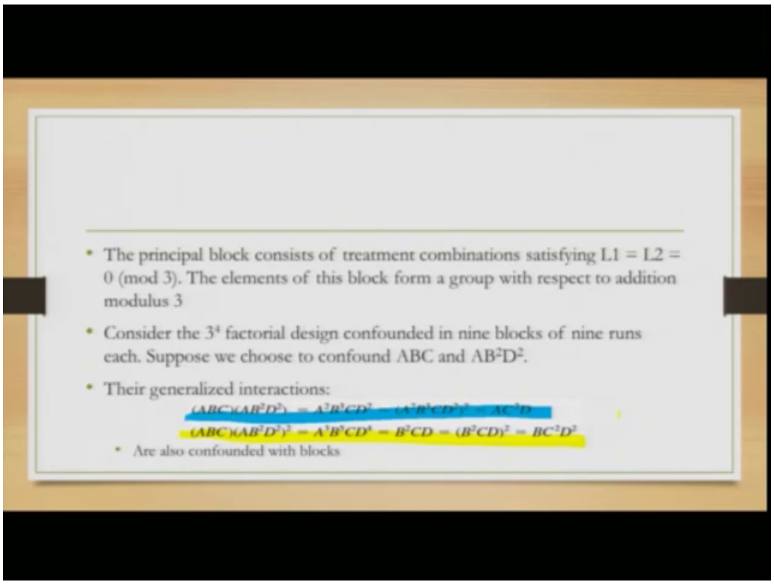

So, the principal blocks so basically you are trying to utilize some concept of principal come to analysis, I would not go into the details for that. So, this principal block consist of treatment combinations satisfying the L1, L2 and so can. So, called this effects you has the mod 3 level the elements of this blocks form a group with respect addition of the on the mod 3, mod modulus 3 consider the 3 to the power 4 factorial design confounded in 9 blocks of 9 runs and we choose the confounded as basically as ABC or AB square D square.

So, what you want to find out is the overall effect, their generalized interactions would be given as stated here I will just highlight them. So, this is the first effect and then I will use the highlighter colour, this is the second one. So, all these are the confounded block based on which you can proceed.

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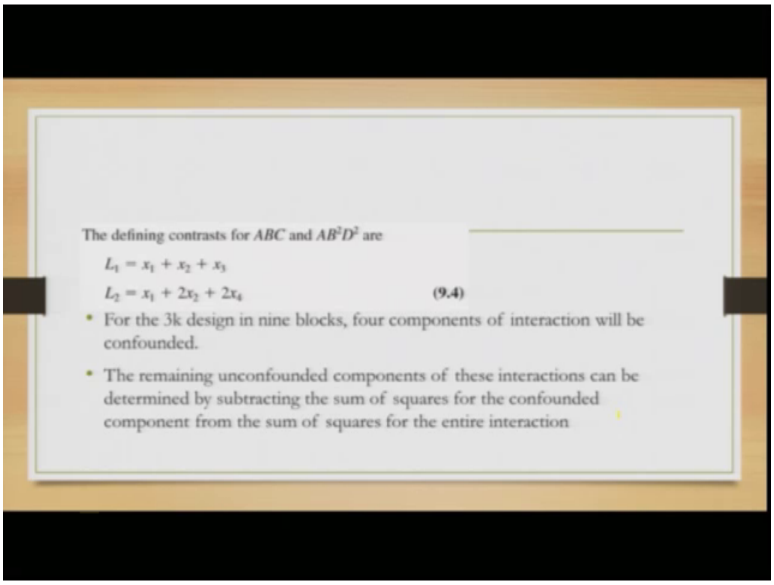

The defining contrast for ABC and AB square D square are here you will have L1, L2 x 1, x 2, x 3 and x 1, 2 x 1 pus 2 x 4 would be anyone considered and you can take the combinations. For the 3 k design in 9 blocks 4 components of interaction will be confounded and the remaining confounded components of the interactions can be determined by subtracting the sum of squares for the confounded component from the sum of the squares for the entire interaction. So, you want to basically differentiate the overall effects into blocks or in confounded such combinations of ABC is depending on number of factors you have and the level of interactions each of them have and based on that you can find out the best effect.

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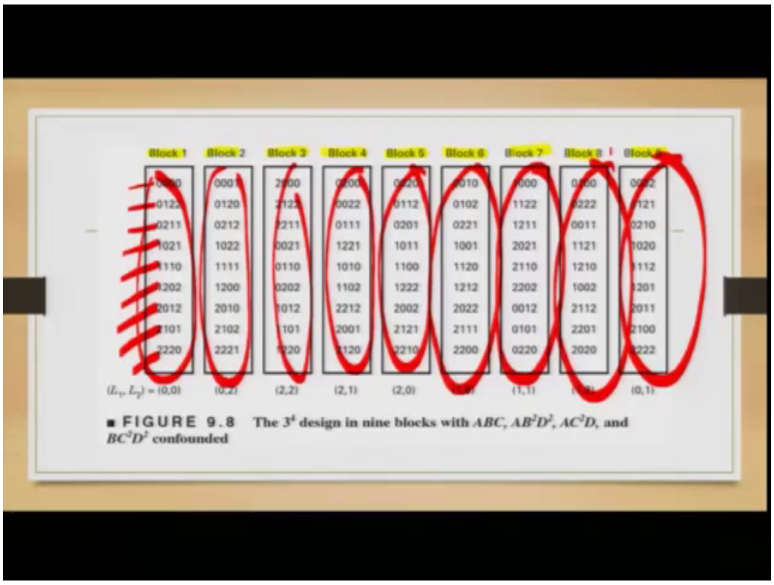

So, in this case you have basically 9 blocks starting from boc block 1 to block 9 and the values I will just mark the blocks as 1, 2, 3, 4, 5, 6, 7, 8, 9 and if you note down. So, basically it would be, when I use the effects. So, it will be as given as block 1 you will basically have their values as this note down the numbers. So, it is 1, 2, 3, 4, 5, 6, 7, 8, 9 similarly from block 2 9, block 3 9, block 4 9, block 5 9, block 6 9, block 7 9, block 8 9 and block 9 9.

So, this is basically for the 3 to the power 4 design and based on that you can differentiate the blocks and do your analysis accordingly. So, you have main task if you remember time and again I am mentioning is basically to find on the maximum amount of dependence such that you are able to predict estimate using this models and such that the sum the errors are minimized.

So, with this we I will end the 33rd lecture and I will continue discussing more about on this factorial fractional factorial models in more details the confounded the blocks the role over effects the folding effects in more details in the remaining 34th, 35th, 36th, 37th, 38th, 39th, 40th lecture. And obviously, in the last fag end of the last we lectures or I will try to take the 40th lecture in such a way so that I will try to summarize whatever we have discussed in TQM II. So, with this and end this lecture and have a nice day.

Thank you very much.