

Quality Control and Improvement with MINITAB
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Lecture - 39
Fractional Factorial Design

Hello and welcome to session 39 on our course on Quality Control and Improvement with MINITAB. I am Professor Indrajit Mukherjee from Shailesh J. Mehta School of Management, IIT Bombay.

So, we are discussing about multiple response optimizations. So, we will just revisit the problems what we are solving and how to solve it in MINITAB; we will try to see ok.

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Quality Control and Improvement using MINITAB

Example

A chemical engineer is interested in determining the operating conditions that **maximize** the yield (y_1) [LTB], **on-target** viscosity (y_2) [NTB], and **minimize** molecular weight (y_3) [STB] of a process. Two controllable variables that influence process are: reaction time (x_1) and reaction temperature (x_2).

$70 \leq y_1 \leq 80; T_1 = 80$
 $62 \leq y_2 \leq 68; T_2 = 65$
 $3200 \leq y_3 \leq 3400; T_3 = 3200$

Reaction Time (x_1)	Reaction Temperature (x_2)	Yield (y_1)	Viscosity (y_2)	Molecular Weight (y_3)
-1	-1	76.5	62	2940
-1	1	77	60	3470
1	-1	78	66	3680
1	1	79.5	59	3890
0	0	79.9	72	3480
0	0	80.3	69	3200
0	0	80	68	3410
0	0	79.7	70	3290
0	0	79.8	71	3500
1.414	0	78.4	68	3360
-1.414	0	75.6	71	3020
0	1.414	78.5	58	3630
0	-1.414	77	57	3150

Source: Montgomery, D. C. (2004). *Design and analysis of experiments*. John Wiley & Sons

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So, this is the example where we want to maximize the yield over here. So, one is maximization of yield over here; what we are this is to be maximized. Then, viscosity should be on the target and molecular weight should be lower the better type of scenarios.

So, one is maximization; so this is known as larger the better functions like that. And on target is known as nominal the best and another one is minimization, i.e., smaller the better type of functions like that. So, all this response function we can define like above. If I have the response surface, then either one of them has to be maximized minimized or

to be on the target like that; any specification is like that only engineering specification we can think of ok.

So, in this case the boundary condition is taken from Montgomery's book. So, this is given y_1 should be within 70 to 80 and the target value is around 80. Then y_2 should be between 62 and 68, the target value is 65 over here; so y_1 is we want to maximize; so target value is 80.

And the third one y_3 is minimization problem anything below 3400 is fine, but we are giving some boundary conditions over here, but any solutions less than this. So, and we have defined some target values over here. So, minimization; smaller the better means any values the problem statement is 3400, anything below that is sufficient like that.

So, in this case now we have to solve this problem; so this is the CCD design that was experimentation was done. So, we have a factorial points, we have a center points and we have the axial points like that.

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Desirability function:
 The desirability function transforms each response variable to a corresponding **scale free desirability value** (say d_j), which lies between zero and one ($0 \leq d_j \leq 1$). The value of increases as the "desirability" of the corresponding response increases.

Composite Desirability:
 It (say, D) is defined as **geometric mean** of the individual desirability values and mathematically it is defined as.

$$D = \left(\prod_{j=1}^r d_j \right)^{1/r}$$

The composite desirability is also lies between 0 and 1.


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So, this is the experimental setup that we are having and we are using a composite desirability to reach to the optimal solution. So, MINITAB will we will use a response optimizer or MINITAB which will use heuristics to solve the problems of multiple response and it will give you the final solution of setting of x_1 and x_2 ok.

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Alternative Desirability Function Approach

For STB Response

$$d_j = \begin{cases} 1 & \text{if } \hat{y}_j(\mathbf{X}) \leq y_j^{\min} \\ \left[\frac{y_j^{\max} - \hat{y}_j(\mathbf{X})}{y_j^{\max} - y_j^{\min}} \right]^n & \text{if } y_j^{\min} < \hat{y}_j(\mathbf{X}) < y_j^{\max} \\ 0 & \text{if } \hat{y}_j(\mathbf{X}) \geq y_j^{\max} \end{cases}$$

For NTB Response

$$d_j = \begin{cases} 0 & \text{if } \hat{y}_j(\mathbf{X}) < y_j^{\min} \text{ or } \hat{y}_j(\mathbf{X}) > y_j^{\max} \\ \left[\frac{\hat{y}_j(\mathbf{X}) - y_j^{\min}}{\tau_j - y_j^{\min}} \right]^n & \text{if } y_j^{\min} \leq \hat{y}_j(\mathbf{X}) \leq \tau_j \\ \left[\frac{y_j^{\max} - \hat{y}_j(\mathbf{X})}{y_j^{\max} - \tau_j} \right]^n & \text{if } \tau_j < \hat{y}_j(\mathbf{X}) \leq y_j^{\max} \end{cases}$$

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And it is using desirability functions that I told that for smaller, the better type it will use a desirability function and nominal the better there is a different desirability function and this is given by the ranger; so the for the larger the better the function is like this.

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Desirability Function Approach

For LTB Response

$$d_j = \begin{cases} 0 & \text{if } \hat{y}_j(\mathbf{X}) \leq y_j^{\min} \\ \left[\frac{\hat{y}_j(\mathbf{X}) - y_j^{\max}}{y_j^{\max} - y_j^{\min}} \right]^n & \text{if } y_j^{\min} < \hat{y}_j(\mathbf{X}) < y_j^{\max} \\ 1 & \text{if } \hat{y}_j(\mathbf{X}) \geq y_j^{\max} \end{cases}$$

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So, if I can reach the target immediately the what we will get is that desirability d_j value will be equals to 1 and the composite score desirability can be calculated which is nothing, but the geometric mean of the of the values of d_j 's over here. There are

different ways of doing this, but one of these is geometric mean that we are referring over here ok.

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Regression Equation

$$y1 = 79.940 + 0.9950 x1 + 0.5152 x2 - 1.376 x1*x1 - 1.001 x2*x2 + 0.250 x1*x2$$

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.266000	98.28%	97.05%	91.84%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	79.940	0.119	672.00	0.000	
x1	0.9950	0.0940	10.58	0.000	1.00
x2	0.5152	0.0940	5.48	0.001	1.00
x1*x1	-1.376	0.101	-13.65	0.000	1.02
x2*x2	-1.001	0.101	-9.93	0.000	1.02
x1*x2	0.250	0.133	1.88	0.102	1.00

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	28.2478	5.6496	79.85	0.000
x1	1	7.9198	7.9198	111.93	0.000
x2	1	2.1232	2.1232	30.01	0.001
x1*x1	1	13.1761	13.1761	186.22	0.000
x2*x2	1	6.9739	6.9739	98.56	0.000
x1*x2	1	0.2500	0.2500	3.53	0.102
Error	7	0.4953	0.0708		
Lack-of-Fit	3	0.2833	0.0944	1.78	0.290
Pure Error	4	0.2120	0.0530		
Total	12	28.7431			

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So, in this case MINITAB will do it automatically for you; it will generate the x1 and x2 conditions and it will see the desirability and composite desirability and based on that final solutions will be derived, after the complete iteration process like that ok; of the algorithm. So, what; how do we how do we MINITAB; so this is very easy.

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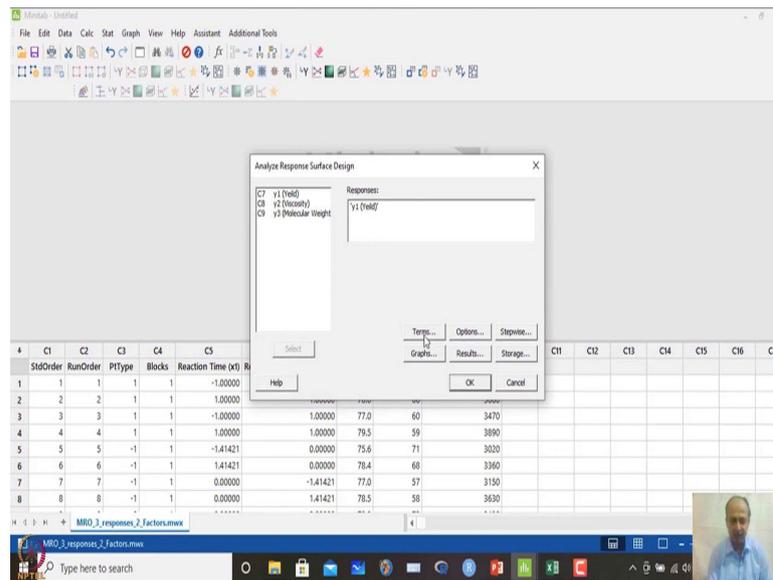
The screenshot shows the Minitab software interface. The 'Doe' menu is open, and the 'Response Surface' sub-menu is selected. The 'Analyze Response Surface Design...' option is highlighted. Below the menu, a data table is visible with columns for RunOrder, PTType, Blocks, Reaction Time (x1), Reaction Temperature (x2), y1 (Yield), y2 (Viscosity), and y3 (Molecular Weight). The table contains 8 rows of data.

RunOrder	PTType	Blocks	Reaction Time (x1)	Reaction Temperature (x2)	y1 (Yield)	y2 (Viscosity)	y3 (Molecular Weight)
1	1	1	-1.00000	-1.00000	76.3	62	2940
2	2	1	1.00000	-1.00000	78.0	66	3660
3	3	1	-1.00000	1.00000	77.0	60	3470
4	4	1	1.00000	1.00000	79.5	59	3890
5	5	-1	-1.41421	0.00000	75.6	71	3020
6	6	-1	1.41421	0.00000	78.4	68	3360
7	7	-1	0.00000	-1.41421	77.0	57	3150
8	8	-1	0.00000	1.41421	78.5	58	3630

So, what we will do is; that I am taking the data set over here and this is the excel, this is the worksheet file that we have the data and C5 and C6 is the experimentation; design matrix and this is the outcomes y1, y2 and y3.

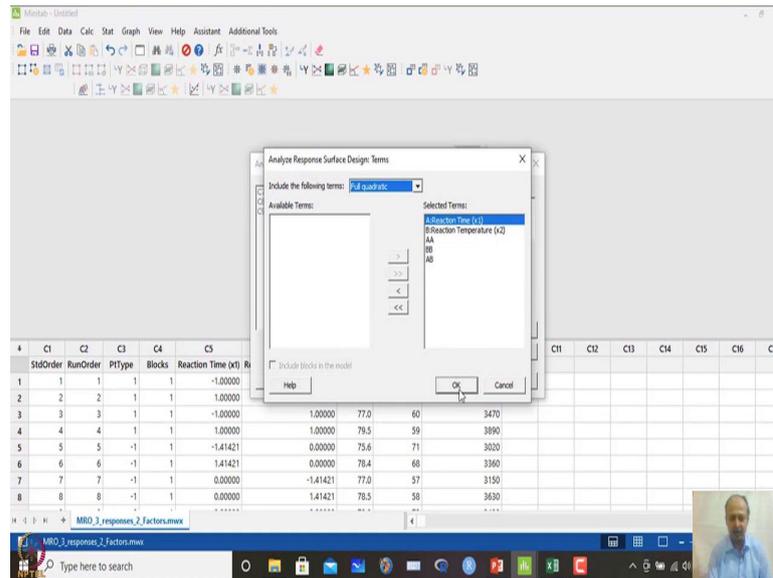
So, what we have to do is that; first we have to generate the response surface over here for y1, y2 and y3. How do I do that? Design of experiments; response surface and then analyze response surface over here.

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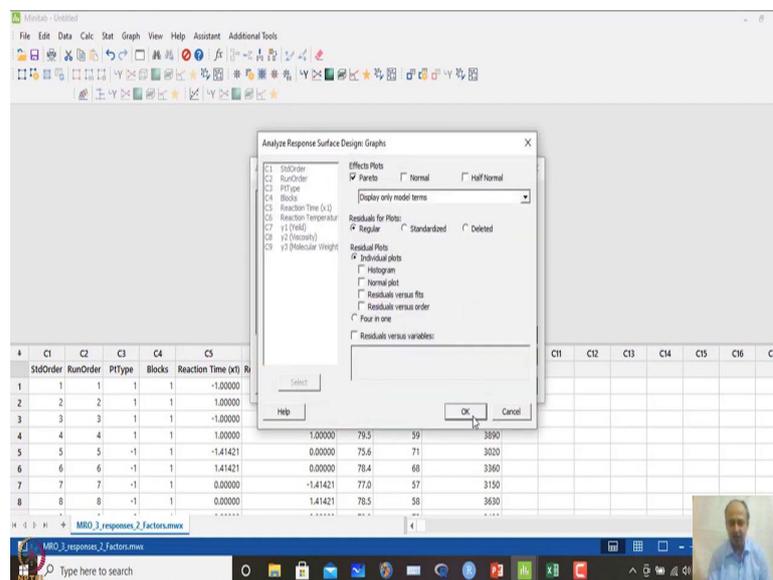
Then, you mentioned that I want to see y1 yield; I want to develop a response surface for this.

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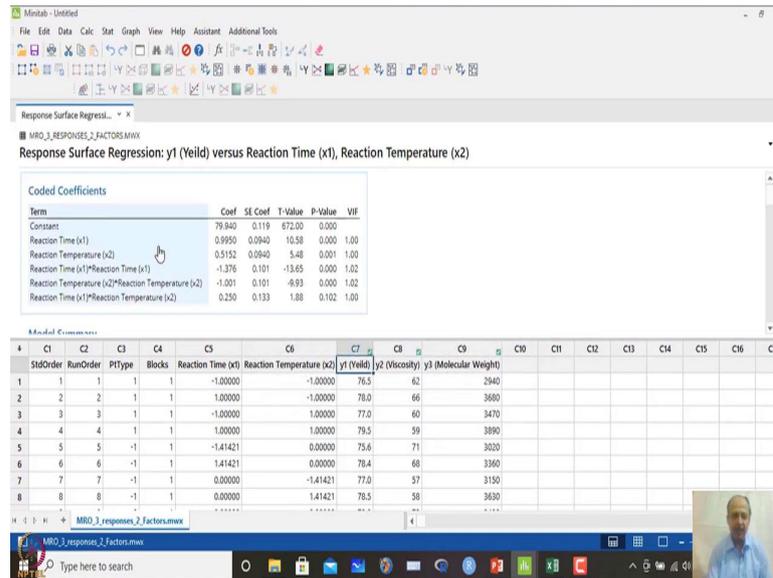


And go to terms because we are using CCD design; we can we can go for full quadratic equations and then we click ok.

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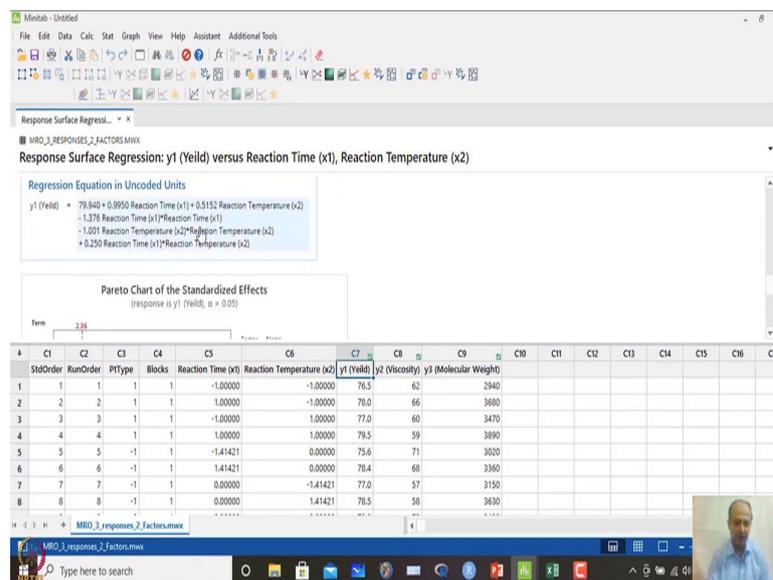


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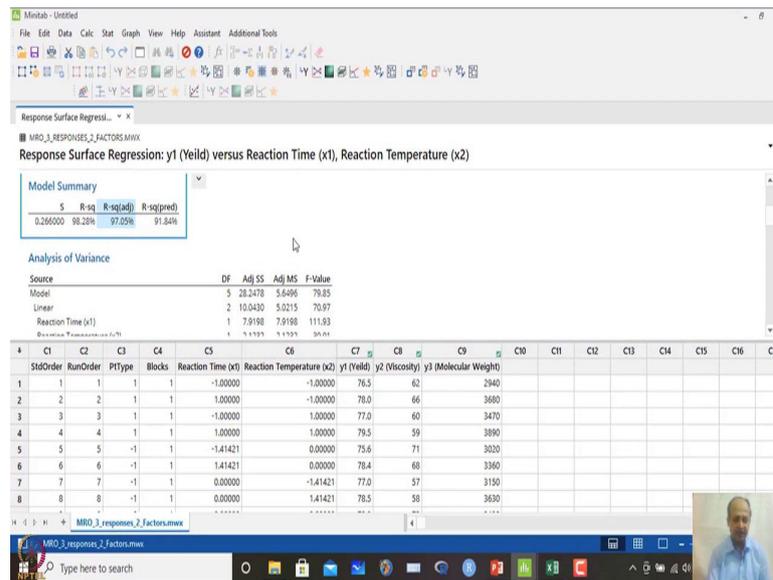


And in graphs, you can see the Pareto plots also and then click and you will get the equations of the final equations for this.

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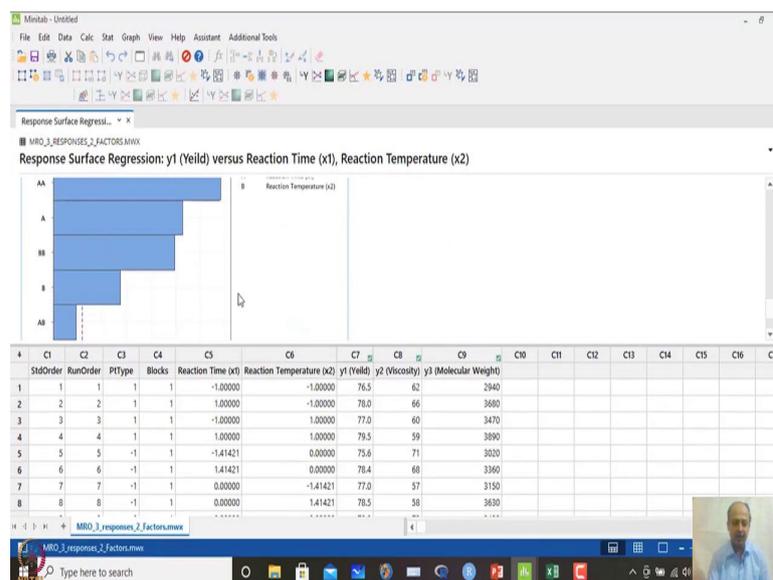
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And the R square value will tell me whether the model is adequate. So, R square 97.05 is adjusted; R square is quite significant, so we can retain the model. So, this is a quadratic model that we are retaining over here.

And in this slide, you can see this is the model for y1; this is the model that is used and MINITAB also generates this model. So, and in that case, also the Pareto plot will show you which is important, which is not.

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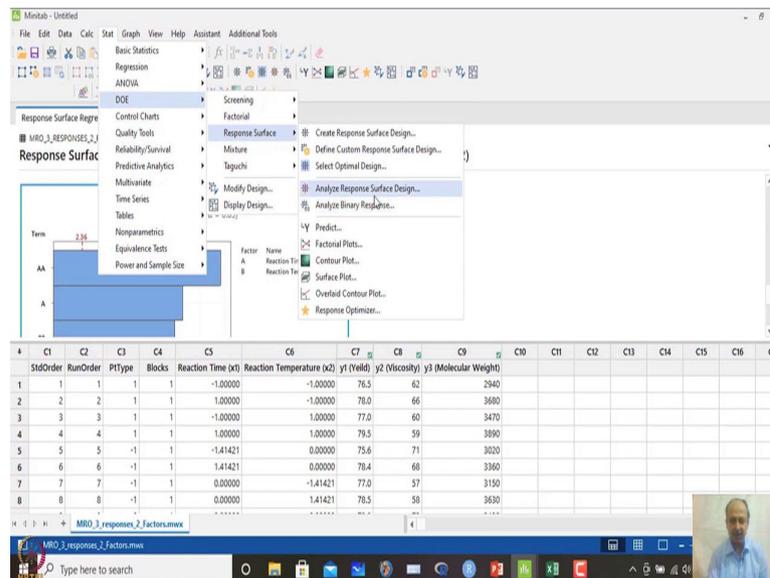


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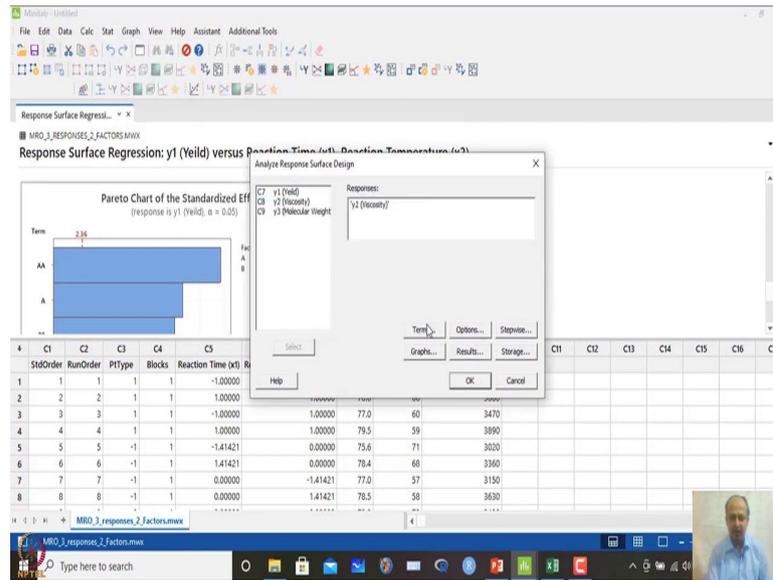
So, in this case we can see that Pareto plots indicates that square; A square is important, A is important, B square is important, B is important. Although maybe AB is not so significant, but we will retain this one because in the book example. It is retained like that, for the response surface; so we are retaining this one.

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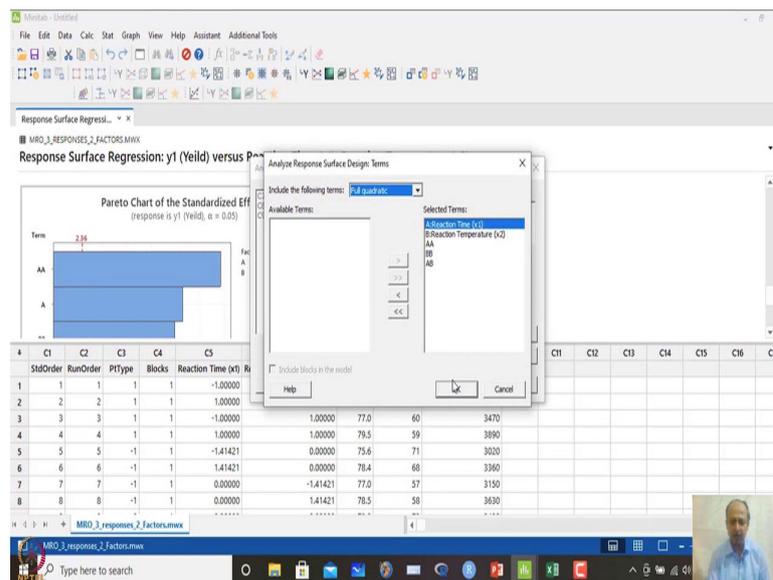


Similarly, for the second y2 variable viscosity, we have to generate the response surface and I am generating the response surface; analyze response surface over here.

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I will take y2 over here and terms again full quadratic model we are using and then click ok.

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Response Surface Regression: y2 (Viscosity) versus Reaction Time (x1), Reaction Temperature (x2)

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	70.00	1.02	68.79	0.000	
Reaction Time (x1)	-0.155	0.004	-0.19	0.852	1.00
Reaction Temperature (x2)	-0.848	0.004	-1.18	0.277	1.00
Reaction Time (x1)*Reaction Time (x1)	-0.687	0.063	-0.80	0.429	1.02
Reaction Temperature (x2)*Reaction Temperature (x2)	-6.688	0.063	-7.75	0.000	1.02
Reaction Time (x1)*Reaction Temperature (x2)	-1.25	1.14	-1.10	0.308	1.00

Model Coefficients

StdOrder	RunOrder	PTType	Blocks	Reaction Time (x1)	Reaction Temperature (x2)	y1 (Yield)	y2 (Viscosity)	y3 (Molecular Weight)
1	1	1	1	-1.00000	-1.00000	76.5	62	2940
2	2	2	1	1.00000	-1.00000	78.0	66	3680
3	3	3	1	-1.00000	1.00000	77.0	60	3470
4	4	4	1	1.00000	1.00000	79.5	59	3890
5	5	5	-1	-1.41421	0.00000	75.6	71	3020
6	6	6	-1	1.41421	0.00000	78.4	68	3360
7	7	7	-1	0.00000	-1.41421	77.0	57	3150
8	8	8	-1	0.00000	1.41421	78.5	58	3630

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Response Surface Regression: y2 (Viscosity) versus Reaction Time (x1), Reaction Temperature (x2)

S	R-sq	R-sq(Adj)	R-sq(Pred)
2.27530	89.67%	82.80%	44.02%

Analysis of Variance

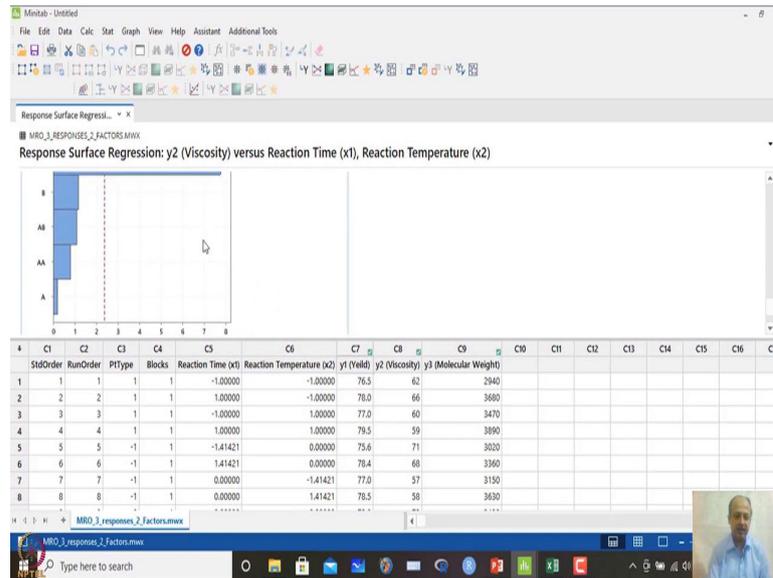
Source	Df	Adj SS	Adj MS	F-Value
Model	5	324.992	64.998	12.56
Linear	2	7.386	3.693	0.71
Reaction Time (x1)	1	0.193	0.193	0.04
Reaction Temperature (x2)	1	7.193	7.193	1.39

Model Coefficients

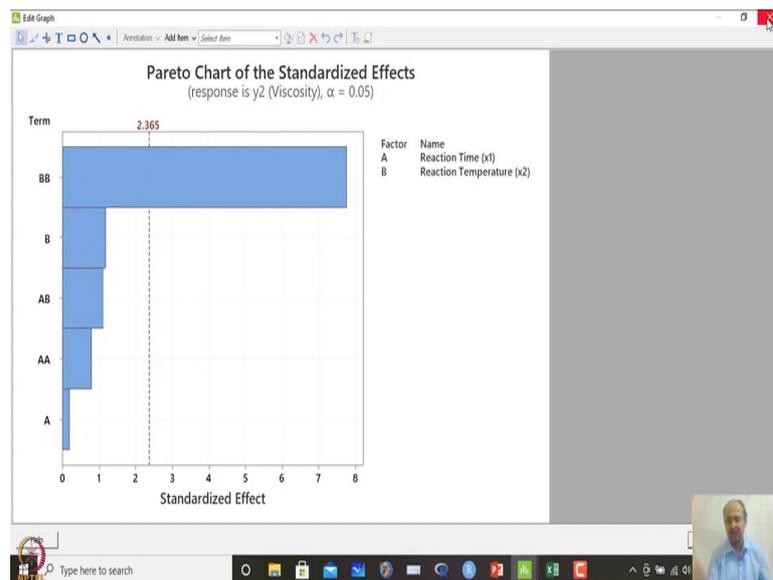
StdOrder	RunOrder	PTType	Blocks	Reaction Time (x1)	Reaction Temperature (x2)	y1 (Yield)	y2 (Viscosity)	y3 (Molecular Weight)
1	1	1	1	-1.00000	-1.00000	76.5	62	2940
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3	3	3	1	-1.00000	1.00000	77.0	60	3470
4	4	4	1	1.00000	1.00000	79.5	59	3890
5	5	5	-1	-1.41421	0.00000	75.6	71	3020
6	6	6	-1	1.41421	0.00000	78.4	68	3360
7	7	7	-1	0.00000	-1.41421	77.0	57	3150
8	8	8	-1	0.00000	1.41421	78.5	58	3630

And then again, you will find that R square value; adjusted value is 82.8 ok; we can retain this one.

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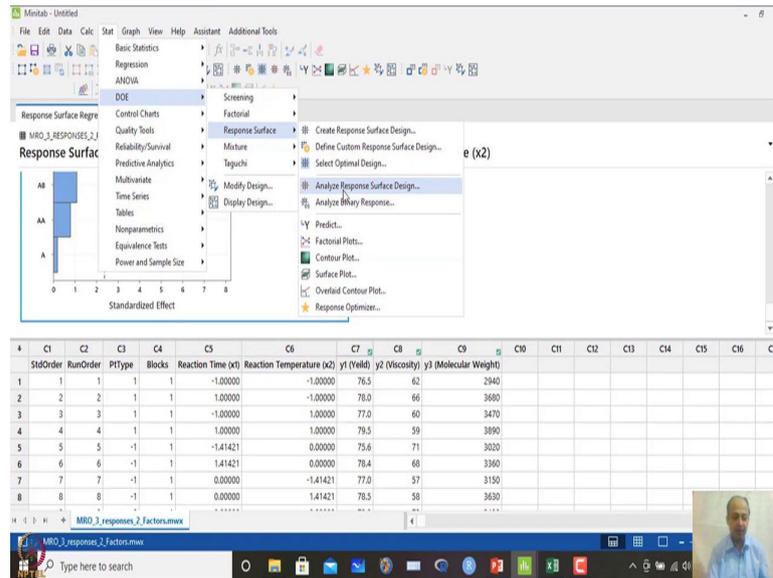


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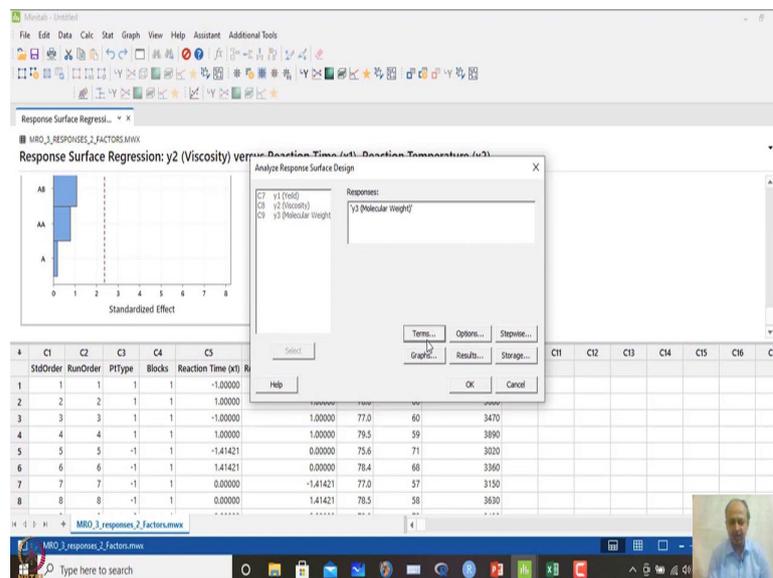
And then in that case also Pareto over here; what it shows is that, this Pareto plot says only B square is important, but because then we have to retain B and we are retaining all the variables. Because in the in the example, this Montgomery's example this equation was retained like that ok. We can change the equations also, but I am retaining this one; full quadratic model we are retaining over here.

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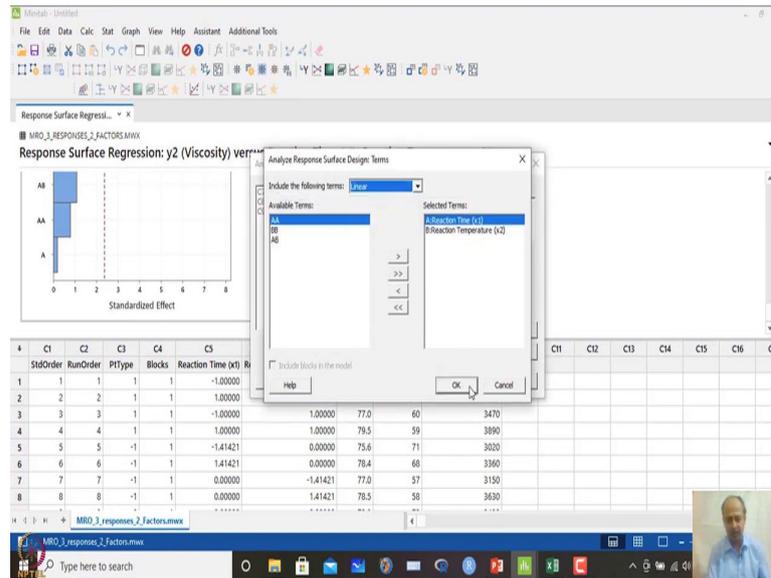
So, then the third one is the molecular weight; so I am going to response surface.

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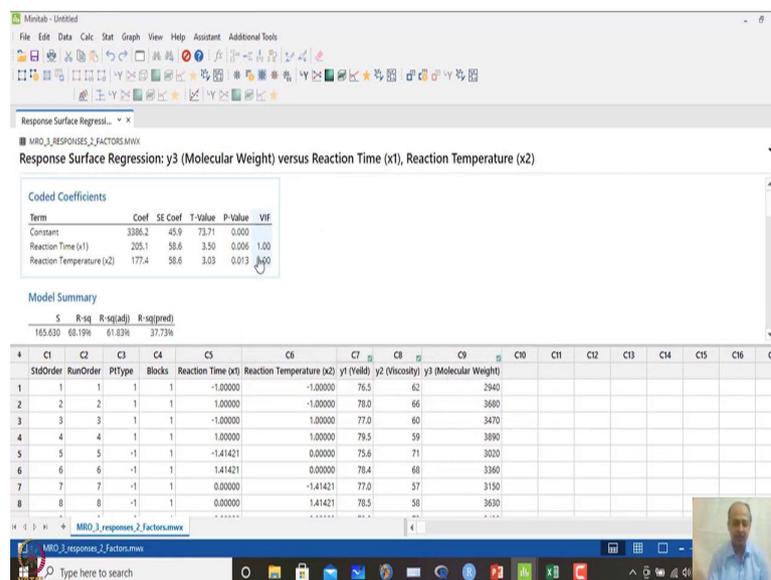
And I am using what Montgomery has recommended the response surface that is that that equation that was taken.

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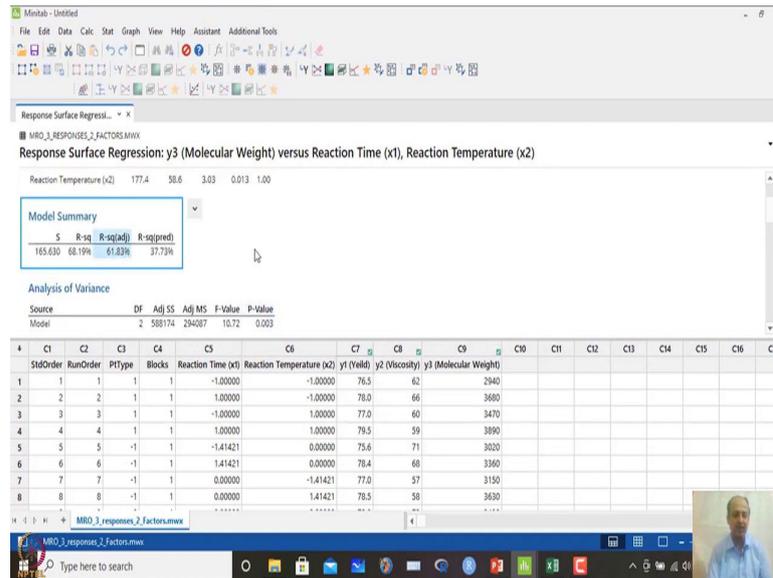


So, y_3 I am taking over here; then the term condition what we will so, we will only use the linear model over here because in the book, it is linear model only that is considered; when you consider only x_1 and x_2 and no interaction.

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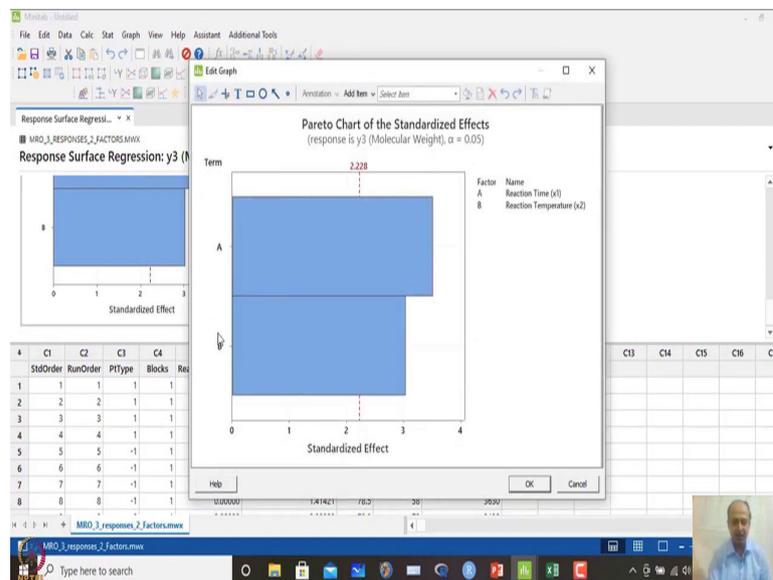


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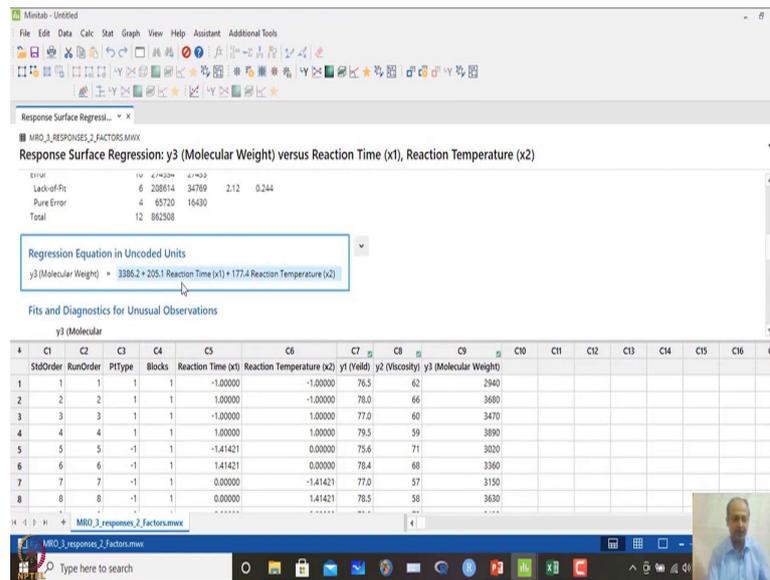


So, that is the model we are using over here. So, this is around 61 percent explained variability and this you can see that all the variables are important A and B is important over here.

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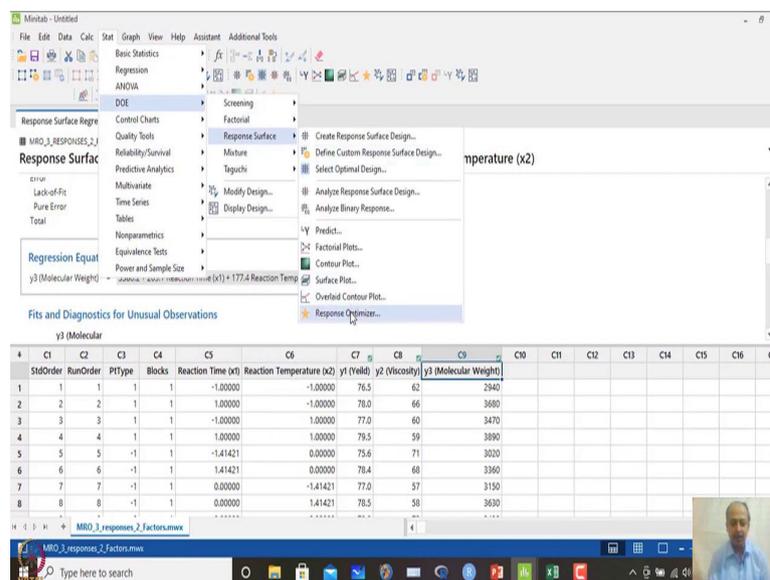


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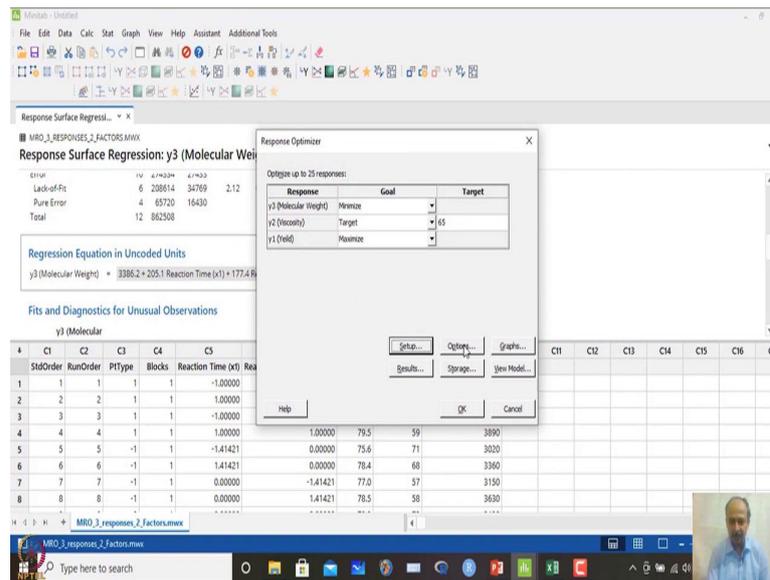
And the final equation is given over here; so this is y3. So, what we have done is that; we have developed the response surface for y1, y2, and y3 and now we can we can just optimize using response optimizer of MINITAB.

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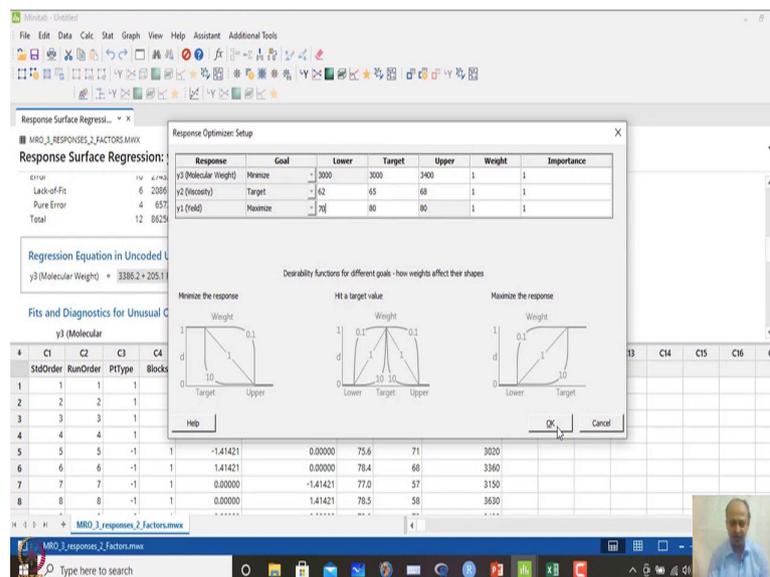
So, what we will do is that now we will go to stat and then we will go to DOE response surface; go to response optimizer over here.

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When you go there, what it will ask is that it can optimize up to 25 response. So, how do you like to optimize these three variables three responses? So, y3 is molecular weight we want to minimize this one; y2 is viscosity. So, should be on the target and the target value is 65; that we know and yield we have to maximize. So we are just mentioning that one.

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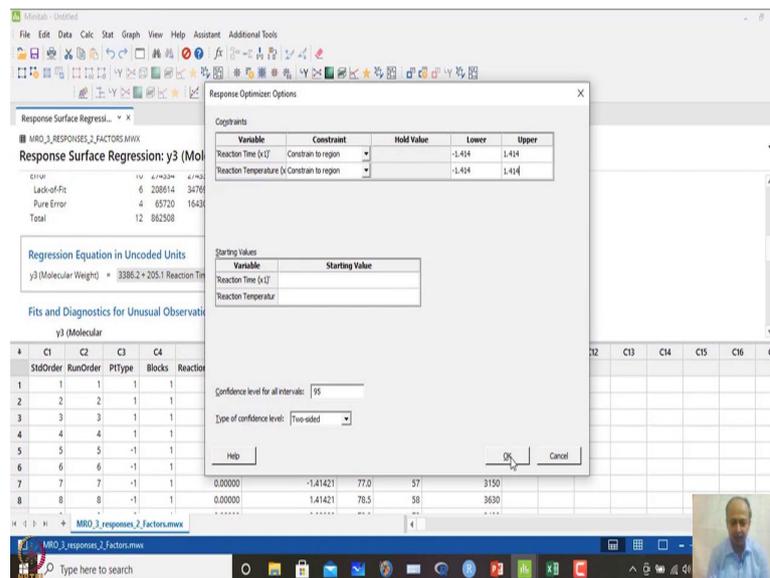
In setup what we will do is that molecular weights we can take; let us say the target value is around this is approximately 3200. What we have written, but we can mention

something lower than that also, it does not matter like that. So, we have to minimize this one; upper bound that is given is 3400 like that; so this is taken like that.

And then the target value of y_2 viscosity is 62 to this is taken as 68 and this is 65; that is taken as correctly and yield has to be maximized. So, target value is 80 over here and this varies from 70 to 80 like that we have taken like that. And the weights for is; that means, what is the importance of this variable y_1, y_2 ; we can keep equal weights to this weights will be equals to 1 and we can change the desirability function over here.

So, in this case; we can change the shape of the desirability function, we are not doing that. So, importance matrix is also taken as 1 like that; we can keep it same or linear desirability function we are keeping over here.

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So, in this case we click and then in options what we can do is that; you can give restrictions to the constraint to the region. So, region is -1.414 that you have defined to +1.414 and similarly for the second one also we are giving a constraint to the region. So, maybe -1.414 to +1.414.

So, solutions can differ because it is using heuristics to solve the problem because there is no single way of solving this problem. So, in this case because there are multiple response over here; so we cannot reach global optimal solutions for all variables and it is not possible. So, in this case we have a tradeoff solution; then for that we are using a

some kind of heuristics to get the solution; MINITAB will use that one internally and but desirability function will be used and geometric mean will be used like that.

So, we will click and let us try to see the solutions like that; this is one example I have taken, you can you can also change this option and change it to like this options. In settings, what you can do is that we have taken 3 to 3000, you can change this to 3200s like that that is also possible and I am just giving a wider range over here for the solutions.

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The screenshot shows the Minitab software interface for Response Optimization. The title bar reads "Minitab - Untitled". The menu bar includes File, Edit, Data, Calc, Stat, Graph, View, Help, Assistant, and Additional Tools. The main window title is "Response Optimization: y3...". Below this, the response variables are listed: "Response Optimization: y3 (Molecular Weight), y2 (Viscosity), y1 (Yield)".

Parameters

Response	Goal	Lower	Target	Upper	Weight	Importance
y3 (Molecular Weight)	Minimum	3000	3400		1	1
y2 (Viscosity)	Target	62	65	68	1	1
y1 (Yield)	Maximum	70	80		1	1

Variable Ranges

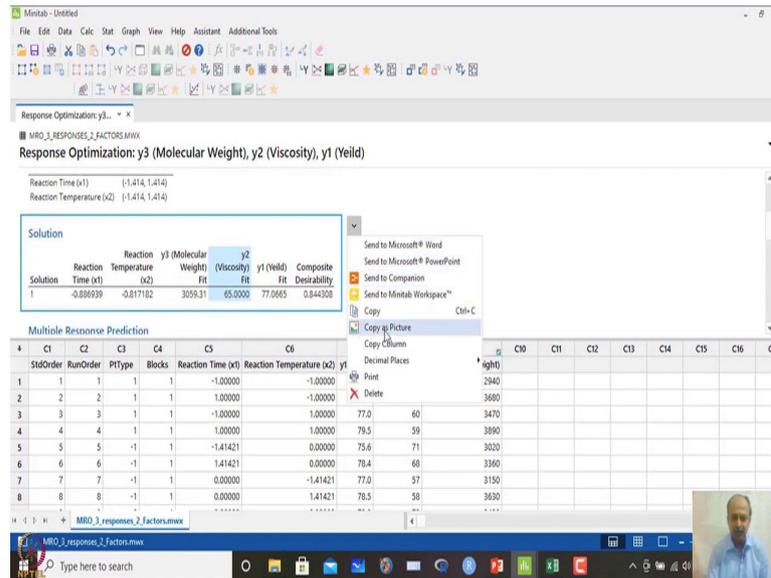
Variable	Values
Reaction Time (x1)	(-1.414, 1.414)

The main data table is as follows:

StdOrder	RunOrder	PTType	Blocks	Reaction Time (x1)	Reaction Temperature (x2)	y1 (Yield)	y2 (Viscosity)	y3 (Molecular Weight)
1	1	1	1	-1.00000	-1.00000	76.5	62	2940
2	2	2	1	1.00000	-1.00000	78.0	66	3680
3	3	3	1	-1.00000	1.00000	77.0	60	3470
4	4	4	1	1.00000	1.00000	79.5	59	3890
5	5	5	-1	-1.41421	0.00000	75.6	71	3020
6	6	6	-1	1.41421	0.00000	78.4	68	3380
7	7	7	-1	0.00000	-1.41421	77.0	57	3150
8	8	8	-1	0.00000	1.41421	78.5	58	3630

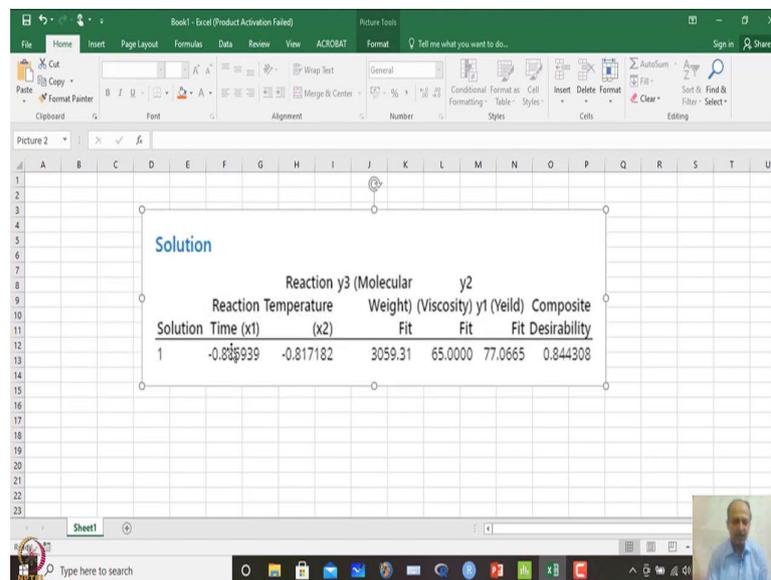
So, in this case what I will do is that I will click ok.

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And when you click ok; what happens let us try to see the solutions over here. So, this is the solutions that we are getting over here. So, I can just copy this as and paste it in excel.

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The solutions; one of the solutions that we have got minute I was derived one of the solutions over here. So, solution is reaction time is minus 8.886 and I am assuming continuity of the variables x1 and x2. So, other one is minus 8.816; this is the coded information; we can convert into two variables that I have told earlier also.

And the molecular weight that we are generating over here is approximately this solution is giving a predicted value of 3059 and y2 is predicted around 65; so it is hitting the target for y2. And y1 is around 78; so we wanted 80 to be maximized, but it has not reached to that point. So there will be a; so all the desirability values of this will not reach to 1.

So, when you take the take the geometric mean over here; the composite desirability is coming out to be 0.84. So, it is lying between 0 to 1 also, higher the better type of scenarios; if composite desirability reaches 1; that means, it will hit all the target values that is defined over here for y1, y2 and y3.

So, then only the composite desirability will be equals to 1. So, it is not the case because y1 it has not reached and also molecular weight, the target value is not reached. So, in this case what composite desirability that we are getting is 844 like that. So, this is what we wanted to explain in this in this session on multiple response optimization. So, you can change the equations; you can change the equations and you can you can just derive the solutions like that.

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Quality Control and Improvement using MINTAB

Regression Equation

$y_2 = 70.00 - 0.155 x_1 - 0.948 x_2 - 0.687 x_1 * x_1 - 6.687 x_2 * x_2 - 1.25 x_1 * x_2$

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.27530	89.97%	82.80%	44.02%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	70.00	1.02	68.79	0.000	
x1	-0.155	0.804	-0.19	0.852	1.00
x2	-0.948	0.804	-1.18	0.277	1.00
x1*x1	-0.687	0.863	-0.80	0.452	1.02
x2*x2	-6.687	0.863	-7.75	0.000	1.02
x1*x2	-1.25	1.14	-1.10	0.308	1.00

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	324.992	64.998	12.56	0.002
x1	1	0.193	0.193	0.04	0.852
x2	1	7.193	7.193	1.39	0.277
x1*x1	1	3.288	3.288	0.64	0.452
x2*x2	1	311.114	311.114	60.10	0.000
x1*x2	1	6.250	6.250	1.21	0.308
Error	7	36.239	5.177		
Lack-of-Fit	3	26.239	8.746	3.50	0.129
Pure Error	4	10.000	2.500		
Total	12	361.231			





Because response surface changes and in that case algorithm also finds different points and in that case it will be a different solution.

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Quality Control and Improvement using MINITAB

Regression Equation

$$y_3 = 3386.2 + 205.1 x_1 + 177.4 x_2$$

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
165.630	68.19%	61.83%	37.73%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	3386.2	45.9	73.71	0.000	
x1	205.1	58.6	3.50	0.006	1.00
x2	177.4	58.6	3.03	0.013	1.00

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	588174	294087	10.72	0.003
x1	1	336541	336541	12.27	0.006
x2	1	251632	251632	9.17	0.013
Error	10	274334	27433		
Lack-of-Fit	6	208614	34769	2.12	0.244
Pure Error	4	65720	16430		
Total	12	862508			

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So, whichever the more accurate the response surface; more accurate will be the results like that is the thing that we can say.

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Solution

Solution	Reaction y3 (Molecular Weight)		y2 (Viscosity)		Composite Fit Desirability	
	Time (x1)	(x2)	Fit	Fit		
1	-0.104555	-0.928709	3200	65.0000	78.4789	0.946484

Multiple Response Prediction

Variable	Setting
Reaction Time (x1)	-0.104555
Reaction Temperature (x2)	-0.928709

} Confirmatory Trial

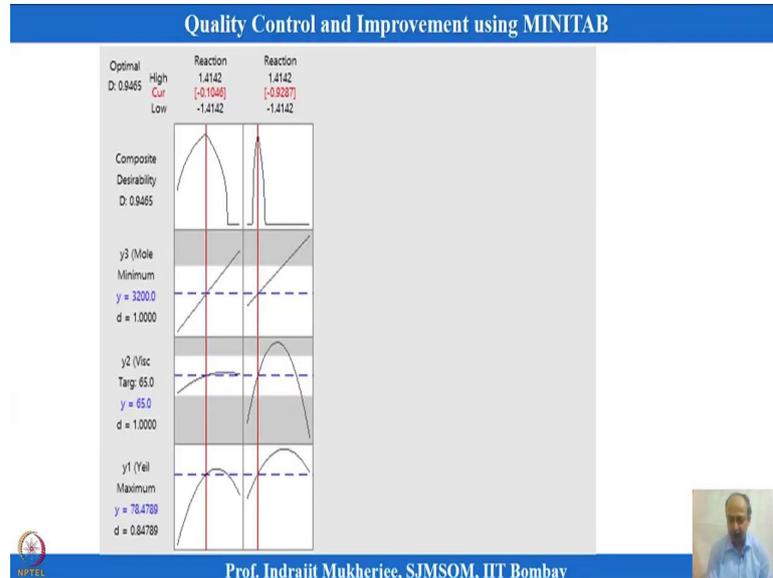
Response	Fit	SE Fit	95% CI	95% PI
y3 (Molecular Weight)	3200.0	71.5	(3040.8, 3359.2)	(2798.1, 3601.9)
y2 (Viscosity)	65.00	1.12	(62.34, 67.66)	(59.00, 71.00)
y1 (Yield)	78.479	0.150	(78.133, 78.825)	(77.695, 79.263)

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And finally, what we can do is that; this is another recommended settings that we are getting in MINITAB over here. And this is giving me a composite desirability of 0.94 like that and so solutions can change also. So, finally, what you have to do is that you have to make a confirmatory trial over here. So, this is the reaction time setting condition

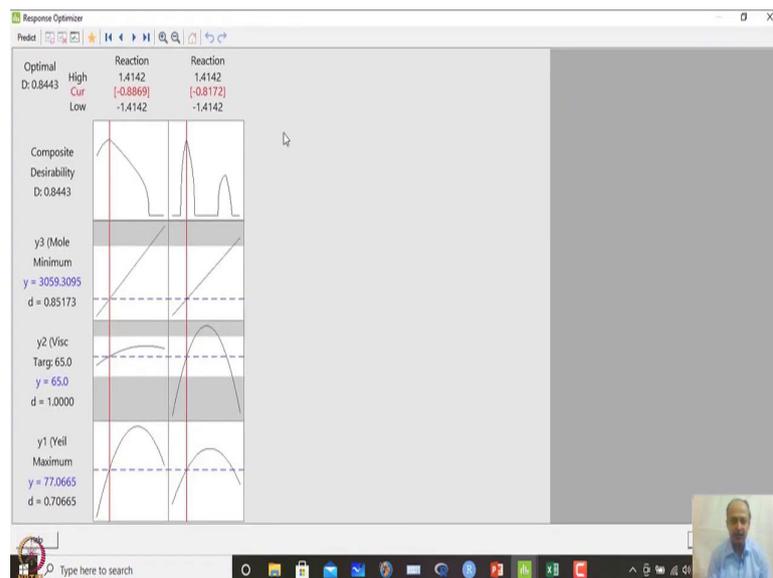
that is MINITAB is predicting; let us let us do trial after that and let us figure out what is the predictor, what is the actual outputs like that.

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So, this is the predicted one; solutions that is predicted values over here. So, actual values can only be seen, when we are doing confirmatory trials like that. So, this is the way we should do and this is the response surface optimizer plot that you can see in MINITAB also.

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So, this is in MINITAB you can see at the end of the when we come down; this is the solutions. So, what you can see; this is the final solutions, current solution minus 8; this one is the value that is that is giving MINITAB.

So, red one you can see this is the solutions like that; composite desirability values are individual desirability values are given an individual values are also mentioned over here. So, this is the response optimizer that diagrammatically, you can also see like that what is the solution. Although, the solution is given over here; this is the solution that is given over here ok.

(Refer Slide Time: 11:57)

Quality Control and Improvement using MINITAB

Fractional Factorial Designs

- Motivation for fractional factorials is obvious; as the **number of factors becomes large** enough to be “interesting”, the size of the designs grows very quickly
- Emphasis is on **factor screening**; efficiently identify the factors with large effects

Why do Fractional Factorial Designs Work?

- The **‘sparsity of effects-principle’**

- There may be lots of factors, but **few are important**
- System is dominated by main effects and low-order interactions

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So, we will move to some different topics which is also important that we have to mention over here is known as fractional factorial design. So, motivation behind this is sparsity of effect principle; that means, if you have n number of factors; many number of factors and you do not want to do full factorial experiments for all the factors because you do not know which is important which is not.

So, to reduce the number of factors what we do is that; we do screening, screening of the factors in design of experiments. Before we go for optimization, what we do is that; we do screening of the factors. There can be n number of factors, there are seven factors; all may not be influencing like that, there can be 10 factors, 20 factors ok.

So, what we have to do is that we have to reduce the number of factors; based on the importance like that. So, there is an important principle that I have earlier mentioned also; sparsity of effect principle that there are lots of factors, but very few are important over here.

So, system is dominated by the main effects and lower order interactions like that. So, maybe if you have factor A and B like that; these main effects are important and maybe AB interactions. So, if you are let us say A, B, C over here; so this is the first order interactions over here and then we can also see interactions third-order interactions ABC.

So, in this case what happens is that more and more you go for higher-order interaction; they become insignificant basically; so that is known as sparsity of effects. So, individual factors are important sometimes and second-order; up to second-order interactions can be seen like that and third-order interactions. This may be insignificant like that we try to afford that one. So, this can be a drop down many other.

So, screening experimentation is and out of these factors; all may not be important like that; so we can screen maybe only one factor is important over here. So, this will help me in the final experimentation. So, sequentially what we can do? Take three or four factors, see which is important then think another sequence, another experimentation and we do and we screen the factors like that.

So, we do not go all at a time; we may be we do sequentially and then we try to extract the information which is required; that means, which are the factors are important like that ok.

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Quality Control and Improvement using MINITAB

One Half (1/2) Fraction of the 2^k Design 2^k

Consider a situation where 3 factors are 2 levels are of interest, but the experimenter can not afford to run all $2^3=8$ treatment combinations. The experimenter can only afford to run 4 experiments.

Treatment Combination	I	A	B	C	AB	AC	BC	ABC
a	+	+	+	-	-	-	+	(++)
b	+	+	-	-	-	+	-	(+-)
c	+	-	-	+	+	-	-	(-+)
abc	+	-	+	+	+	+	+	(+++)
ab	+	+	+	-	+	-	-	-
ac	+	+	-	+	-	+	-	-
bc	+	-	+	+	-	-	+	-
(1)	+	-	-	-	+	+	+	-

Note that $I=ABC$ (Design Generator)-Defining Relationship

$2^3 = 8$
 $2^{3-1} = 2^2$





So, how do we do that basically? This is fractional factorial; name is fractional factorial design and the complete experimental if you have k factors; we told that it is 2 to the power k design like that. But we are not interested in running the full experimental trial let us say, we want to reduce we want to see that which factor is important.

So, in that case in fractional factorial will run only. So, there are let us say 2 cube designs; so this is three factors A,B,C. So, if you are doing full factorial design; this is 2 cube over here, that is 8 experimental trial. But as a researcher I am not interested or as an experimenter.

I am not interested to run the full trial because maybe because of economic constraints, I do not want to do that. I want to figure out which factors are important or not which is; so, I want to run only half of this trial like that. I want to run only half of this trial, this is one half fraction this is known as one half fraction over here.

So, this may be written symbolically; it will be written as 3 minus 1; that means, I will only run a 2 square trials, let us assume like that. So, if you have done full factorial of this; it would have been all combinations would have been like this. But I am only running half of the trials. I do not want to run full trials over here and based on that, I will I will screen the factors; based on that only I will screen the factors like that ok.

So, then you will find that whenever A is; the first trial is given as this trial experiments A is positive, then B is on the lower level, C is on the lower level like that. So, but if you multiply this symbolically A B C; what will happen is that, you will get a ABC over here, that is that is positive over here.

So, similarly when combination is b; only b is at high level and other a and c is a low level, what will happen is that this is the combination that I am running over here. And but if you multiply this A B and C; what will happen is that ABC is positive over here. Again for c; ABC will be positive like that. When ABC; all at high level, what will happen is that; ABC symbolically, if you multiply the plus signs over here, that is also positive like that.

So, all positive signs; if you accumulate; that means, these are the trials you have to run a, b, c and abc; if you run this trial, this; only this block this block is known as the principal block, this is known as the principal block and there is a alternate block over here.

So, either I can run this one or I can run this one like that both you can run any of this; you can run any of this. So, my idea is that I do not want to run the full trial; I am only running the half of the trials like that, half of the experimental trials like this. And I will lose some information; I will always lose some information because I will not be able to calculate all interaction independently like that.

So, in this case some information I will lose , but in this case I am not going for optimization over here; my idea is to screen the factors, to screen the with minimum number of trials like that. So, in this case; what we do is that, so this is known as design generator over here. So, if I have to make it into fractions which fraction we should run and which fraction we can omit like that and for that we use a design generator; here ABC is used as design generator.

So, when I multiply the signs of A B and C and it will give some either plus sign or minus sign. So, one block will be plus sign, one block will be minus sign like that; I can take the first block which is known as principal block and only carry out this trials over here that is combination a, b, c and abc like that.

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Quality Control and Improvement using MINITAB

Notation: because the design has $2^{3/2}$ runs, it's referred to as a 2^{3-1} 2³⁻¹

Treatment Combination	Factorial							
	I	A	B	C	AB	AC	BC	ABC
a	+	+	-	-	-	-	+	+
b	+	-	+	-	-	+	-	+
c	+	-	-	+	+	-	-	+
abc	+	+	+	+	+	+	+	+
ab	+	+	+	-	+	-	-	-
ac	+	+	-	+	-	+	-	-
bc	+	-	+	+	-	-	+	-
(1)	+	-	-	-	+	+	+	-

Appealing because they allow for study of many factors at many levels
 - Reduced number of runs at the cost of confounding effects (aliasing)



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So, and this is the combination that we will run; so this is basically half of the full trials over here 2 to the power k by 2; so 2 to the power k minus 1 that I mentioned. So, this is 2 to the power 3 minus 1 that we are running, this symbolically we write like that ok.

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Quality Control and Improvement using MINITAB

Aliasing in the One-Half Fraction of the 2^3

$A = BC, B = AC, C = AB$
 Aliases can be found from the defining relation $I = ABC$ by multiplication:
 $A.I = A.(ABC) = A^2BC = BC$
 $BI = B(ABC) = AC$
 $CI = C(ABC) = AB$
 Textbook notation for aliased effects:

Treatment Combination	Factorial							
	I	A	B	C	AB	AC	BC	ABC
a	+	+	-	-	-	-	+	+
b	+	-	+	-	-	+	-	+
c	+	-	-	+	+	-	-	+
abc	+	+	+	+	+	+	+	+
ab	+	+	+	-	+	-	-	-
ac	+	+	-	+	-	+	-	-
bc	+	-	+	-	-	-	+	-
(1)	+	-	-	-	+	+	+	-

Principal Fraction

Alternate Fraction

$[A] \rightarrow \check{A} + \check{BC}$ $[B] \rightarrow \check{B} + \check{AC}$ $[C] \rightarrow \check{C} + \check{AB}$



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So, when you run this; there will be some confounding, there will be some confounding in the effects. That means, when I am estimating A basically, I am not estimating alone A because there will be some alias; that means, confounded because I have not run the full trial; so this some of the information.

So, over here when I estimate the effect of A; basically I will estimate A plus BC like that. So, this is A plus BC I will estimate; so how it is coming? So, A if you multiply with ABC; what happens is that, this is A square and BC. So, I can I can figure out; this is the main effect A with which it is confounded with; that means, it is together with another interaction effects.

So, when I am estimating A; basically I am estimating A plus BC over here, when I am estimating B; I am estimating basically B plus AC over here, when I am estimating C; I am estimating C plus AB over here. So, this is known as confounding that will happen inevitable in case of fractional factorial design; that means, some information independent, independently I cannot calculate BC interaction effect.

So, this will be inside when I am calculating the effects of A like that. So, this is confounded over here A and BC is confounded and B and AC is confounded; like this C and AB will be confounded. And I told, this is the principal block that I am running all plus signs is a principal block and this is the alternate fractions that you can run also. So, either I run principal or I run the alternate fraction; it does not matter like that ok.

So, alias structure will be shown by MINITAB that whenever you select a design like that; a fractional factorial design, it will show like that. And so, we will have this confounding effects that I told; that means, there will be some effects which will be when I estimate one; it is not estimating that one, it is estimating something along with that some other interactions like that.

When I am calculating main effects; I am from the four experimental trials I am doing. So, I am estimating a from over here; it is not a; it is a plus bc that I am estimating basically. So, this is also estimated when I am estimating a, is effect basically ok. So, that is the idea; so some all information, but if you run the full trial what will happen is that; I can independently estimate a, I can independently estimate bc also.

So, that is the idea that goes and that is known as alias structure over here what you see and any time will give you automatically which is aliased with which one like that which is confounded with each other interaction effects or main effects like that that will be shown.

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Quality Control and Improvement using MINITAB

A chemical product is produced in a pressure vessel. A factorial experiment is carried out in the pilot plant to study the factors thought to influence the filtration rate of this product. The four factors are temperature (A), pressure (B), concentration of formaldehyde (C), and stirring rate (D). Each factor is present at two levels. The design matrix and the response data obtained from a single replicate of the 2^4 design are:

$2^4 = 16$

Run Number	A	B	C	D	Run Label	Filtration Rate
1	-	-	-	-	(1)	45
2	+	-	-	-	a	71
3	-	+	-	-	b	48
4	+	+	-	-	ab	65
5	-	-	+	-	c	68
6	+	-	+	-	ac	60
7	-	+	+	-	bc	80
8	+	+	+	-	abc	65
9	-	-	-	+	d	43
10	+	-	-	+	ad	100
11	-	+	-	+	bd	45
12	+	+	-	+	abd	104
13	-	-	+	+	cd	75
14	+	-	+	+	acd	86
15	-	+	+	+	bcd	70
16	+	+	+	+	abcd	70

Source: Montgomery, D. C. (2004). *Design and analysis of experiments*. John Wiley & Sons

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So, we are taking one of the examples over here where it is a 2 to the power 4; four factor experimentation temperature, pressure, concentration and stirring rate.

So, 2 to the power 4 is 16 trials; complete experimentation over here. So, I do not run full factorial over here; so I will only run half of the fraction over here. So, 2 to the power 4 minus 1 half of the fraction, we want to run over here and which are the runs that we will take? So, that is 1 will be taken, ab will be taken.

So, you can just multiply A, B, C, D over here and principal block will be whenever A, B, C, D multiplication is positive plus 1; so that will be the that will come in the principal block. So, over here minus; this will come in the principal block; here also if you multiply that will be positive, A, B, C, D will be positive. So, this will come, this will come like this you can see which are the trials that will come over here.

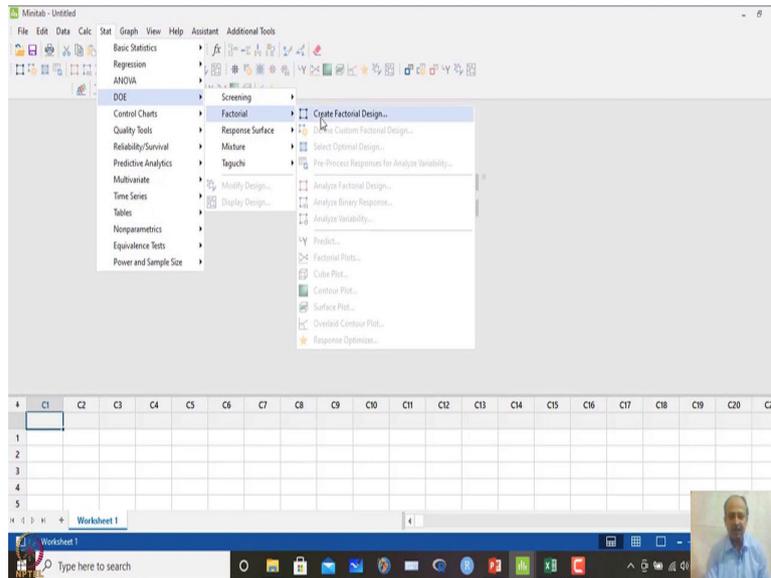
So, out of 16 trials; only this 8 trials if you run, in that case we will be having a having an interpretation that we will have some information that which is basically prominent out of A, B, C and D. And that sparsity of effect principle we are using over here that if you consider all interaction. It is you have to run the full experimental trials like that factorial design like that.

We do not want to do that; we have run half of the fraction and based on that we will just screen the variables over here. So, how do you run the trials? So, we will only run this

one and MINITAB will do it automatically for you, you do not have to multiply anything like that; so, MINITAB will do it.

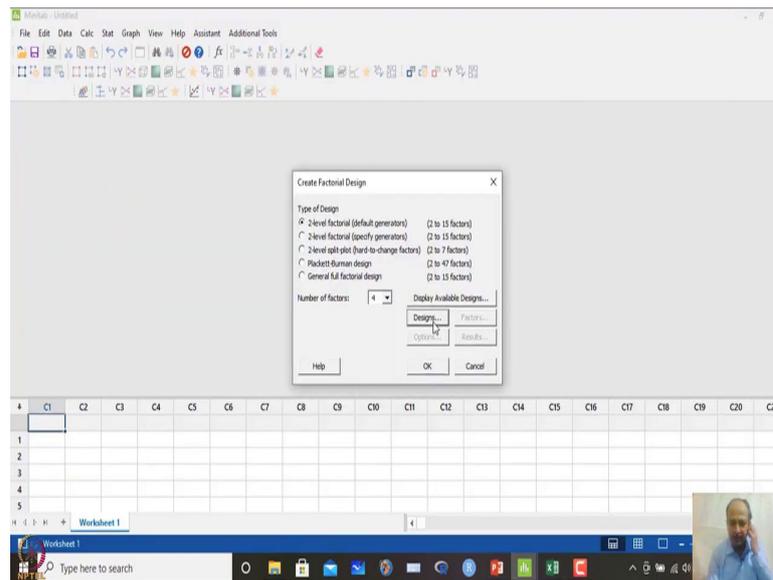
So, this is the experimental run where I want to see that which are the factors affecting the filter rate which is y basically over here and we want to analyze that one. So, (Refer Time: 21:41) see in the experimental design; so I have to create a fractional factorial design.

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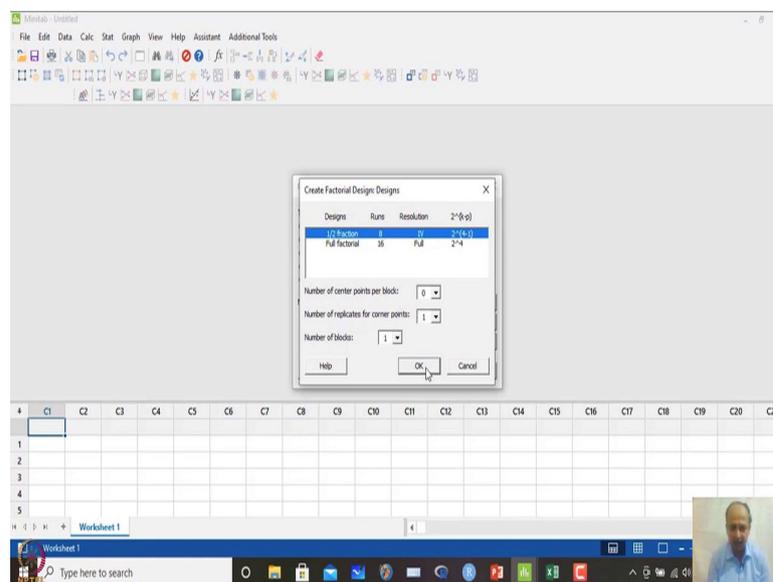
So, what I will do is that stat design of experiment; factorial design, create factorial design.

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So, I will go to create factorial design; 2 level factors, number of factor is 4. Because I have four variables over here that is A B C D and then what I have to do is that 4 factor, then design.

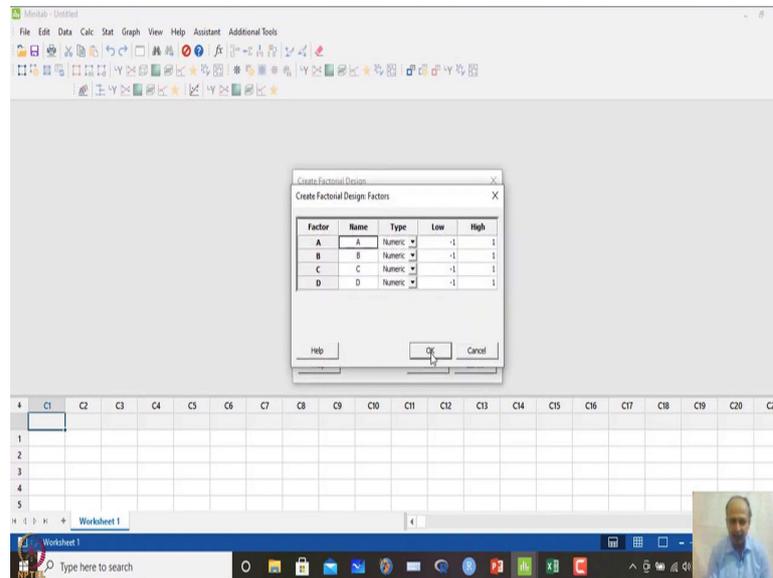
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I will go to design. So, it will say full factorial or fractional factorial. So, I will say half of the fraction 8 trials, I will run over here and it will give you some resolution; resolution IV over here. So this is the resolution IV design basically in fractional factorial. There is a term which is known as resolution of the design ok; so higher the

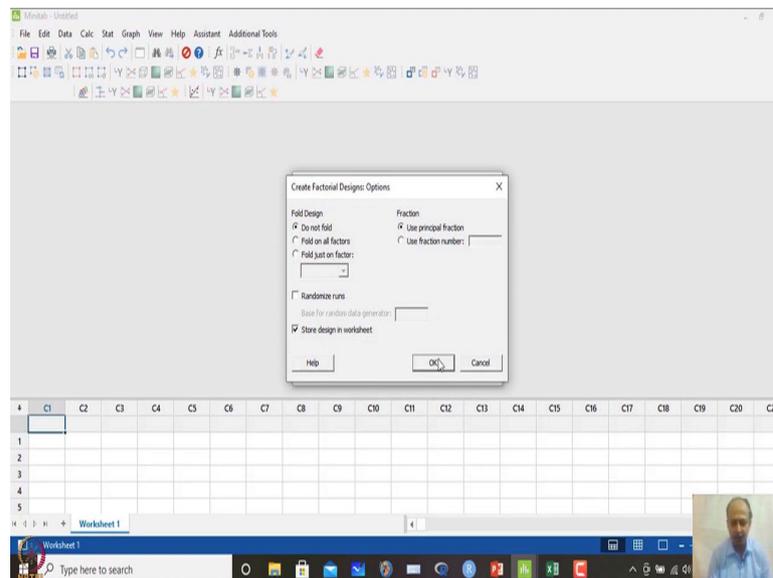
resolution better is the fractional factorial design ok. So, in this case; what I am doing is that number of replicates corner point is nothing over here.

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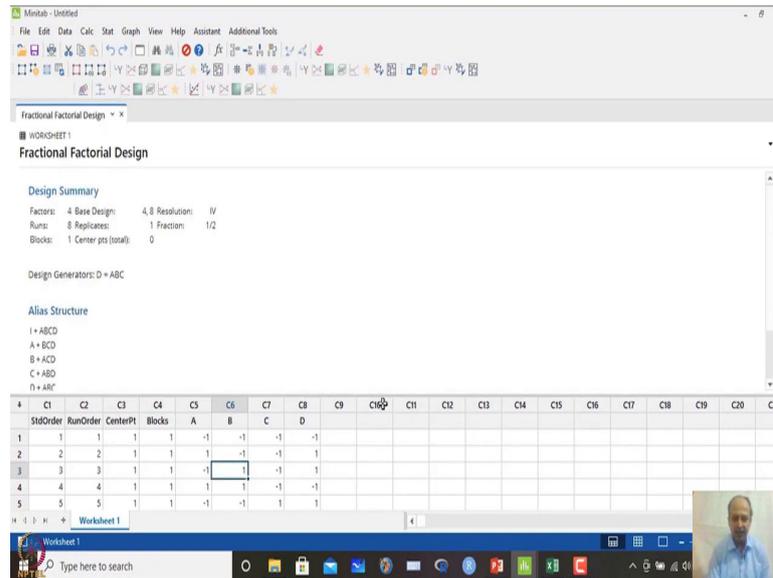
So I will click and then factors; we can add over here, so this is factors we can write.

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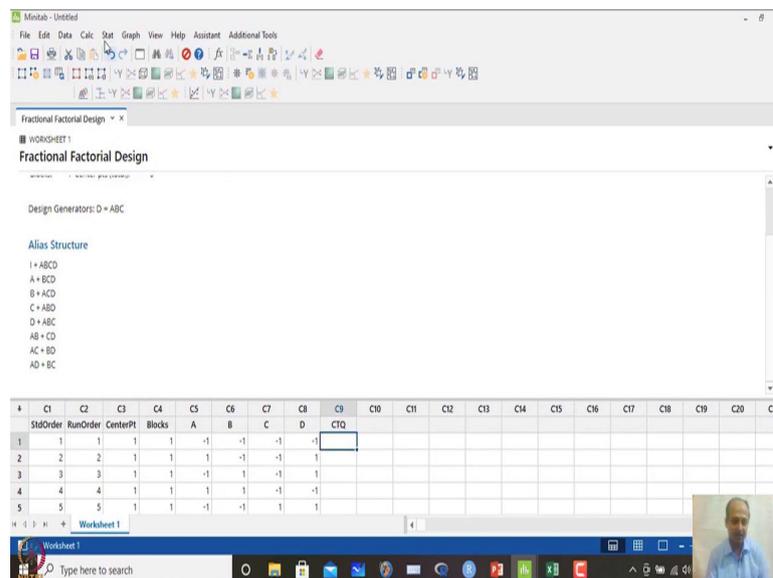
Then in options; what we do not fold; randomize, we do not randomize; use principal block. So, either you can use fractions over here, use principal block; I am using the principal block over here which is all positive that is the principal block.

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So, in options; so then you click ok, the design will be created.

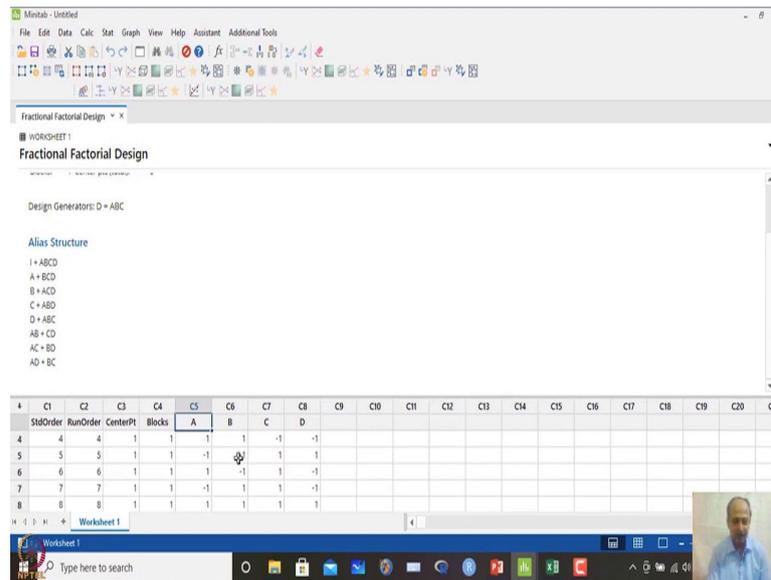
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So, if you see that this is the design and it will say which is aliased with which one like that; this is the alias structure that is given. So, A is confounded with BCD, B is with ACD like that and AB is confounded with CD. So, when you estimate AB; AB interaction, it is actually estimating AB plus CD interaction basically, when you estimate A, it is estimating A plus BCD basically ok.

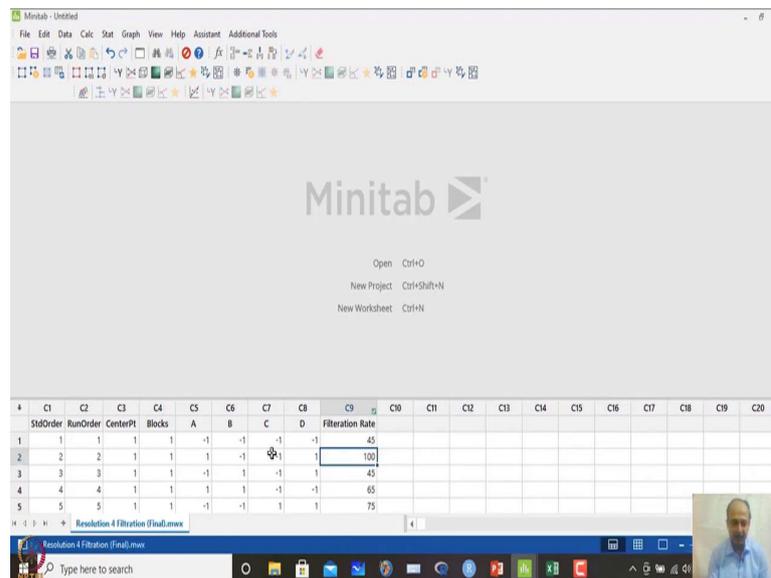
And we told sparsity of effects BCD is may be ignored; so effectively you are making a good judgment and only the estimation will be correct and close to effect of A basically. So, because higher order interaction has little impact in the system; so in that case, we can consider that one as ignoring; we can ignore that one basically ok.

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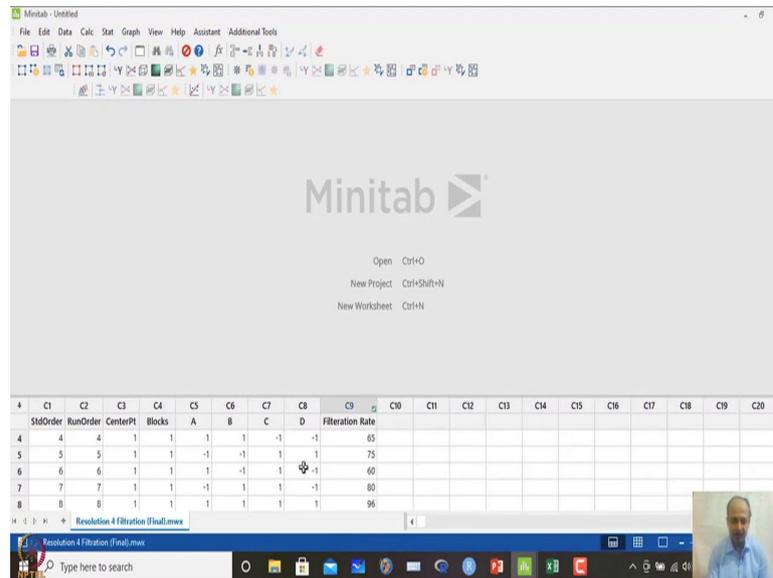
So, when you run this; so 8 trials are given over here, so there is no blocking over here. So, in this case 8 trials and we have got the design. So, I have to response CTQ, you write over here and then run the trial and then run the analyze factorial design like that.

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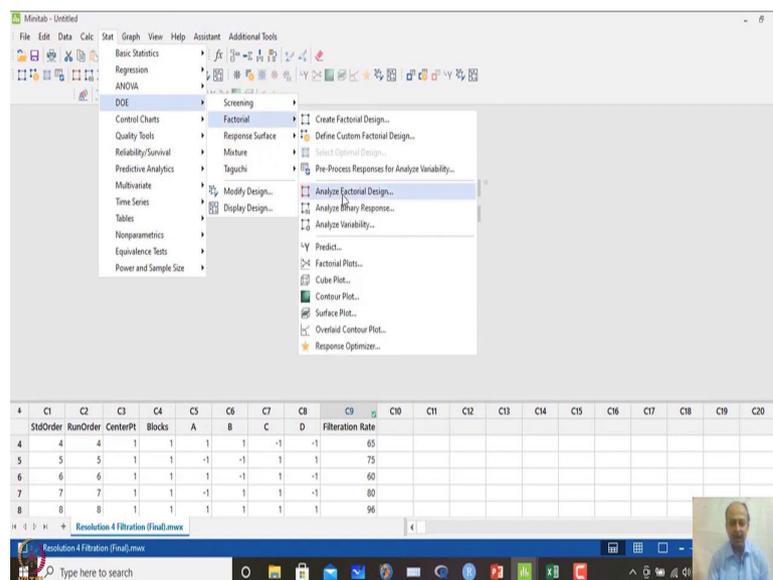
So, this is already created; so if I go to the examples factorial design, it is already created.

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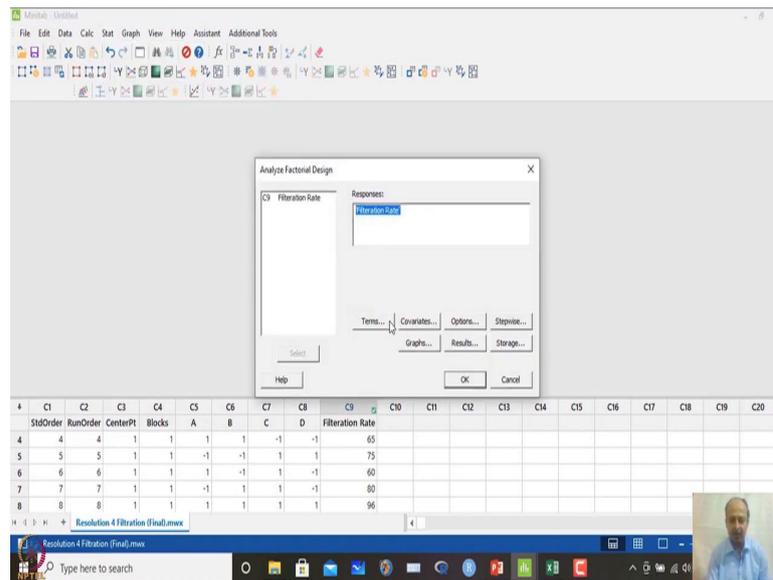
So, in this case filter rate experimentation; so this is the principal block is given over here, so this is the trial.

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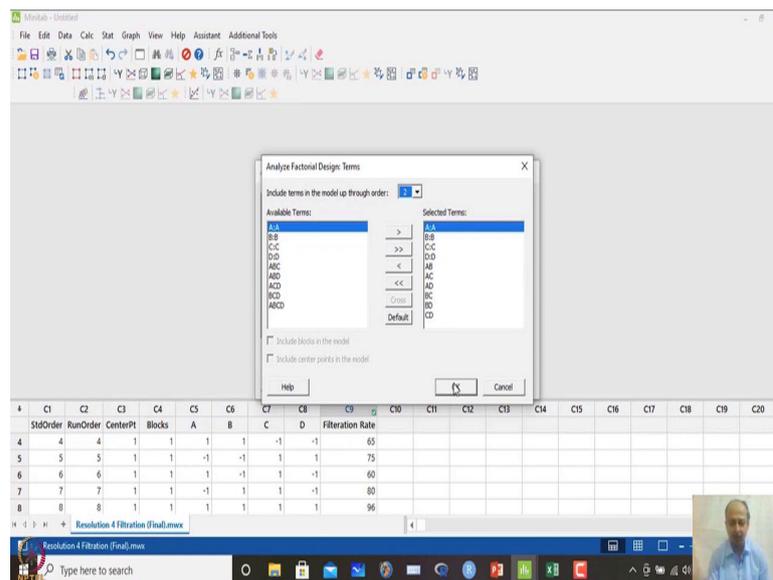


So, how do I analyze this one? I go to stat, DOE, factorial design, analyze factorial design.

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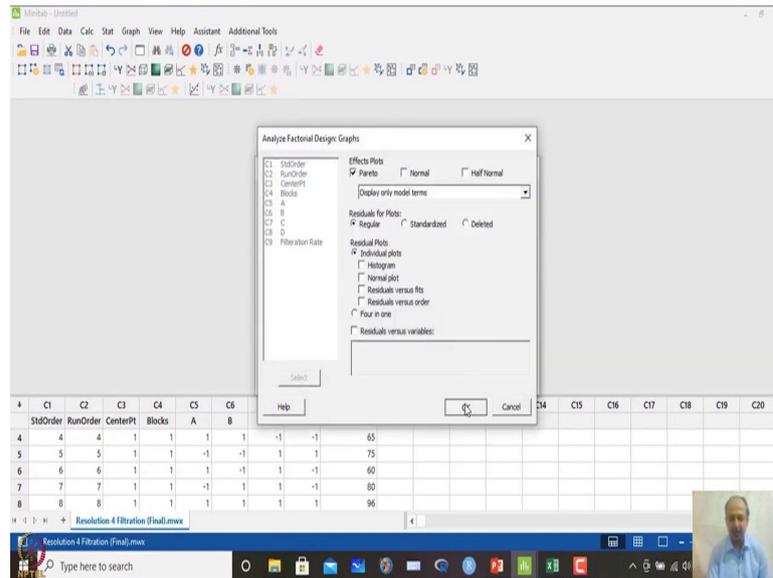


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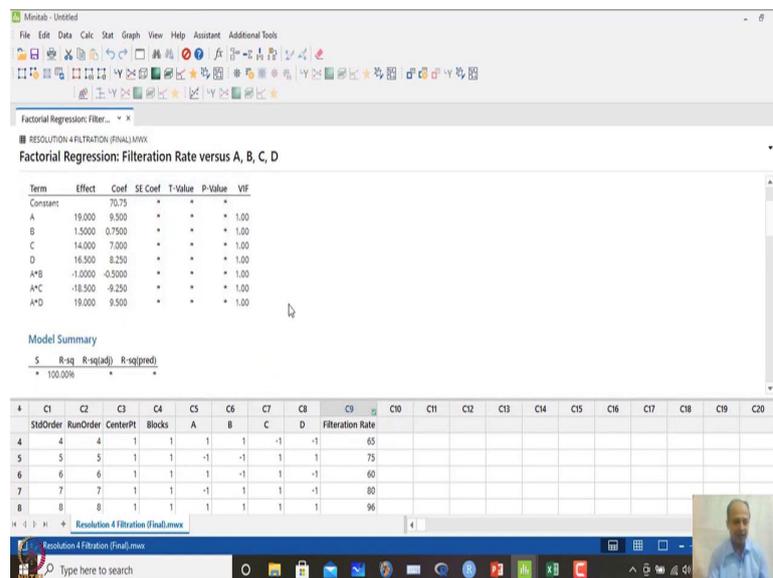
So, you have to write what is the response; then you go to terms and in this case. What we are interested in that; we can go to second order terms all terms, we can see over here.

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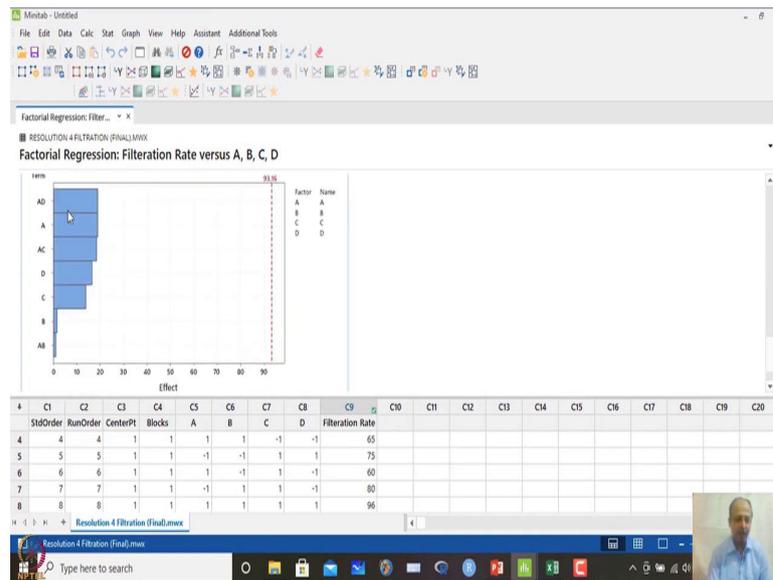
And in graphs, we can see the Pareto plots over here and then click over here.

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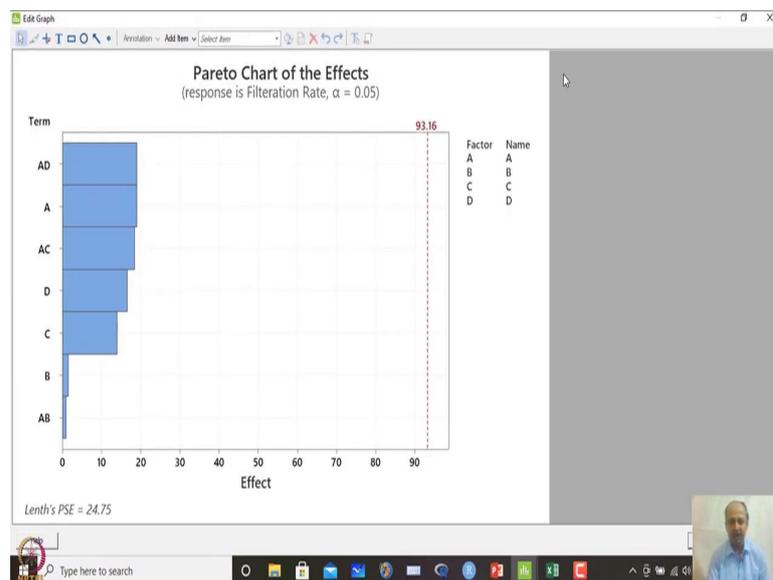
What you see that; estimation is not done because we do not have degree of freedom error over here.

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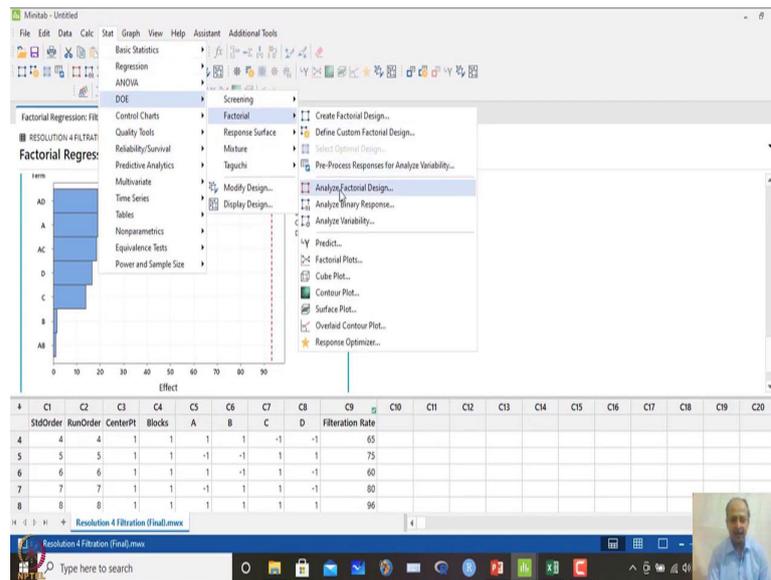
But which is prominent we can see AC D; AC and D is prominent over here and AC and AD is prominent over here. So, B is not prominent, AB interaction is not prominent.

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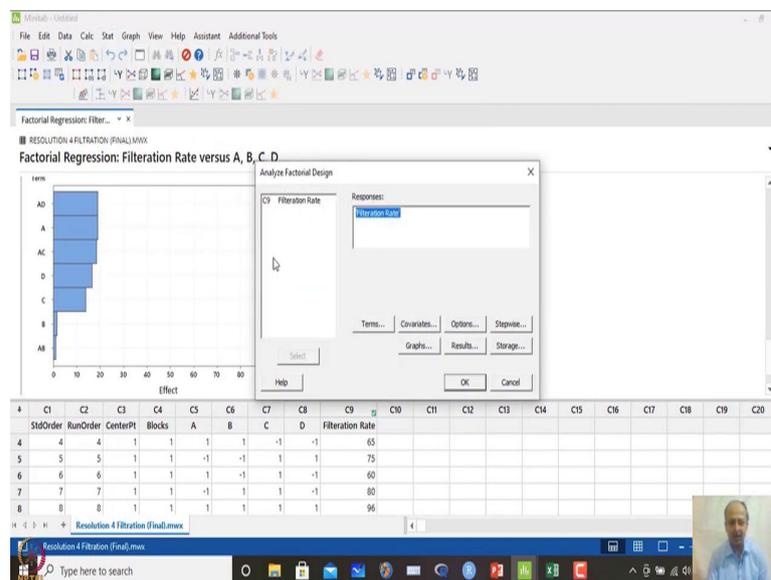
So, what we can do is that; this is the plot what we are seeing. So, initially what we are seeing is that magnitude of AD, A, AC and D and C is quite prominent. So, we will keep only these variables in our next analysis.

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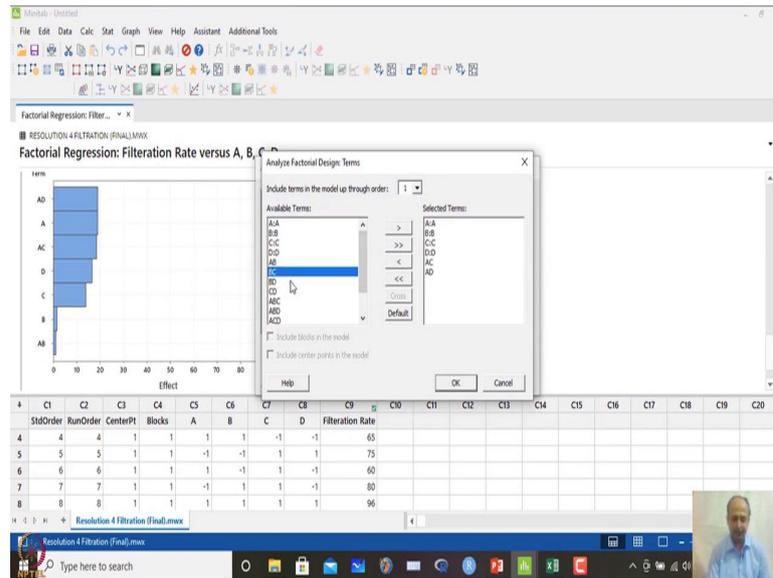


So, what we will do is that; go to stat, design a factorial design; analyze factorial design over here.

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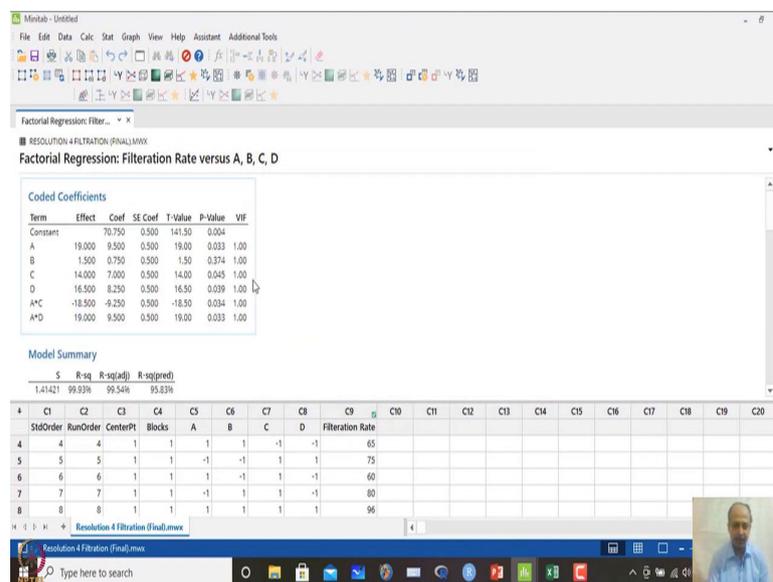


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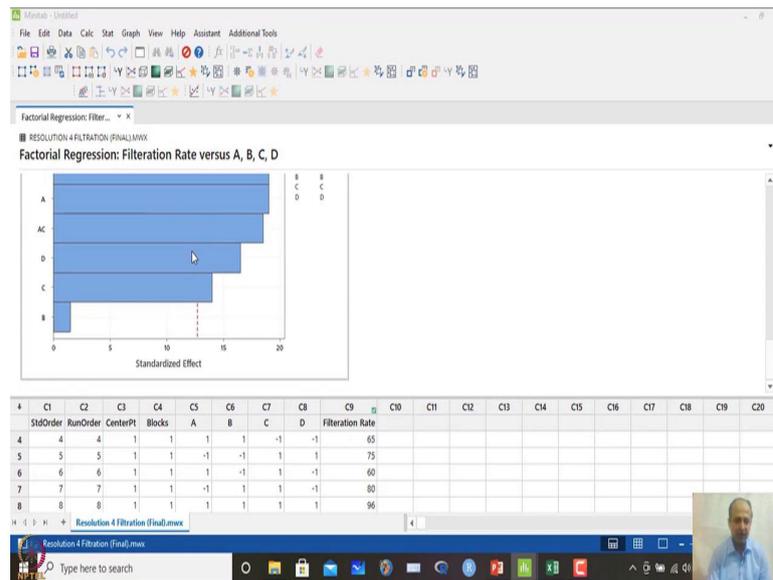


And in terms; what we will do is that, we will go to first order interactions and first order factor effects and then AC: we will put, AD; we will put, AC, AD. We will put and based on that we will see AC and AD. So, we will click and we will click ok.

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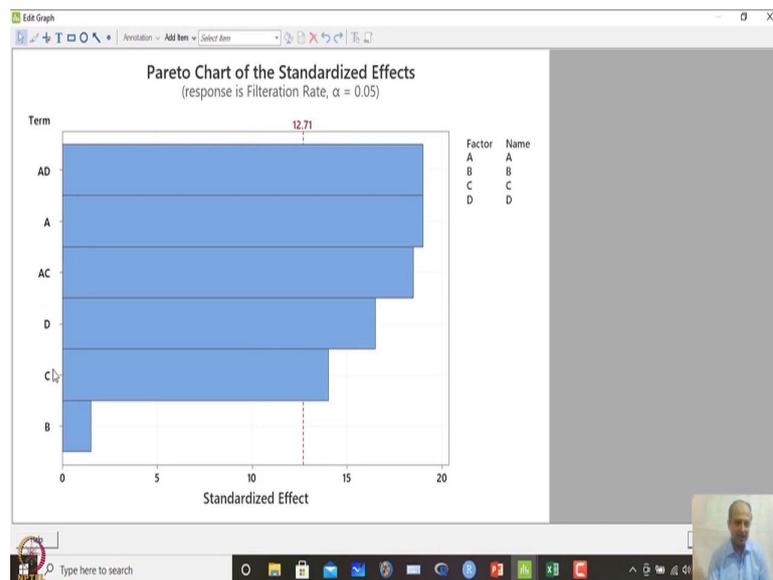


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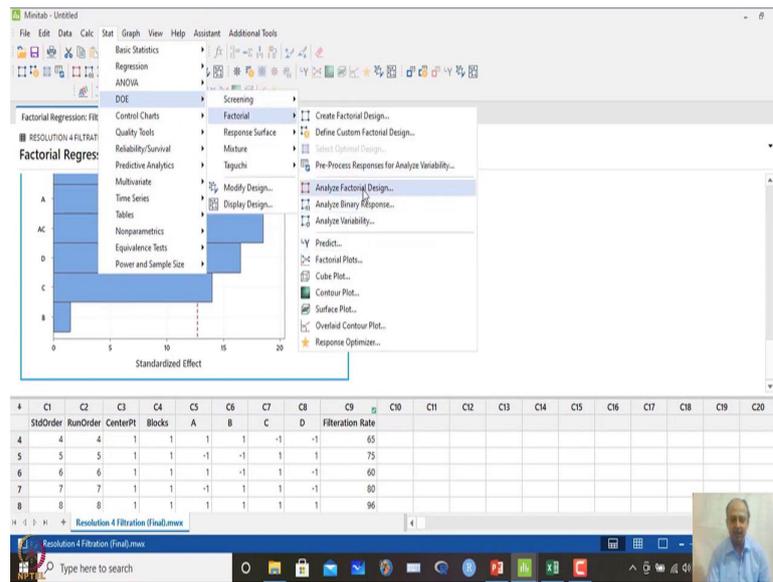
And then we can see the estimations over here.

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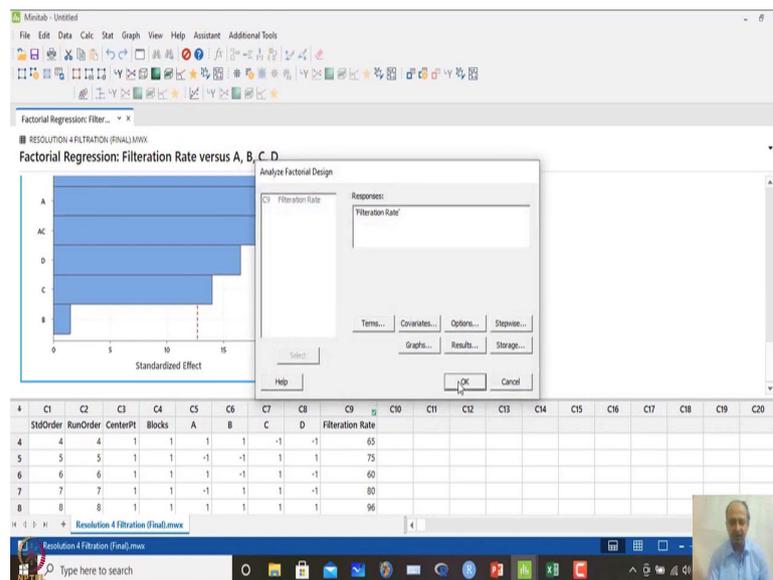


And what we see is that B is not coming out to be prominent; so we can eliminate B like that, so B is very very insignificant. So, what is important over here from this analysis? So, what we are seeing is that A AD, AC, D, C and only B, we can eliminate.

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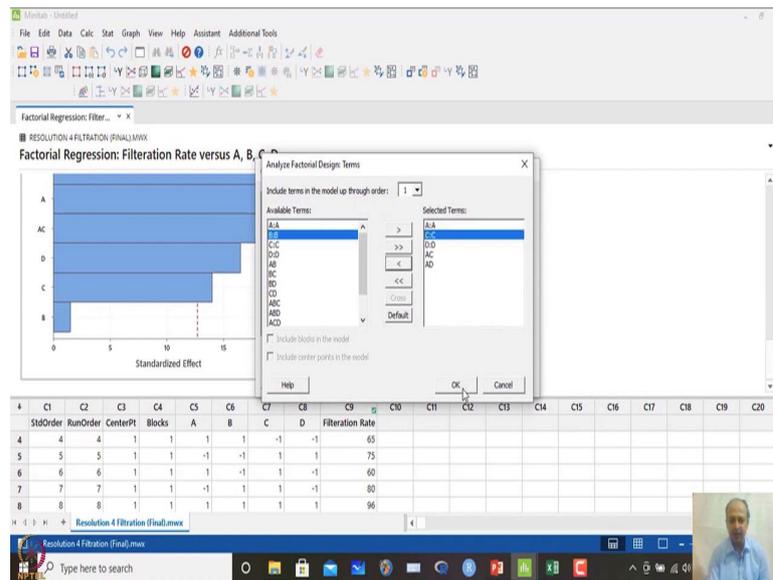


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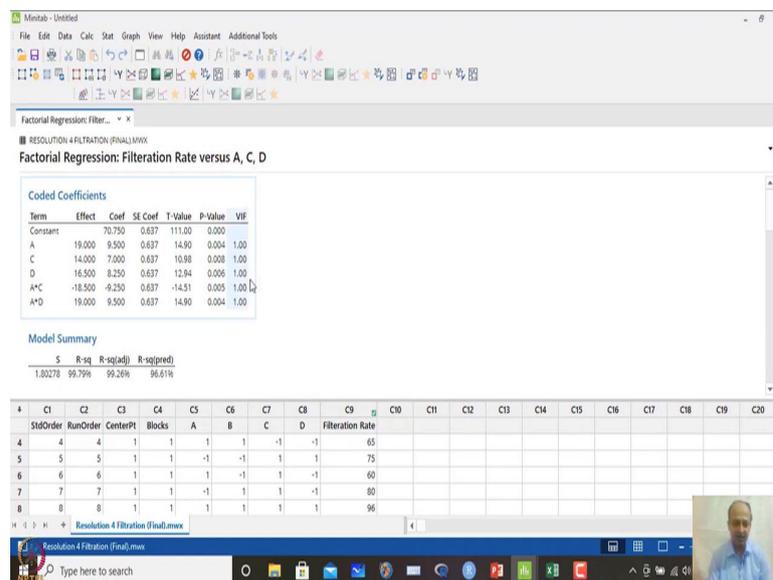
So, next we what we can do is that; we can eliminate this one factorial design and then we can analyze.

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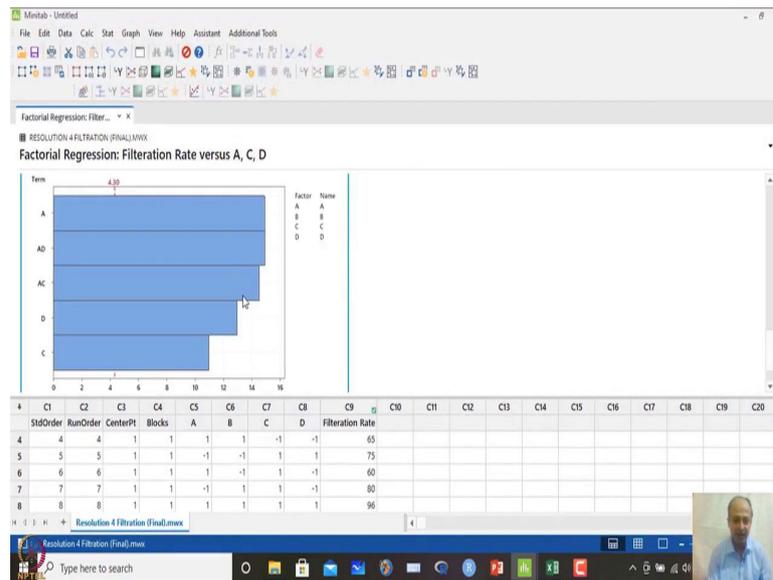
And then in terms of we have to remove; let us say B, I will remove from here; only A, C and D is important and their interactions.

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So, in this case the model explains R square value is around 99.26; that is very good; that means, other terms. Even if you ignored all other terms like second order, third order; all other interactions, higher order interactions like that; I am getting a model which is having a predictivity around 99.26 like that.

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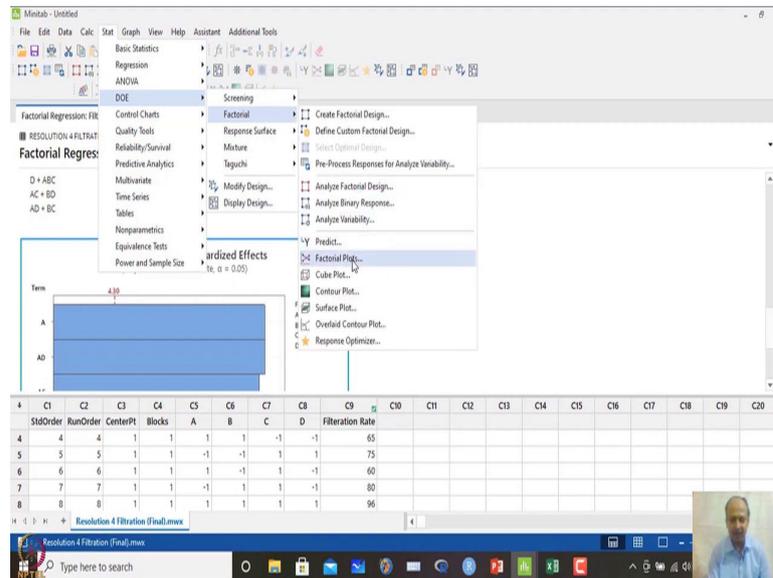


So, you see the factors which are important over here; what you can see is that only main effect A, D and C and AD interaction and AC is important, others are ignored over here. So, you could have done up to four interaction effects; A multiplied by B and C and D like that; if I have done full trial.

So, I do not need full trials over here; out of these four factors, what I am seeing is that only A, D and C is important; so I can eliminate B from here. So, that is the objective of

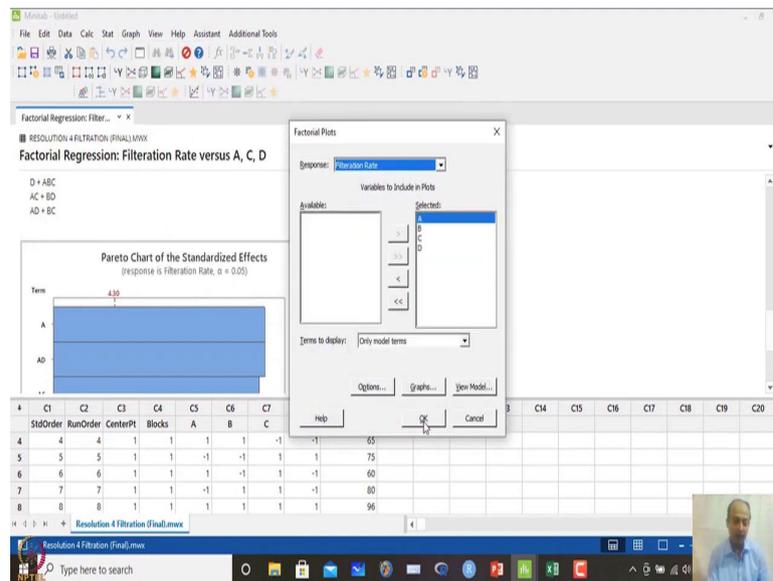
fractional factorial design that is the objective of fractional factorial design. What we can do is that, we can we can eliminate this one.

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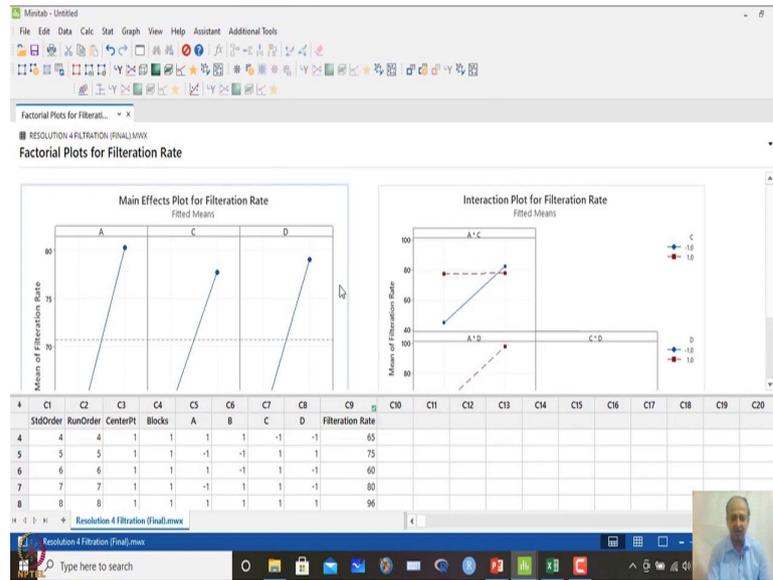
And what we can do is that; when we have done this.

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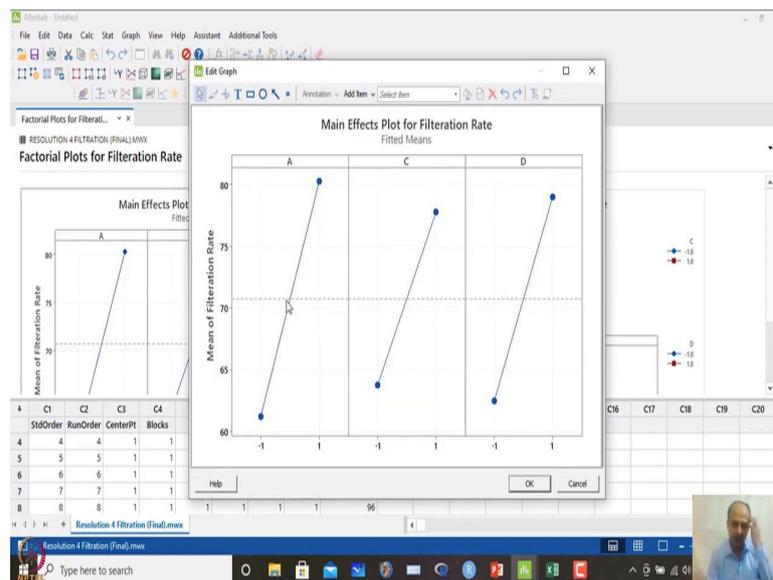


Then, also we can see the factor effects, factorial plots over here and if you want to get the settings like that.

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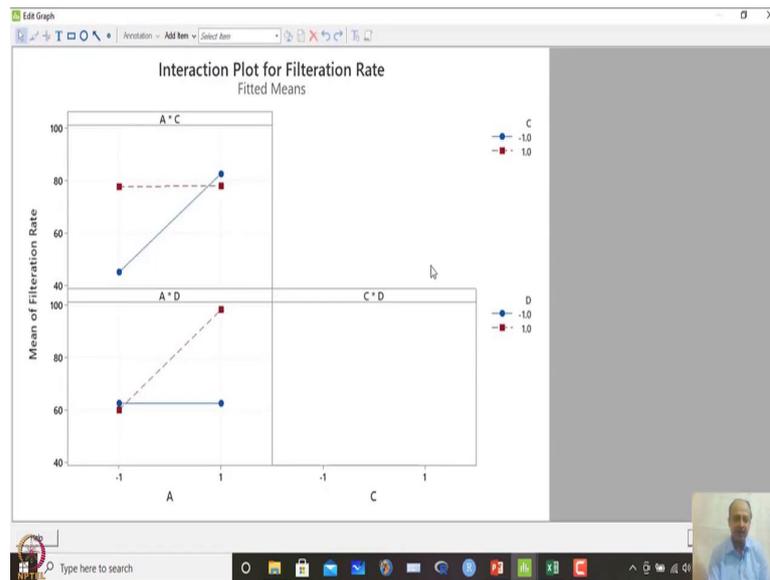


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So, what you can see is that; main effect plots interaction is prominent.

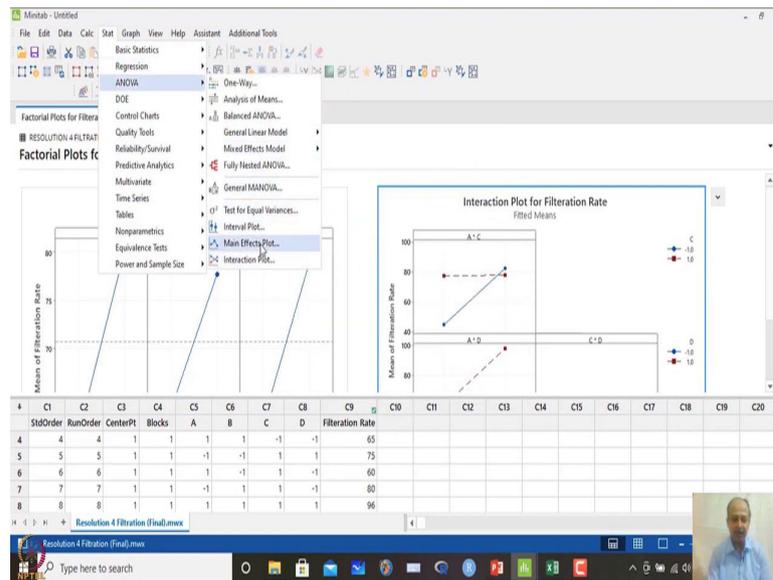
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So, we will see interaction plots over here and based on that we can get the settings like that. So, in this case filter rate; if you have to maximize. So, in this case what is required is that C is minus 1 and also we can see that A is at plus 1 like that; A and C at; A at plus 1, C at minus 1; that is the combination that we are seeing over here ok.

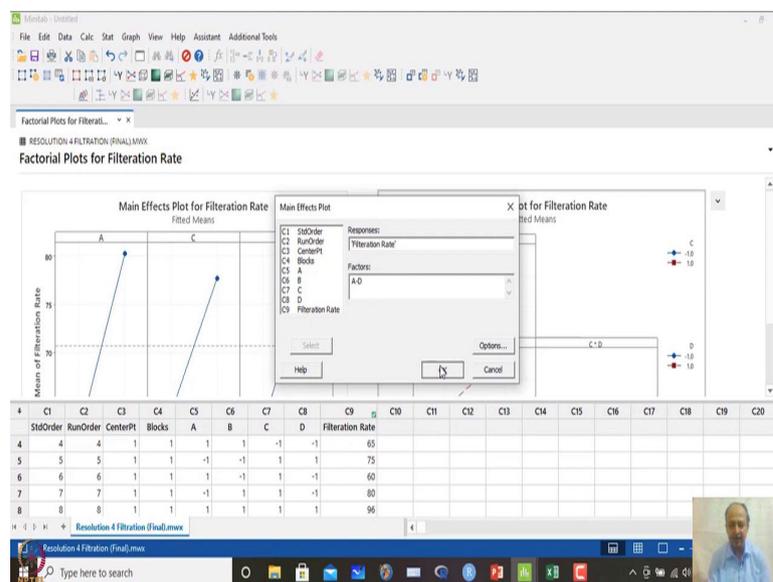
And in this case; for case of D. D is at plus 1 over here. So, A was at plus 1; that is the condition A plus 1, D plus 1 and C at minus 1; there is a combination that we are getting over here and B is not significant. So in that case; it can be at any level, it can be at any level and which we can see.

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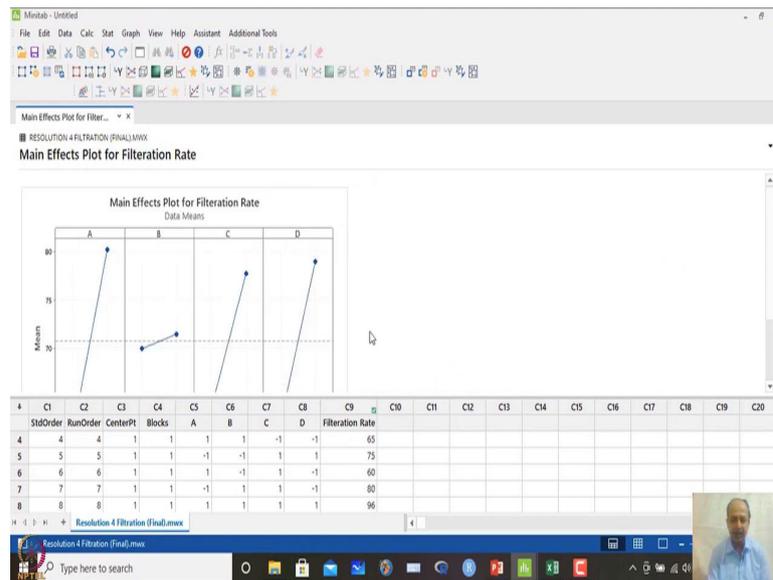
So, if you want to see the B's effect; so what we can do is that ANOVA analysis, you go to sorry; you go to main effect plots that will give you the idea. So, in this case what we are doing is that; so main effect plots over here and we can consider filter rates over here.

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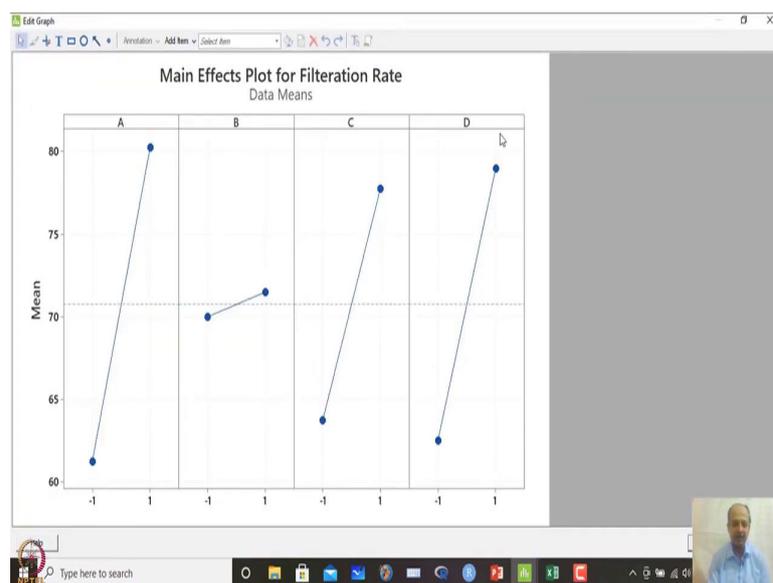


And the factors are considered from A to D like that; you click ok.

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What you will see is that this is the main effect plots that you are seeing. Do not go by settings. What I wanted to show is that I want to show B's effect over here, that it is quite flat over here; quite flat and the slope is quite flat over here.

So, in this case B's effect is not significant that is also reflected after the fractional factorial design; that is also reflected after the fractional factorial design. So, this experimentation is done for screening fractional, factorial design is used for screening.

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Quality Control and Improvement using MINITAB

One Half (1/2) Fraction of the 2⁴ Design

Run Number	A	B	C	D=ABC	Treatment Combination	Filtration Rate
1	-	-	-	-	(1)	45
2	+	-	-	+	ad	100
3	-	+	-	+	bd	45
4	+	+	-	-	ab	65
5	-	-	+	+	cd	75
6	+	-	+	-	ac	60
7	-	+	+	-	bc	80
8	+	+	+	+	abcd	96

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So, out of four factors; we can see AC and D is important and we can screen that one. So, this is the fraction that we have run; so this is the experimental trial that is done and all the combinations. So, this is principal blocks; all will be plus 1 over here. So, if you multiply ABC and D, it will be plus 1 like that ok.

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Quality Control and Improvement using MINITAB

Regression Equation in Coded Units

Filtration rate = 70.750 + 9.500 A + 7.000 C + 8.250 D - 9.250 A*C + 9.500 A*D

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant	70.750	0.637	111.00	0.000		
A	19.000	9.500	0.637	14.90	0.004	1.00
C	14.000	7.000	0.637	10.98	0.008	1.00
D	16.500	8.250	0.637	12.94	0.006	1.00
A*C	-18.500	-9.250	0.637	-14.51	0.005	1.00
A*D	19.000	9.500	0.637	14.90	0.004	1.00

Alias Structure

Factor Name	Alias
A	A
B	B
C	C
D	D

Aliases

I + ABCD
~~B~~ + BCD ✓
 C + ABD
 D + ABC
~~AC~~ + BD ✓
 AD + BC

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So, this is the interaction plot that I showed to you and based on that. This is the equation filter rate equation and this is about 99 percent explained variability that we have seen;

when we have ignored B and higher order terms over here; so that is the sparsity of effects.

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Quality Control and Improvement using MINITAB

Design Resolution

- Resolution III Designs:
 - example

$$2_{III}^{3-1}$$

- Resolution IV Designs:
 - example

$$2_{IV}^{4-1}$$

Design resolution

III : Main effects not aliased, Main effect aliased with 2 factor interaction, 2 factor interactions may be aliased.

IV : Main effects not aliased, main effect not aliased with two factor interactions, **two factor interactions aliased, main effect aliased with three factor interaction**





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So, then there is design resolution over here; it can be resolution III and resolution IV. We have used 2 to the power 4 minus 1 is resolution IV design, where two factor interactions are aliased with each other and the main effects are aliased with three factor interaction that we have seen.

That means, alias structure if you see; so, A is aliased with BCD, that is third order interactions and AC is aliased with BD like that two; two second order interactions are aliased with each other like that ok; two factor interactions are aliased with each other like that; so this is resolution IV.

So, MINITAB will generate automatically; so, higher the resolution better is the interpretation you can have a better interpretation better screening like that. So, resolution IV and resolution V designs are also possible; so this is the resolution of design.

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Quality Control and Improvement using MINITAB

Resolution III Design Example

A human performance analyst is conducting an experiment to study **eye focus time** and has built an apparatus in which several factors can be controlled during the test. The factors he initially regards as important are acuity or **sharpness of vision (A)**, **distance from target to eye (B)**, **target shape (C)**, **illumination level (D)**, **target size (E)**, **target density (F)**, and **subject (G)**. *Two levels of each factor are considered.* He suspects that only a **few of these seven factors are of major importance** and that high-order interactions between the factors can be neglected. On the basis of this assumption, the analyst decides to run a screening experiment to **identify the most important factors** and then to concentrate further study on those. To screen these seven factors, he runs the treatment combinations from the design in random order, obtaining the **focus times in milliseconds**, as shown in table

Run Number	A	B	C	D=AB	E=AC	F=BC	G=ABC	Treatment Combination	Focus Time
1	-	-	-	+	+	+	-	def	85
2	+	-	-	-	-	+	+	afg	75.1
3	-	+	-	-	+	-	+	beg	93.2
4	+	+	-	+	-	-	-	abd	145.4
5	-	-	+	+	-	-	+	cdg	83.7
6	+	-	+	-	+	-	-	ace	77.6
7	-	+	+	-	-	+	-	bcf	95
8	+	+	+	+	+	+	+	abcdefg	141.8

Source: Montgomery, D. C. (2004). *Design and analysis of experiments*. John Wiley & Sons

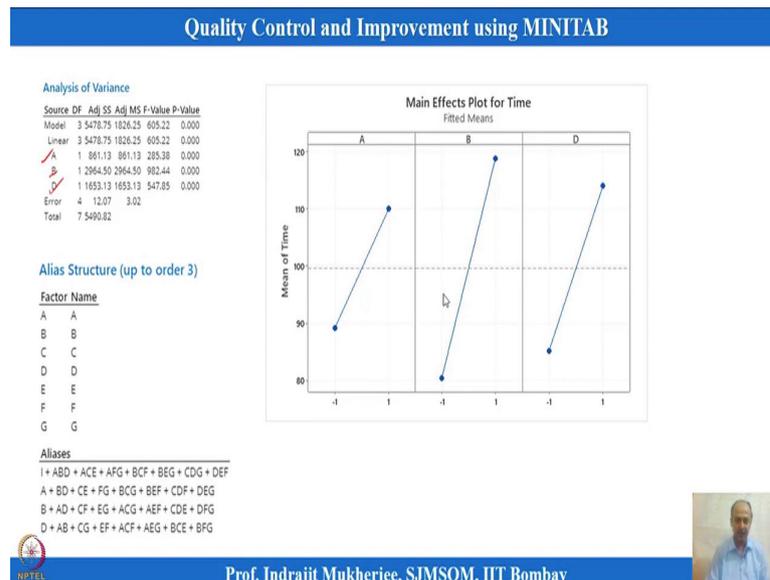
$2^7 = 128$
 $2^{7-4}_{III} =$


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So, resolution; this is a resolution III design where you can see that there are seven factors; A B C D E F G like that and if you run the full factorial trial; what happens is that this is a 2 to the power 7, experimental trials. So, 128 trials has to be run to see all effects and their interaction.

But if I use the theory of that sparsity of effects principle so in that case; I can use fraction of this. So, in this example; what was used is that 2 to the power 7 minus 4; so only 8 trials was considered over here to find out which factor is important and which is not.

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So, in this case; what was figured out is that now only A, B and D is significant over here; only main effects are significant, no order interaction nothing was significant like that. So, we have arrived at a conclusion that only A, B, D is important and that can be screened like that; so that is also possible ok.

So, this is fractional factorial design; more you study, you can see resolution III, resolution V designs like that and how to define resolutions like that. But I told that if you go to the higher resolution. It is better always because then main effects are not aliased with lower-order interactions like that.

So, that is the principle we will use so that we can safely assume that this effect is significant like that this effect or this interaction lower-order interaction is significant like that. We go for higher resolutions like that. So, we will continue discussion on, different topics that is known as Taguchi's experimentation and that will be the final topic that we will cover ok.

So thank you for listening.