

Managerial Economics
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Lecture - 40
Theory of Cost – II

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Managerial Economics

Private and Social Costs

Private and social costs are those costs which arise as a result of the functioning of a firm, but neither are normally reflected in the business decisions nor are explicitly borne by the firm.

Costs in this category are borne by the society.

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Then, we will talk about the last category that comes as the private cost and the social cost. Private and social cost are those cost which arise as a result of function of a firm but, neither are normally reflected in the business decision nor are explicitly borne by the firm. Cost in this category are borne by the society.

So private and social cost are those cost generally, it comes as a result of the function a of the firm but, neither normally reflected in the business decision. So, typically if you look at this is a part of the bi product or sometimes during the process of production. Whatever the cost incurs, this is not strictly decided by the firm. That this is the cost has to be incur or neither the firm explicitly takes care of this cost. Generally, it pass to the society and this cost generally borne by the society and generally, this is related to the bi product of the firm.

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Private and Social Costs

Total cost generated in the course of doing business may be divided into two categories:

- (i) those paid out by the firm; and,
- (ii) those not paid or borne by the firm, including the use of resources that are freely available plus the disutility created in the process of production.

Costs under the first category are known as *private costs*. Those of the second category are known as *external or social costs*.

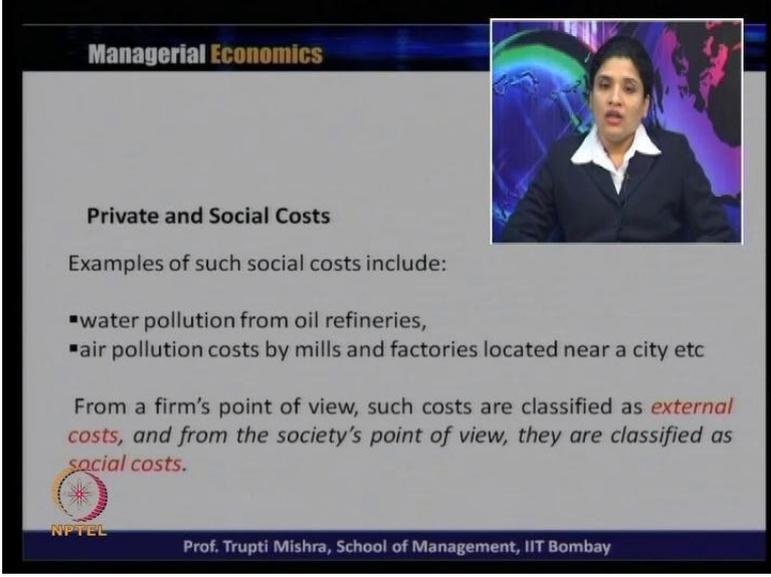
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So, total cost is generated in the course of doing business, may be, divided into two category; one those paid out by the firm and those are and those not paid or borne by the firm, including the use of resources that are freely available plus the disutility created in the process of production. So, the total cost if you look at one is those paid out by firm so these are strictly the private cost.

They are incurring the expenses and the firms also paying for it and second not paid or the borne by the firm, including the use of resources that is freely available plus disutility created in the process of production. And this second category generally known as the external cost or the social cost.

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Private and Social Costs

Examples of such social costs include:

- water pollution from oil refineries,
- air pollution costs by mills and factories located near a city etc

From a firm's point of view, such costs are classified as *external costs*, and from the society's point of view, they are classified as *social costs*.

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Now, if you look at the water pollution or the air pollution, water pollution from a oil refinery, air pollution caused by mills, factory located near a city etcetera from the firm point of view such cost are classified as the external cost. This is not strictly part of a business decision and from the society point of view they are classified as the social cost.

So, pollution if you look at, they are the bi product. It's not that its decided in the business. It is a part of the business decision that the pollution has to be taken care of; so, water pollution from oil refinery here the main product is oil but, the bi product in that process is the waste or since the discharges are into the local water body that creating a water pollution.

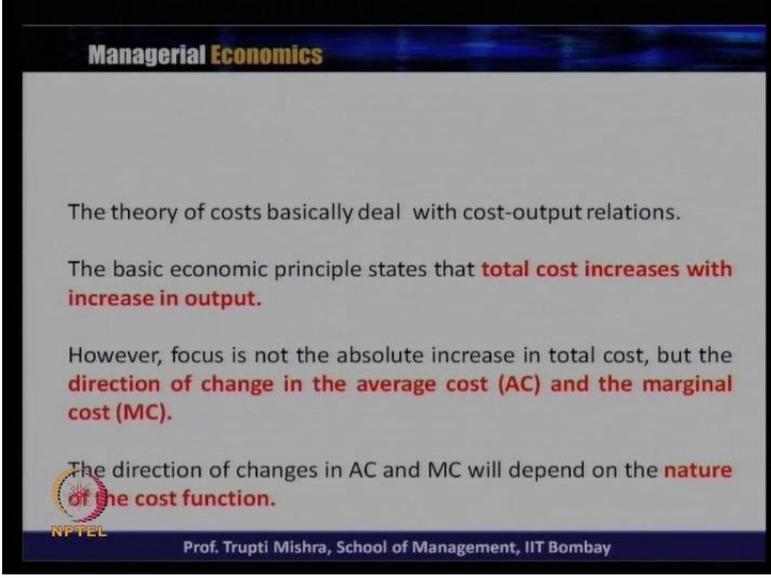
Now, firm tries to say that these are external cause because this is discharge and somehow we have to make the discharge somewhere. And that targeting the local water bodies for that but, who is getting affected by that the society, the locality who is staying near the oil refinery. They are getting affected by that..

And they are incurring a cost weight. So, for the firms view point this is external cost, but, from the society since even if they are not producing the product they are exposed to the bi product of the firm, bi product of the producer and that is the reason they are paying a cost to it so that is this is known as the social cost.

Similarly, air pollution caused by the mills and the factory located near a city air gets polluted. This is part of external cost for the firm because this is not a part strictly part of their business decision. But for the society those who are developing a respiratory diseases because of air pollution, because of their staying in the locality where the factory is located and the factory discharging creating a air pollution. Because of that their getting a respiratory disease. This is the social cost to the since they are staying in a society since, they are staying near to a factory they are incurring a cost to it.

But for the firm it is always the external cost. So, social cost and the private cost is one. Private that is one strictly paid by the firm but, when it comes to the external cost or the social cost it is not paid by the firm. May be, it happens that sometimes the firm spends some amount of money in order to in order to reduce the in order to treat the pollutant. Treat the effluent but, when it comes to major chunk of the cost it is always paid by the paid by the paid by the society. Because since, they are staying at the part staying in a society and it is a part of their social cost.

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The theory of costs basically deal with cost-output relations.

The basic economic principle states that **total cost increases with increase in output.**

However, focus is not the absolute increase in total cost, but the **direction of change in the average cost (AC) and the marginal cost (MC).**

The direction of changes in AC and MC will depend on the **nature of the cost function.**

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So, if you look at in the last few slides we discussed this about the category of the cost in two cases; one where there is a accounting accounting perspective, other when it is the case of the economic analysis. So, if you look at the theory of cost basically deals with the cost and output relation.

It is cause and effect since the output is there. That's the reason there is a expenses of the there is a cost. So, the basic economic principle states that total cost increases with the increase in the output. But here the focus is not the absolute increase in the total cost but, the direction of change in the average cost and the marginal cost.

And the direction of change in the average cost and marginal cost will depends up on the nature of the cost function. So, essentially if you look at this is a constant output relationship; whenever the output increases that leads to increase in the cost.

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Cost – Output Relationship

A cost function is a symbolic statement of the technological relationship between the cost and output.

$C = TC = f(Q), \text{ and } \Delta Q > 0,$

The specific form of the cost function depends on the time framework for cost analysis: **short-or long-run.**

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Whenever the output decreases, that leads to decrease in decreases the cost. Here, the focus in case of cost output relationship. The focus is not on the absolute increase in the cost due to increase in the output. Here, the focus is that what is the average cost when the output increases.

What is the average cost when the output decreases? What is marginal cost when decreases? When the firm operates at different scale of the output, different scale of the production, what happens to the marginal cost? When the firms operates in the different scale and the focus here is to know the direction of change in the average cost and the marginal cost, and which we can know from the nature of the cost function.

So, cost function, if you look at it is a symbolic statement of the technological relationship between the cost and output where c is the cost $t c$ is the total cost, showing is the function of Q and the change in the Q , is always greater than zero. Then only it will lead to increase in

the cost the specific form of the cost. Function depends on the time framework for cost analysis that is the short run and long run..

So, the specific form of the cost function depends on the time framework of the cost line whether, it is the short run function, whether it is a long run cost function. That's depends up on the specific form.

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The slide is titled "Managerial Economics" and features a video inset of Prof. Trupti Mishra. The main content is under the heading "Short Run Costs". It lists two types of costs:

- **Total Variable cost (TVC)**
 - Total amount paid for variable inputs
 - Increases as output increases
- **Total Fixed Cost (TFC)**
 - Total amount paid for fixed inputs
 - Does not vary with output

At the bottom, the formula is given as $\text{Total Cost (TC)} = \text{TVC} + \text{TFC}$. The NPTEL logo is in the bottom left, and the professor's name and affiliation are in the bottom right.

So, total variable cost, if you look at short run cost, we get total cost, which is a combination of total variable cost and total fixed cost. Total variable cost is total amount paid for variable inputs and it increases as output increases. So, total variable cost is the path cost incur from the variable inputs total amount paid for the variable inputs..

And it increases whenever there is a increase in the output. Total fixed cost is the total amount paid for the fixed input of production. It does not vary with the output. Generally, this is the part of the short run because, this total fixed total cost is always a part. Essentially related with a fixed input and fixed input, is the feature of a short run cost analysis and total cost is the combination of the total fixed cost and total variable cost.

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Short-Run Total Cost Schedules

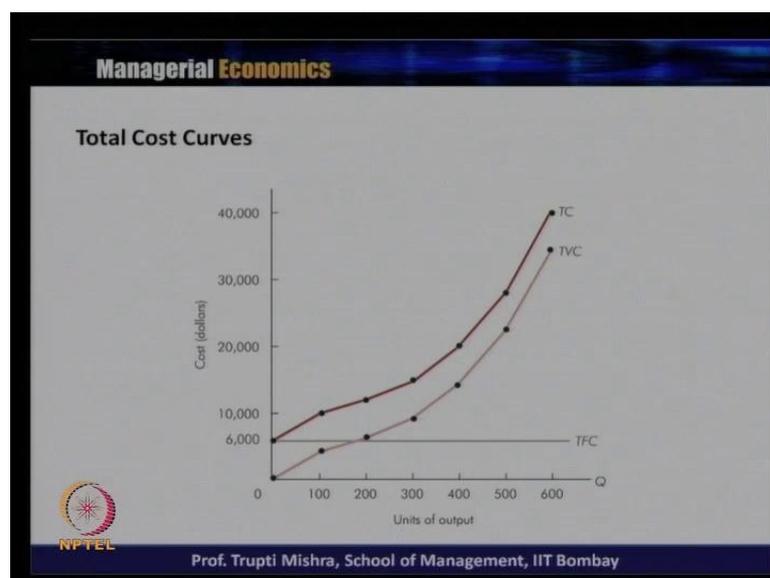
Output (Q)	Total Fixed cost (TFC)	Total Variable cost (TVC)	Total Cost (TC=TFC+TVC)
0	6,000	0	6,000
100	6,000	4,000	10,000
200	6,000	6,000	12,000
300	6,000	9,000	15,000
400	6,000	14,000	20,000
500	6,000	22,000	28,000
600	6,000	34,000	40,000

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So, this if you look at, this is the short run total cost schedule where ,column one talks about the output, column two talks about the total fixed cost, column three is total variable cost and total cost is total fixed cost plus total variable cost.

So, this is just a hypothetical example so if you look at from 0 unit to 600 unit, the total fixed cost is 600. So, there is no change in the fixed input from 0 unit to 600 unit. But from hundred unit to 600 unit the variable cost is changing and that leads to all this changes in case of the total cost.

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So, this is the graphical representation of the total cost. Total variable cost and total fixed cost since, total fixed cost is fixed up to 600 units of output. This is just a horizontal straight line parallel to X axis where, X axis represents the unit of output and y axis represent the cost and total variable cost is, total variable cost is starting from origin and it goes on increasing when there is a increase in the output. Total cost is summation of total variable cost and total fixed cost. That is the reason the total cost starts from the 6000 unit of input which is at the total fixed cost.

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Average Costs

$$AVC = \frac{TVC}{Q}$$
$$AFC = \frac{TFC}{Q}$$
$$ATC = \frac{TC}{Q} = AVC + AFC$$

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Then, we will look at the average average variable cost, is total variable cost divided by the unit of output that is, represented by Q average fixed cost is the total fixed cost divided by Q which is, unit of output and average total cost is T C. That is total cost which is a summation of total variable cost and total fixed cost divide by Q and that leads to the average total cost is equal to the average variable cost plus average fixed cost.

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Short Run Marginal Cost

- Short run marginal cost (*SMC*) measures rate of change in total cost (*TC*) as output varies

$$SMC = \frac{\Delta TC}{\Delta Q} = \frac{\Delta TVC}{\Delta Q}$$

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Marginal cost measures the rate of change in the total cost as output increases or decreases. So, short run marginal cost is change in the total cost with respect to change in Q. So, change in the if you look at in the short run marginal cost curve when you talk about the change, there is no change in the fixed input. There is only change in the variable factor. So, that is the reason when you say that change is the total cost strictly, there is a change in the total variable cost and with respect to change in the output because, there is no change in the fixed input.

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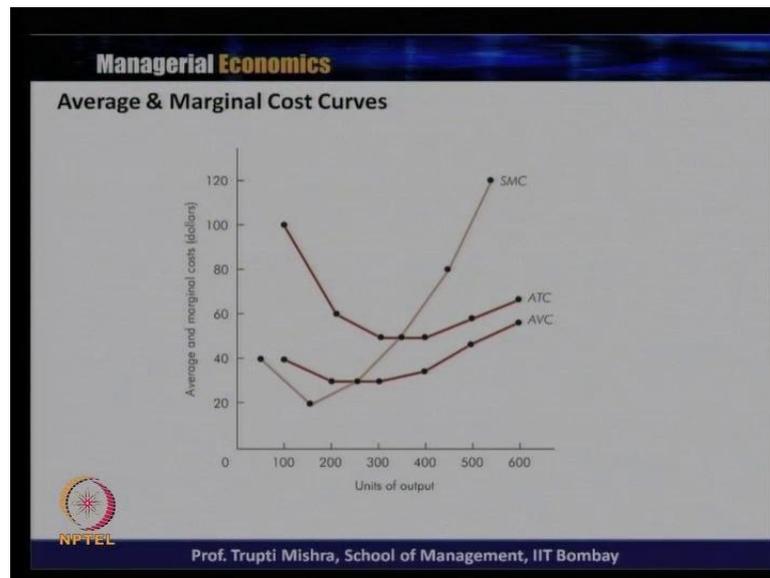
Average & Marginal Cost Schedules

Output (Q)	Average fixed cost (AFC=TFC/Q)	Average variable cost (AVC=TVC/Q)	Average total cost (ATC=TC/Q= AFC+AVC)	Short-run marginal cost (SMC=ΔTC/ΔQ)
0	--	--	--	--
100	60	40	100	40
200	30	30	60	20
300	20	30	50	30
400	15	35	50	50
500	12	44	56	80
600	10	56.7	66.7	120

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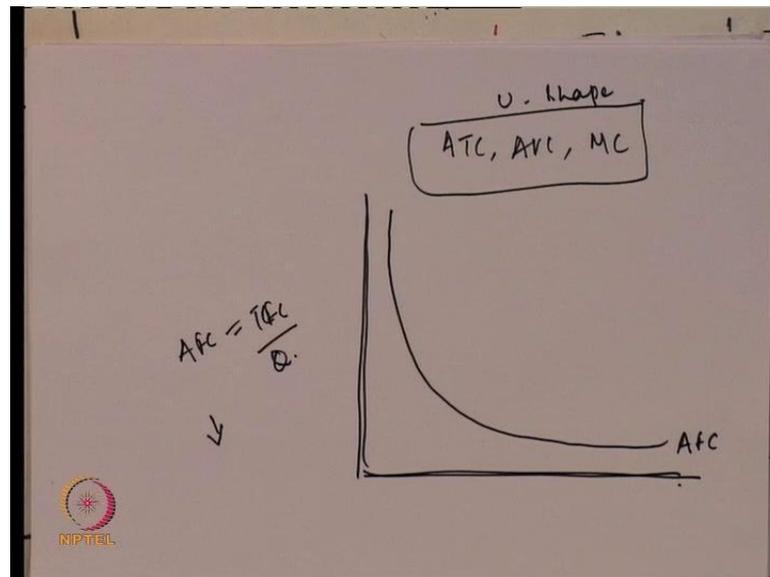
So, this is the example of average and marginal cost schedule. So, the first one is output, second one is the average fixed cost, the third one is the average variable cost, fourth one is average total cost and last one is the marginal cost curve and since fixed cost is constant. The average fixed cost goes on decreasing when there is increase in the output. This is the graphical representation of average fixed cost. Average variable cost marginal cost curve and average total cost curve.

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So, if you look at, whether it is a average variable cost average total cost or short run marginal total cost curve all it follows A U shape and here one thing is missing. We need to find out what is the shape of a average fixed cost which is not u shaped as compared to the other.

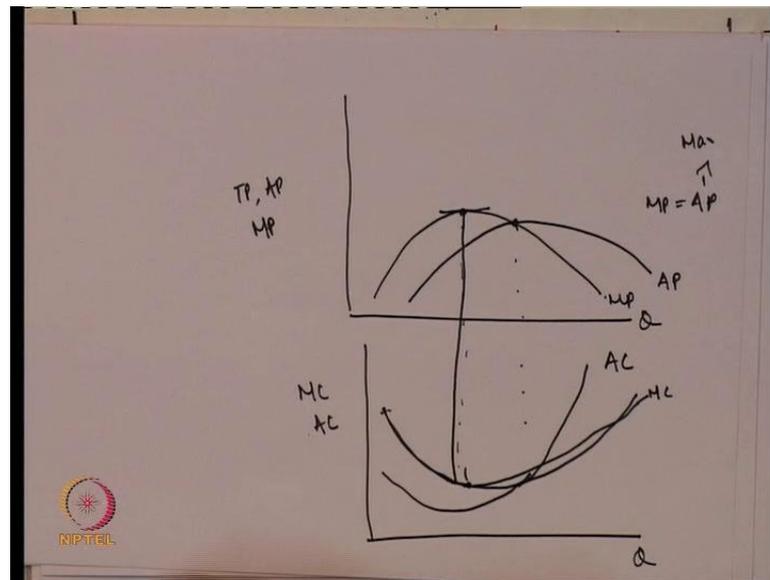
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So, as we know average total cost, average variable cost and marginal cost. All these three cost curves are U-shaped. However, when it comes to the average fixed cost, average fixed cost is nothing but the total fixed cost divided by Q and which goes on decreasing and that is the reason we get a rectangular hyperbola shape for the average fixed cost even if it is close to both the y-axis and x-axis. But it never touches any of these axes. It can be never 0 and that is the reason it cannot touch either the y-axis or the x-axis.

So, if you look at all these cost curves, all the average total cost, average variable cost, average short-run cost curve, they can be used as a U-shaped curve but the average fixed cost curve is a rectangular hyperbola. It never touches the axes even if it is close to the x-axis and y-axis. It never touches the axes and it goes on decreasing when the output increases till the specific level because there is no specific input which leads to no change in the fixed cost of production. Then, we will come to the relationship between the production and the short-run production and short-run cost.

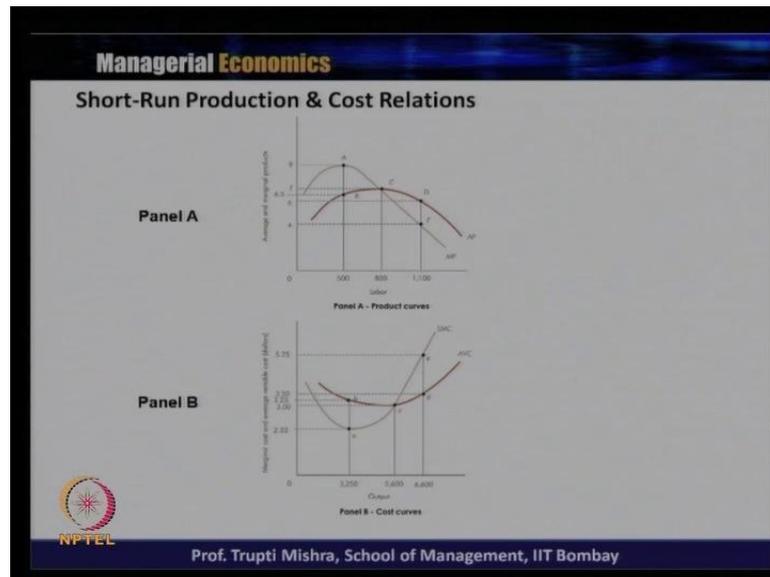
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So, if you remember the law of variable proportion where we essentially discuss about the relationship between the total product, average product and marginal product. So, if you remember how the average product and marginal product, they are related. So, this is our marginal product, this our average product. Average product is equal to the marginal product where average product is maximum corresponding to this. We will see how this is related to the marginal cost and average cost of production. So, if you remember when marginal product is highest this is the point when the law of diminishing return takes place because beyond this there is no increase in the output whenever there is no increase in the input no increasing return to scale..

So, in this case if you look at corresponding to this we will see that, our marginal cost is minimum. Corresponding to the marginal product of maximum of the marginal product similarly, corresponding to the equality between the marginal product and the average product our average cost will be equal to the marginal cost.

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So, from this if you look at the figure over here also, we are showing the average product and marginal product at the upper part of this graph and marginal cost and the average variable cost at the lower part of this graph.

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Relations Between Short-Run Costs & Production

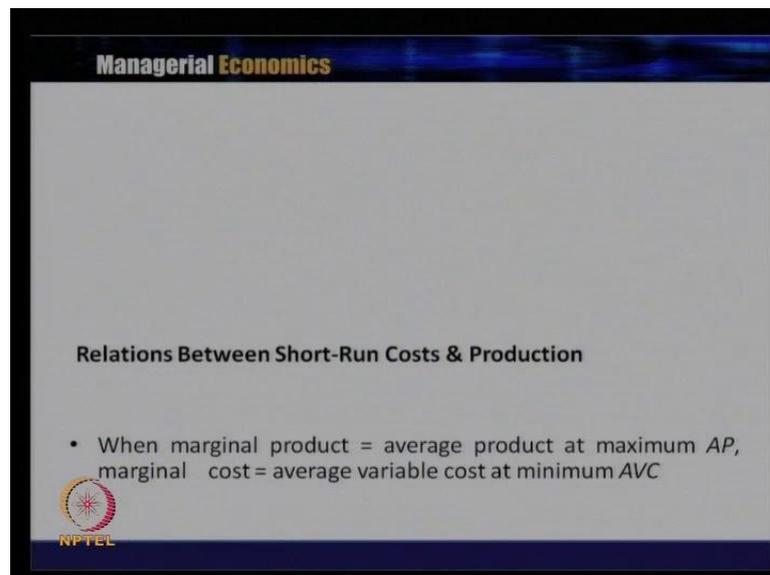
- When marginal product (average product) is increasing, marginal cost (average cost) is decreasing
- When marginal product (average product) is decreasing, marginal cost (average variable cost) is increasing

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So, what is the relationship between this short run cost and production, when marginal product is increasing? Marginal cost is decreasing that is, from the first part of the curve. So, if you look at if marginal product is increasing, marginal cost is decreasing when, average product is increasing average cost is decreasing..

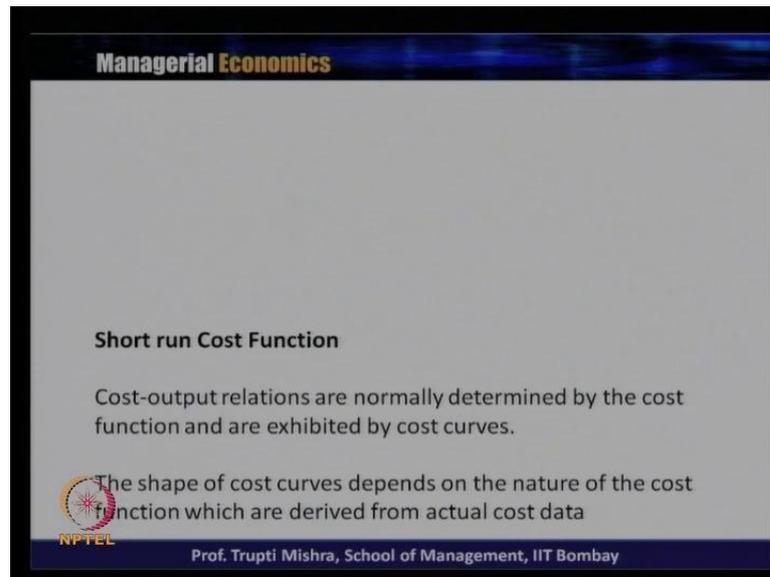
When marginal product is decreasing; marginal cost is increasing. When average product is decreasing, average cost is increasing. So, marginal product, average product, marginal cost, average cost they are inversely related. So, when marginal product is increasing; marginal cost is decreasing. When marginal product is maximum; marginal cost is minimum. When marginal product is decreasing; marginal cost is increasing. Similarly, when average product is increasing, average cost is decreasing. When average product is decreasing; average cost is increasing. When marginal product is equal to average product.

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And, at that point average product is maximum corresponding to that marginal cost is equal to the average cost and at this point the average variable cost is also minimum.

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Short run Cost Function

Cost-output relations are normally determined by the cost function and are exhibited by cost curves.

The shape of cost curves depends on the nature of the cost function which are derived from actual cost data

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So, to summarize if you look at the average cost and the marginal cost and the average product and the marginal product, they are related in an inverse way. Whenever the average product, marginal product is increasing average cost marginal cost is decreasing. Whenever marginal product, average product is increasing, marginal cost and average cost is decreasing. Marginal cost is equal to average cost. At the minimum point of the average cost. So, the relationship is such that of the product is doing well then, that has to be at the minimum cost.

And that is the reason, we say, that whether it is the case of increasing return or whether it is a case of decreasing return or whether it is the case of the constant return. So, cost and output relationship are normally returned by the cost function and exit weight by the cost function. Whether it is the short run or whether it is a long run and the shape of the cost curve depends up on the nature of the cost function which derive from the actual cost data.

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Linear Cost Function.
 $TC = C = a + bQ$
where $a =$ Total Fix Cost (TFC), $bQ =$ Total Variable Cost (TVC)
The Average and Marginal cost functions can be obtained from the Total Cost Function as follows:
Average Cost (AC) = $TC = a + bQ = a/Q + b Q / Q$
Marginal Cost (MC) = $dTC/dQ = b$

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So, we will talk about three different kind of cost function typically, in the case of a specifically in case of the short run cost.

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$TC = a + bQ$ Linear Cost-function.
Fixed Cost TVC.
 $AC = \frac{TC}{Q} = \frac{a + bQ}{Q} = \frac{a}{Q} + b$
MC
↳ $\frac{dTC}{dQ} \cong \frac{\Delta TC}{\Delta Q}$
 $= \frac{d(a + bQ)}{dQ} = b \cdot \frac{dQ}{dQ} = b$

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So, first one is the linear cost function where it takes a functional form that is TC is equal to $a + bQ$ where, this is the fixed cost and this is the total variable cost. Now, how to find out the average cost and marginal cost from here? So, average cost is total cost divided by Q . So, total cost divided by Q , so Q is $a + bQ$ by Q which comes to $a/Q + b$. So, average cost is $a/Q + b$. How you find out the marginal cost.

Marginal cost is change in total cost with respect to change in Q. So, that way we take the first order derivative of total cost with respect to Q and that is d a plus b Q with d q, which is b d Q by d Q and you get b as the marginal cost. So, b is marginal cost and average cost a by Q plus b if, it is a case of a linear cost function where, the cost function takes a functional form of a plus b Q. So, a is the intercept which talks about the fixed cost b is the slope and b Q is the total variable cost.

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Quadratic Cost Function.
 $TC = C = a + bQ + Q^2$
 $AC = TC = a + bQ + Q^2 = a/Q + bQ/Q + Q$
 $MC = dTC/dQ = b + 2Q$
 Example, if $TC = C = 150 + 10Q + Q^2$
 Then, $AC = 150/Q + 10Q + Q^2$
 $= 150/Q + 10 + Q$
 $MC = dTC/dQ = 10 + 2Q$

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Then, we will see the case of the quadratic cost function and quadratic cost function. It takes a value that is the total cost is a plus b Q plus Q square.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, the total cost function is given as $TC = a + bq + q^2$. Below this, the average cost function is derived as $TC/q = AC = a/q + bq/q + q^2/q$, which simplifies to $a/q + b + q$. The marginal cost function is derived as $MC = \frac{dTC}{dq} = b + 2q$. A specific example is provided where $a=150$, $b=10$, and q^2 is the variable term. The total cost is $TC = 150 + 10q + q^2$. The average cost is $AC = TC/q = \frac{150 + 10q + q^2}{q} = \frac{150}{q} + 10 + q$. The marginal cost is $MC = \frac{d}{dq}(150 + 10q + q^2) = 10 + 2q$. The NPTEL logo is visible in the bottom left corner of the whiteboard image.

Now, how to find out the average cost? Here average cost is again total cost by Q . So, this is a by Q plus b Q by Q plus Q square by Q . So, that comes to a by Q plus b plus Q . This is the average cost and for marginal cost. We need to take the derivative with respect to dQ . So, that comes to b plus $2Q$. Now, if you take a functional form or if you add a value here, total cost is equal to 150 plus $10Q$ plus Q square. Then, in this case in order to find the average cost that is AC is equal to TC by Q .

So, this comes to 150 this is 10 and then this is Q . So, this comes to 10 plus Q , is the average cost. And to find out the marginal cost that is T 150 plus $10Q$ plus Q square with respect to dQ so that comes to 10 plus $2Q$. So, this is the this is the marginal cost. So, average cost is 10 plus Q and marginal cost is 10 plus $2Q$.

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Cubic Cost Function

$$TC = C = a + bQ - cQ^2 + dQ^3$$
$$AC = TC = a + bQ - cQ^2 + dQ^3$$
$$= a/Q + b - cQ + dQ^2$$
$$MC = dTC/dQ = b - 2cQ + 3dQ^2$$

Assume that the cost function is empirically and explicitly estimated as:

$$TC = 10 + 6Q - 0.9Q^2 + 0.05Q^3$$

And, $TVC = 6Q - 0.9Q^2 + 0.05Q^3$

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Then, we will take the cubic cost function and in case of cubic cost function, total cost is a plus b Q minus c Q square plus d Q q.

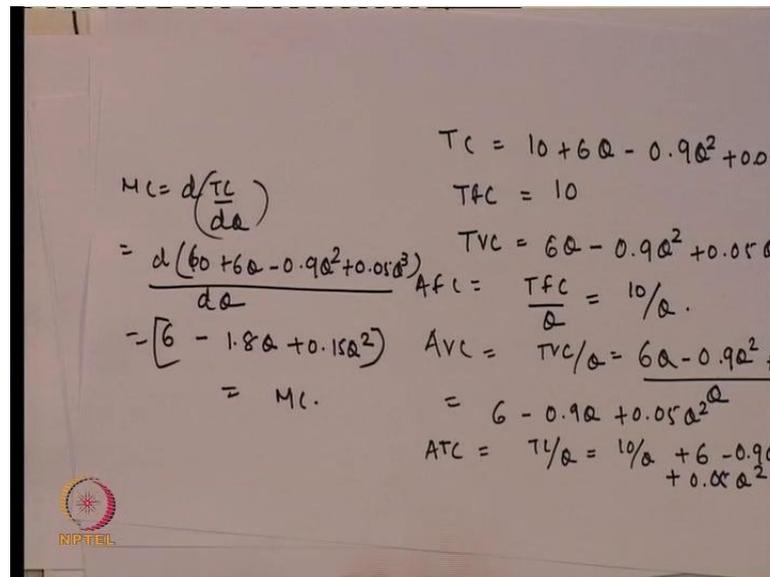
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$$TC = a + bQ - cQ^2 + dQ^3$$
$$AC = TC/Q = \frac{a + bQ - cQ^2 + dQ^3}{Q}$$
$$= \frac{a}{Q} + b - cQ + dQ^2$$
$$MC = \frac{d(a + bQ - cQ^2 + dQ^3)}{dQ}$$
$$= b - 2cQ + 3dQ^2$$

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So, average cost is total cost by Q, which comes to a plus b Q minus c Q square plus d Q Q divided by Q. So, that comes to a by Q plus b minus c Q plus d Q square. And marginal cost will be d a plus b Q minus c Q square plus d Q Q with respect to d Q. So, that comes to b plus b plus 2 c Q plus 3 d Q square.

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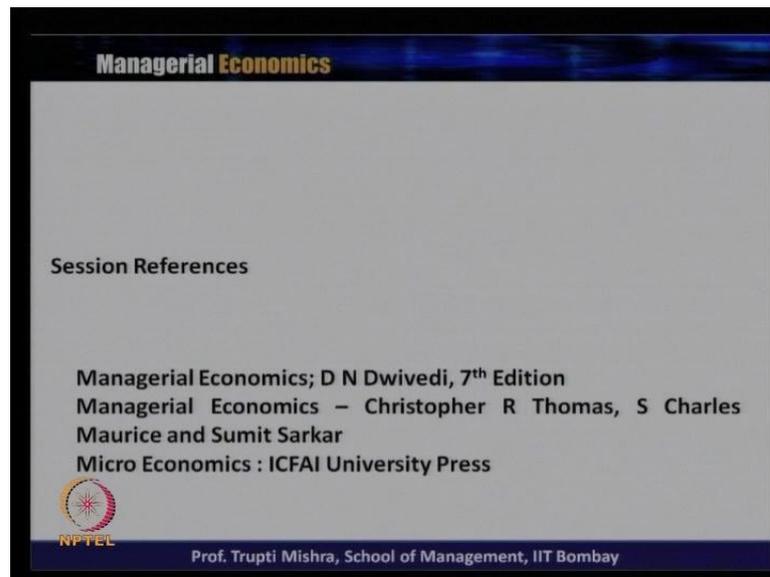
$$\begin{aligned}
 TC &= 10 + 6Q - 0.9Q^2 + 0.05Q^3 \\
 TFC &= 10 \\
 TVC &= 6Q - 0.9Q^2 + 0.05Q^3 \\
 AFC &= \frac{TFC}{Q} = \frac{10}{Q} \\
 AVC &= \frac{TVC}{Q} = \frac{6Q - 0.9Q^2 + 0.05Q^3}{Q} \\
 &= 6 - 0.9Q + 0.05Q^2 \\
 ATC &= \frac{TC}{Q} = \frac{10}{Q} + 6 - 0.9Q + 0.05Q^2 \\
 MC &= \frac{d(TC)}{dQ} \\
 &= \frac{d(10 + 6Q - 0.9Q^2 + 0.05Q^3)}{dQ} \\
 &= [6 - 1.8Q + 0.15Q^2] \\
 &= MC
 \end{aligned}$$

Now, if you will take a numerical value with respect to cubic cost function or we can say that total cost is equal to 10 plus 6 Q minus 0.9 Q square plus 0.05 Q Q. We can make it 2 part so that, we can find out what is T F C we can find out what is T V C. So, ten is at T F C because, this is the intercept value and this is strictly only the fixed cost because it is not associated with the fixed input and total variable cost is six Q minus 0.9 Q square plus 0.05 Q Q. To in order to find a f c here we can take this is as T F C by Q.

So, this is 10 by Q and for A V C. We can find out this is T V C by Q like six Q minus 0.9 Q square plus 0.05 Q q divided by Q which comes to 6 minus 0.9 Q plus 0.05 Q square then, average total cost is T C by Q, that comes to 10 by Q plus 6 minus 0.9 Q plus 0.05 Q square. And to find the marginal cost, that is, D T C with respect to d Q, so that comes to that comes to d 10 plus 6 Q minus 0.9 Q square plus 0.05 Q.

So, that with respect to so that, comes to 6 minus 1.8 Q plus 0.15 Q square. So, this is equal to the marginal cost. So, it depends up on the value of marginal cost. Average cost depends up on that what kind of cost function whether it is a linear cost function, whether it is a quadratic cost function and whether it is a cubic cost function.

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So, we will talk about the long run cost analysis and economic of scale in the next session and these are the session references the materials that is being followed for the preparation of this typical specific session.