

Quantitative Finance
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Module - 08

Lecture – 48

Continuing with value at risk, if you remember I have discussed long time back about omega, delta, vega. So, now you are seeing the real importance of omega, vega, delta, why they are considering trying to calculate the value of var. If you consider linear and non-linear relationship of var with these values.

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Value at Risk (VaR)

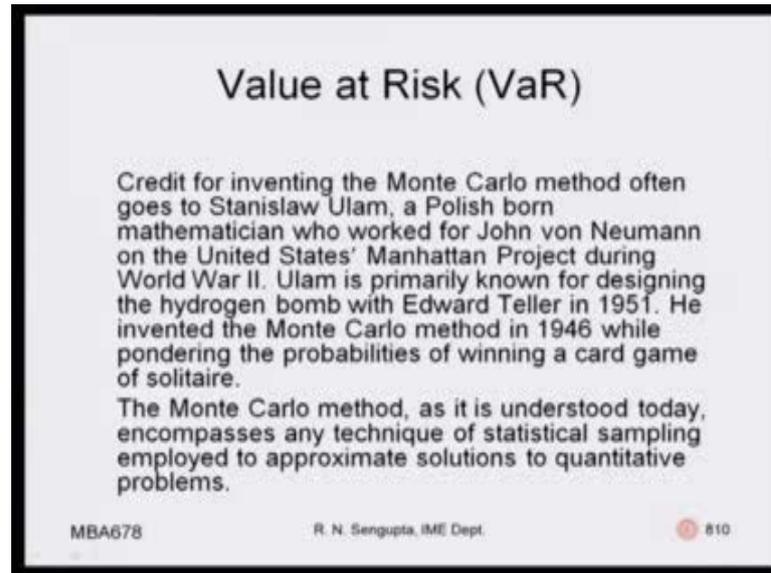
Calculating the change in the portfolio value and hence the volatility of the same is difficult if we have 'n' number of assets and their corresponding options, as the exact mathematical formulation for the following equation is difficult.

$$\Delta P = \sum_{i=1}^n S_i \delta_i \Delta x_i + \sum_{i=1}^n \sum_{j=1}^n \frac{1}{2} S_i S_j \gamma_{ij} \Delta x_i \Delta x_j + \dots$$

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So, calculating the change in the portfolio value, hence the volatility of the same is difficult, if you have n number of assets, because now finding out the cross vars becomes difficult, because you won't have any relationship for information about that. As the exact mathematical formulation of the form of the following equation is difficult to find, because as I mentioned this first part is for the linear part, and second part is the second derivative and obviously there are other terms also which you have omitted them purposefully in our calculations.

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Value at Risk (VaR)

Credit for inventing the Monte Carlo method often goes to Stanislaw Ulam, a Polish born mathematician who worked for John von Neumann on the United States' Manhattan Project during World War II. Ulam is primarily known for designing the hydrogen bomb with Edward Teller in 1951. He invented the Monte Carlo method in 1946 while pondering the probabilities of winning a card game of solitaire.

The Monte Carlo method, as it is understood today, encompasses any technique of statistical sampling employed to approximate solutions to quantitative problems.

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Now, value at risk has to be found out that is true. So, how do you find out? So, there is a huge branch of studies in the simulation, which is known as Monte Carlo simulation. So, credit for inventing the Monte Carlo method often goes to Stanislaw Ulam a polish born mathematician who worked for john von Neumann during the Manhattan project over world war two is basically the bomb one. The hydrogen bomb or the atom bomb which basically was discovered by found an by US. Ulam is primarily known for designing the hydrogen bomb with Edward Teller in 1951, he invented the Monte Carlo method in 1946 while pondering the probabilities of winning a card game for a gamble.

The Monte Carlo method as it is understood today encompasses any techniques or huge set of statistical sampling, which is employed to approximate the solutions to quantitative problems considering the actual distributions may not be known.

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Value at Risk (VaR)

$P_1 = P_0 + \Delta P$
 $P_2 = \Delta P_1$

To calculate VaR using Monte Carlo simulation we follow the steps described below:

- 1) Find the value of the portfolio today, i.e., $P_{t=0}$.
- 2) Sample once from the multivariate distributions of the Δx_i .
- 3) Use the Δx_i to determine market variables at the end of today.
- 4) Reevaluate the portfolio at the end of day, i.e., $P_{t=1}$.

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Now to calculate var using Monte Carlo method; the very simple following steps are found out. So, what you do is that you find out the value. So, say for example, I will first describe it qualitatively, and then come back to those styles. So, what you have is that you have say for example, the value of the portfolio which is there for you at day 0, considering today's day 0. So, now you change consider the change in the overall value of the portfolio considering the prices are changing, you are not considering the investments are changing; that you can also consider, but that would be the next stage of not complication, but next stage of calculation.

So, you consider the alphas of the investments quantum in each and every asset is fixed only the rate of change of the portfolio is happening, because there is a rate of change on the investments as the prices are changing for the assets. So, now you basically find out the value of the portfolio at time t is equal to 0, and then considering the normal distribution to be true for the overall portfolio, what you do is that you will basically have a multi varied normal distribution. Multi varied normal distribution based on the fact that you know so called weights, because alphas are related to weights. So, you have the weights which are given and which are fixed to you find it out.

So, from the multi varied normal distribution you generate a number, and basically that generation that number basically you will give you the overall change of the portfolio. So, how it is done let us consider any one lower level. So, consider if you remember we

have done the Lema, the veneer process in a general veneer process we had considered some alphas and that has a normal distribution with certain mean and certain standard deviation. Consider there are 10 number of assets; each with normal distribution with certain mean and certain standard deviation. So, those means are coming from the rate of change average values, and standard deviations are coming from the standard deviation of the rate of change. So, what you do is that, you basically generate each of them individually from that corresponding normal distribution note down those values.

So, once you note down those values you also have the historical the correlation coefficients of between the assets; use the correlation coefficients and find out the net rate of change of the total portfolio from day 1 from 0 to day 1. Then, what you do is that you increase or decrease your portfolio value by this amount. That means, if your rate of change of the portfolio between day 1 and day 2, 0 is ΔP . So, you add this and find out basically the new rate the portfolio value. Again you simulate individually at one at a time for from a simple univert rate normal distribution for the same assets for the day 2. Again you have already you have the correlation coefficient, find out the change of the portfolio; and once the change of the portfolio is found out. So, consider this is P_1 , you find out P_2 is equal to ΔP_1 . So, these were ΔP_0 .

So, as it will continue doing it you have the rate of change of the portfolios, and you have the value of the portfolio. Now as the value of the portfolio is there they would be different values, it would be positive or negative. So, what you do is that you rang them from the lowest to the highest and find out at what level; that means at one 95 percent or 90 percent or 99 percent, level of value of portfolio you want to have such that var characteristics are met. So, this is what you do? Find the value of the portfolio today that is a p_t is equal to 0. Sample once from the multivariate distribution for all the Δx_i , use the $\Delta x_i \Delta x_i$ to determine the market variable at the end of the today; that means, end of daily 0. Reevaluate the portfolio at the end of day which means finding out the value of p is equal to t_1 by calculating the change c in the value of the individual asset of the portfolios.

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Value at Risk (VaR) 6.9

- 5) Calculate ΔP , using the differences between the portfolio value at the beginning of the day and at the end of the day, i.e., $P_{t=1} - P_{t=0}$.
- 6) Repeat steps # 1-5 many times to build up a probability distribution for ΔP .
- 7) VaR is the appropriate fractile of the distribution times square root of N (number of days).

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Now calculate Δp as I said find out the Δp using the difference between the portfolio value at the beginning of the day, such that the changes of that portfolios are given ΔP 1 ΔP 2 ΔP 3 can be found out. Repeat the steps of 1 to 5 as I said find out the Δp 's to build out the probability distribution as our ΔP . Now what I did not highlight again, but I think you must have understood it that the actual need is not p , it is basically at the ΔP which are important it will give you the distributions of p 's and Δp 's, and also how the var is calculated. Because if you remember whenever you are mentioning ΔP , it is basically sigma of Δp that means the standard deviation the rate of change of the portfolios.

Similarly, if you have done different problems in swaps, and different problems in options. All our concern was the rate of change of the return of the option or rate of change of the return of the portfolio or rate of change of the return of the financial assets and its standard deviation was more important for us.

Repeat steps 1 to 5 many times to build up a probability distribution for ΔP , var is the approximate factor of the distribution based on which you are trying to find out what percentage is it 95 percent, is it 99 percent based on which you will find out the value of var.

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Value at Risk (VaR)

One major **disadvantage** of the Monte Carlo Simulation methodology is that we **assume the market variables to be normally distributed**.

To overcome this problem we use **historical simulation**. In historical simulation we first create a data base consisting of the daily movement of all market variables over the years.

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One major disadvantage of Monte Carlo simulation method is that we assume the market variable to be normal distributed which may not be true. To overcome this problem we use historical simulation; that means, you have the past data and we try to basically do boot strapping in order to estimate that. In historical simulation we first create a database consisting of daily movement of all the market variables over the years and then that do the simulation values accordingly.

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Value at Risk (VaR)

- 1) In the 1st simulation trial the percentage changes in each market variable is assumed to be equal to that of the market variables as in the first day covered by the database.
- 2) In the 2nd simulation trial the percentage changes in each market variable is assumed to be equal to that of the market variables as in the second day covered by the database.
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- n) In the nth simulation trial the percentage changes in each market variable is assumed to be equal to that of the market variables as in the nth day covered by the database.

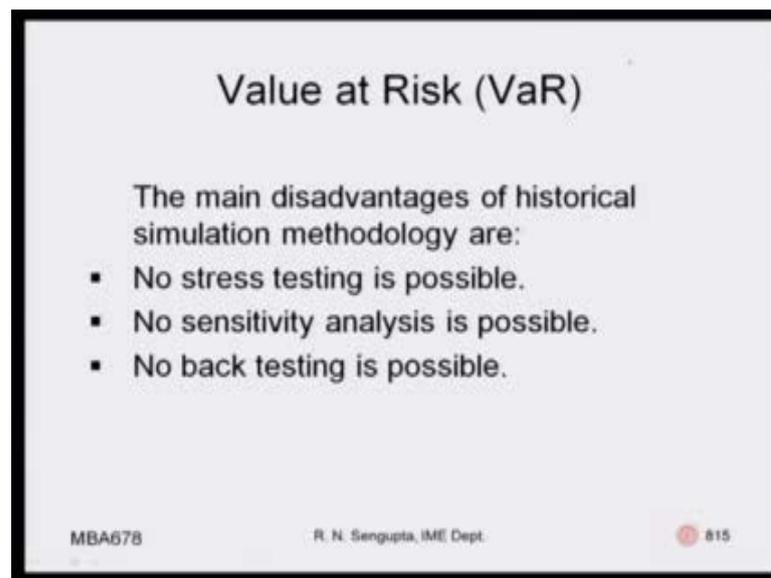
n+1) Using the information from each step we find the ΔP and hence the distribution of ΔP .

n+2) From the distribution of ΔP we calculate the appropriate percentile of the probability distribution for ΔP and finally the appropriate VaR

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In the first simulation, and when you are doing the historical or Monte Carlo in details how we do that. In the first simulation trial the percentage changes in each market variable is assumed to be equal to that of the market variable as in the first day covered in the database in the past historical data. In the second simulation, trial the percentage change in each market variables you assumed to be equal to the market variable as in the second day, then in the third simulation is the third day and so on and so forth. Once you finish that for the n day using that information which is in the n plus 1 step using the information from each step we find out the delta P, hence the distribution of P, and from the distribution of del p we again calculate the values of var, as we have done for the actual history this simulation study based on the fact we consider the normal distribution to be true.

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Value at Risk (VaR)

The main disadvantages of historical simulation methodology are:

- No stress testing is possible.
- No sensitivity analysis is possible.
- No back testing is possible.

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The main disadvantages of the historical simulation method is there is no stress testing; that means, given any untoward incident or extreme cases which are happening, they are not available for us to simulate. No sensitive analysis can be done; that means, rate of change of one variable effects, what great effect or less effect does it have on the portfolio value, that cannot be done, because we do not have that data. And no back testing can be done to in order to basically assume that the model we are using on the historical simulation may using is right or wrong.

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Value at Risk (VaR)

Stress Testing
This involves testing how well a portfolio performs under some of the most extreme market moves seen in the last 10 to 20 years.

Back Testing

- 1) Tests how well VaR estimates would have performed in the past.
- 2) We could ask the question, about how often was the actual 10-day loss greater than the 99% 10 day VaR?

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Stress testing means this involves testing how well a portfolio performs under some most extreme market conditions the last 10 or 20 years, back testing means test how well var estimates would have been performed in the past or historical past is how they would have done in the past or in the coming future considering the past is the best replica of the future which is going to happen. We could ask the question how about how often was the actual 10 day loss greater than 99 percent in a 10 day var; these values even though they can be calculated in a very simple historical or actual theoretical simulation, they may not be exact true, because the overall practical environment is totally different with respect to the Monte Carlo simulation study which you are doing. But having said that believe me that the overall concept of Monte Carlo simulation concept of historical simulation gives us a lot of informations how the var value is changing.

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Value at Risk (VaR)

We usually use Principal Components Analysis to handle risk arising from groups of highly correlated market variables. Using the Principal Component Analysis we try to define a set of factors that can be taken as the market variables.

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We use different types of principle components analysis a statistical multivariate statistical method to handle risk arising from groups of highly correlated assets using pca which is principal component analysis, we try to define a set of factors that to be need that can be taken as the market variables. That means, if you have 10 or 15 different market variables, you want to find out the best 5 or best 6 such that the overall prediction can be done to the maximum possible extent with a least number of variables. So, that is the task of pca concept which you have which we generally use.

And if you remember also in one of the alter Altmans models. If the first class one of its first class when we are discussing the different types of models; Altmans models, o scores, z scores, zeta scores. So, there Altman did use the concept of pca in order to solve this type of different rating concepts.

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Value at Risk (VaR)

Remember a well known characteristic of the loss distributions is that they have fat tails. Fat tails are the extreme sections of the distributions, and indicate that large losses, although unlikely because their probabilities remain low, still they have some likelihood to occur that is not negligible. The fatness of the tail refers to non-zero probabilities over the long end of the distributions. Now remember that for a normal distribution, VaR for a 2.5% one tailed confidence level corresponds to a multiple of loss volatility of 1.96. Therefore if the loss volatility is 100, the unexpected loss will not exceed the upper bound of 196 in more than two or three cases out of 100 scenarios. But if the distribution is not normal then such simple calculations do not apply.

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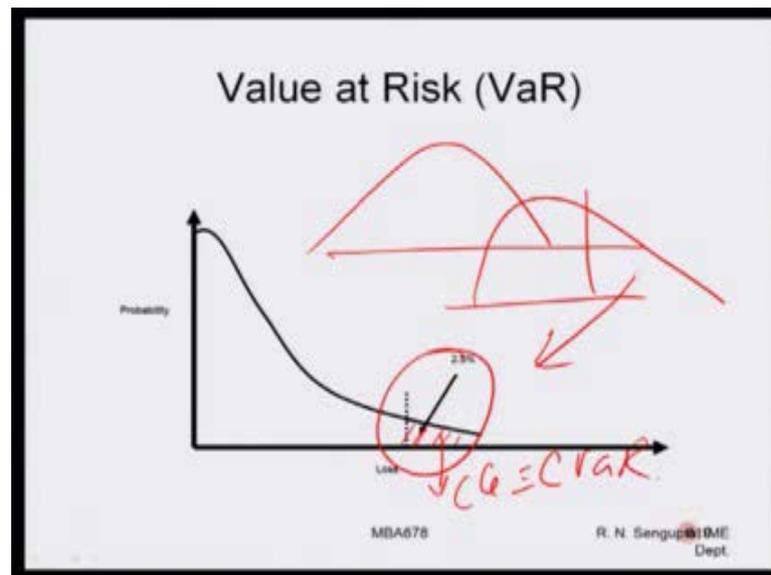
Remember a well known characteristic of the last distribution is that they have the fat tails as I mentioned. Similarly for profit distribution also they have fat tails. Fat tails are the extreme sections of the distribution and indicate that large losses are the unlikely, because the probability remain low still they have some likely to occur that is not negligible that cannot be neglected. The fatness of the tail refers to non zero probabilities over the long end of the distributions.

Now, remember that for a normal distribution var for 2.5 percentage; one tail distribution corresponds to a multiple of losses for values of 1.96. Now as I have mentioned two tailed and one tail that is very important. So, you want to only consider on one side of the distribution, considering this skewed distribution in sales.

Therefore, if the loss volatility is 100, the unexpected loss will not exceed the upper amount of 196 in more than 2 or 3 cases out of those 100 scenarios depending if you are trying to do simulation. But if the distribution is not normal then such simplifications cannot be stated, because non normal distributions even if you generate the probabilities are not as structured or not they cannot be even though they can be explained in the form of a functional form like normal distributions; some extreme value distribution f of x can be given; f of x is the pdf function, but still trying to find out the overall var would not basically hold true, because number 1 historical simulation cannot be done.

Number 2 doing a simulation considering theoretical one may be difficult, and number three is more important is that the 4 characteristics we considered homogeneity in all these things may not hold true, and for the var case may not hold true for the standard deviation case if non symmetric distributions are considered.

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So, this is one type of tail we are considering, basically this is this or this can be this, I am only considering this for which will give you the probability which is over shooting the var value. And obviously again I am repeating, the center of gravity on to the right would give you the conditional value at risk.

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Value at Risk (VaR)

$P[X \leq x] = P\left[\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right] = 1-0.025 = 0.975$
Thus $P[Z \leq z] = 0.975$, hence $z = 1.96$
As $\mu = 0$, hence $x = 1.96 \cdot \sigma$. But remember that for distributions which are not normal, this calculation will not hold.

In case you have the distribution as exponential, then also we can find the cumulative probability of default, or the risk exposure depending on the value of the loss one can sustain. Hence using a value of α , we can find x_1 , which is the loss one can sustain.

$$\int_0^x \exp(-x) dx = \alpha$$

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So, what do we have if we have the normal distribution you can easily find out using the concept of standard normal, and then you can find out the probability. As μ is 0, hence x is 1.996 which is the percentage of deviation; that is the level of confidence, but remember that for distribution which are not normal this calculation won't hold true, in case you have an exponential distribution then you want to find out the default risk. You have to use this formula. So, α would give you the overall probabilities, some of the probabilities and you have to basically integrate it from 0 to x or from a to x ; if say for example, the exponential distribution is starting at a value greater than 0 integrate it, and find all the overall area and equate their area in order to find out what is the value of x ? x is basically the var value which you are talking about.

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Value at Risk (VaR)

When implementing techniques, based on confidence level (CL) and loss percentiles, there is a need for common benchmarks, such as CL, for all the players must be the same. With a very high CL, VaR could be so high that business transactions would soon become limited. On the other hand if the benchmark is different then the banks have different standards for evaluating their VaR. In other words a level playing field should be agreed upon, so that there is no confusion in this respect.

Note, the normal distribution is an approximation for losses in general, over a short period of time, but it cannot be applied for credit risk, for which the loss distribution is highly asymmetric.

In spite of repeating the same yet it is important to note that for any FI it is important to find the distribution of the (i) inflows and (ii) outflows and then try to find the resulting distribution for the net income/loss so that we can find the VaR at a given confidence level and for the particular number of days, as stipulated.

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When implementing techniques based on confidence intervals and loss percent, and there is a need for common benchmark that is this confidence levels are required or with very high confidence level var could be so high that the basic business transactions soon become limited, they cannot because you have to basically keep aside a huge amount of money as a var value. Note the normal distribution is an approximation for losses in general over a short period of time, but it cannot be applied for credit risks for which the loss distributions are highly symmetric.

In spite of the repeating same year, I am still stressing on the fact that the importance of the var cannot be ruled out even for the case when the distributions are non normal, because they give you some feel that how the distributions can be utilized in order to find out var. Let me give you a very simple example why var is important? Like we all solve the Newton's law of motion repeatedly, and we know force is equal to mass into acceleration, and we know that fiction is there still we neglect fiction and solve our problems and still it gives a very good results; in this same way var has many disadvantages, it cannot be used for other types of distributions which are not normal. Still the concept of var in many cases gives us a very good feel that what is the overall risk pertaining to a certain portfolio.

So, with this we have covered a definitely some portions of the var, and considering var this is the end of this course even though we may not have been able to cover many of

the concepts, but considering the 20 hours we had we did trying in our own best effort and I to give a overall view of optimization, overall view of different type of risk measures overall view of different type of averaging techniques which are used, then different types of swaps, different types of options, different concepts how they can be done, and we did deal in many of the facts different concepts of risks.

So, where if you understand the overall emphasis on all our discussion was risk is the main important factor in finance and in quantitative finance which we have to decrease or make it 0. So, I am sure it has been a very exciting class for all of you, and we will be giving uploading as we do understand that the different types of assignments are being uploaded after each week, and we will have two different examinations date and the format and everything will be mentioned to all the students on the website or it would be definitely mentioned through email.

So, I would wish all the best of luck and I am sure that once you go to those slides, once you refer to the different type of references which will be a good learning process for all of you. And obviously, as I did mention in when I have started this class the whole net has huge set of information in the area of finance or the quantitative finance. So, you have to be, we on the lookout and gather all the fact such that at least after completing the course, and if you have been able to open up your mind it will really open up huge amount of knowledge which you have to just pluck and pick in order to understand the concept of quantitative finance.

Thank you very much, and I am sure you will do very well in that.

Thank you very much.