Managerial Economics
Prof. N. Trupti Mishra
S. J. M School of Management
Indian Institute of Technology, Bombay

Lecture - 38
Theory of Production (Contd…) - I

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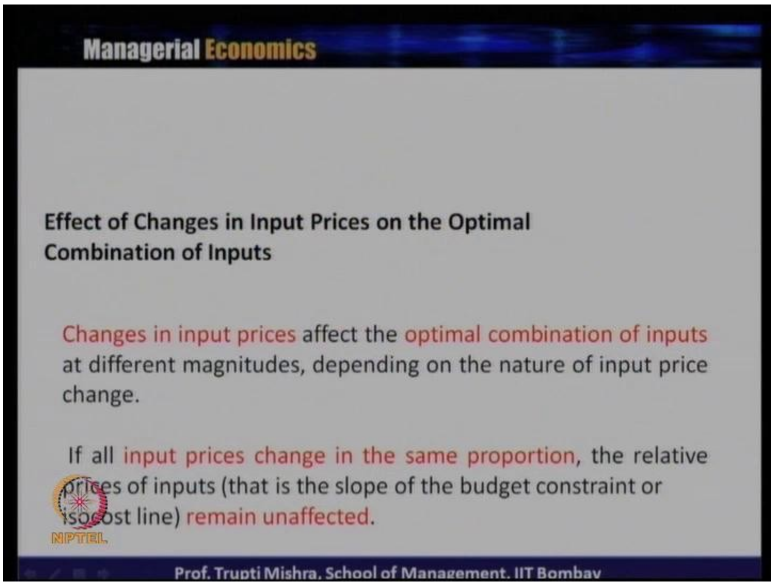

Let us look at the case where, if there is a change in the input prices, how it affects the least cost input combination? So, change in input price, if you look at it affect the optimal combination of inputs, at different magnitude, depending on the nature of the input price change. So, if all input price change in the same proportion, so it bound to happen the optimal combination of inputs, has to change, if there is a change in the input price.

Either in the different magnitude or same magnitude depends on the nature of the input price change. So, if all input price changes, in the same proportion; the relative price of the input that is the slope of the budget constant or they remain unaffected. So, if all input price changes in the same proportion, the relative prices of inputs also, if you look at they move in the same proportion and those are remain unaffected.

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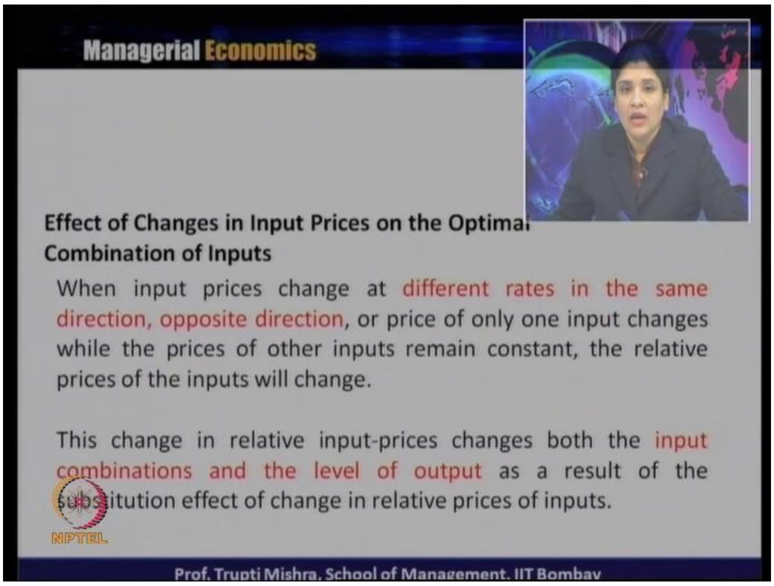

But when the input price is changes at a different rate in the same direction, or the opposite direction, or the price of only one input changes, while the price of other input remains constant, the relative price of the input will change. Input price when it change in the different rate in the same direction; different rate in the opposite direction. Input price of one changes, other remaining constant the relative price of input will change.

This change in the relative input output price change, if both in the input combination and the level of output, as a result of substitution effect of change in the relative prices of input. This change in the relative input prices, changes both the input combination and the level of output. So, whenever there is a change in the input prices, it affects the input combination and the level of output, as a result of substitution effect of change in the relative price.

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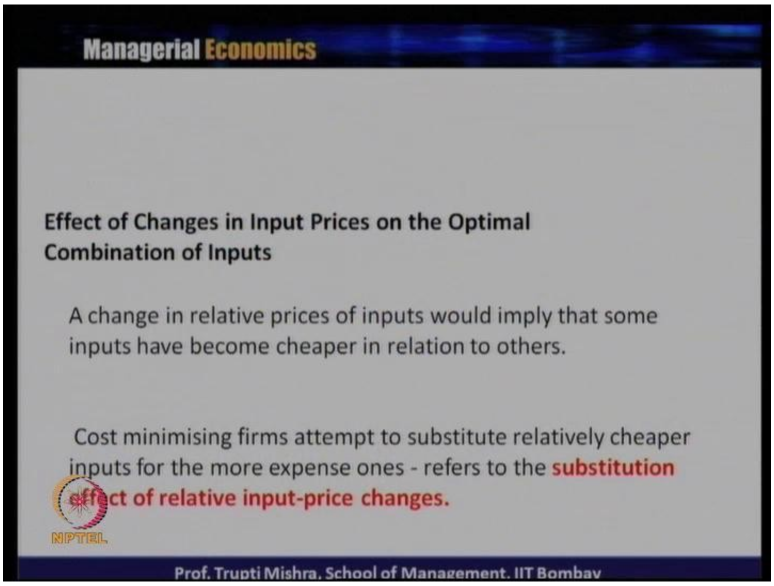

So, if change in the relative price of inputs, either in the same or the opposite, would imply that some input have become cheaper in relation to the others. So, cost minimization firm attempts to substitute relatively cheaper inputs for more expenses one refers to the substitution effect of relative price change. Because whenever the price of one input changes, it becomes cheaper with respect to the other inputs. And what the cost minimizing producers cost minimising firms they do it over here? They generally try to replace the expensive input with respect to the cheaper inputs, and this is generally known as the substitution effect of relative input price change.

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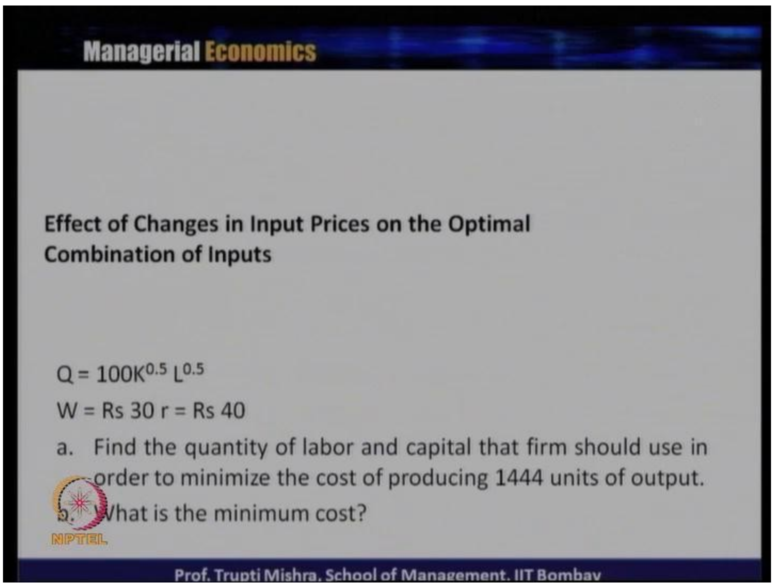

So, this numerical we will just look at later that how the quantity of labour and capital changes. Before that we will look at the graphical representation that when there is a change in the input prices, how it affects the least cost input, how it affects the level of output or how it affects the level of input combination.

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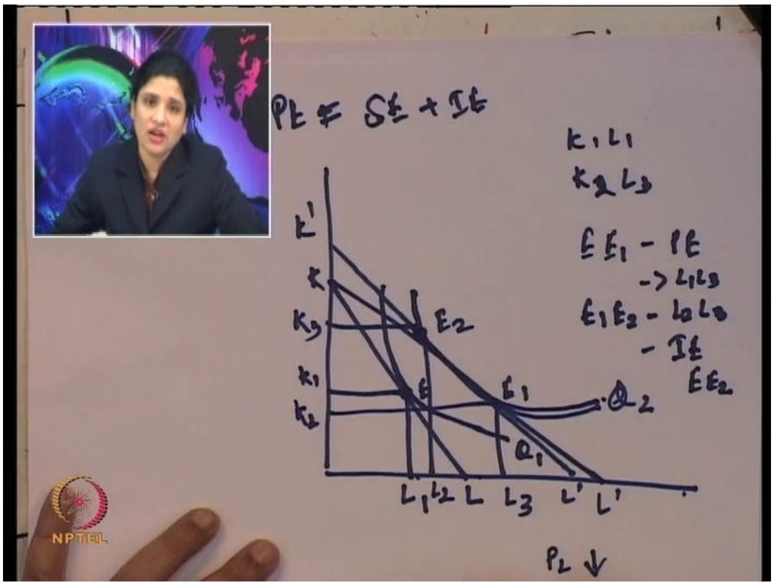

So, initially this is the isocost, K L is the isocost; Q is the output, Q 1 is the isoquant and this is the level of output. Now, price of labour input decreases, with the help of that now the quantity of the, or if you look at the isocost will change from K L to K L dash. So, once it changes from K L to K L dash, then in this case, now what is the new input combination? The new input combination is, if you look at, this becomes L 3, that is L 1 and this is K 2 this is K 1. So, with the change in the input price, now the firm will use more of labour and less of capital that is the reason the combination now already it is K 1 L 1, now it is K 3 K 2 and L 3.

Now, to get, to keep it in the same level what ah the producer will do? May be at least at the same level of output, still want to change the input combination, now what they will do? They will try to draw a parallel line which also a tangent at a point which is at this level. So, suppose this is your point E, this is your point E 1, this is your point E 2. So, at this case if you look at, still it uses a higher level of labour as compared to the previous level, but its use a, also a higher level of capital.

So, the movement from this E to E 1 is the price effect that is in the form of L 1 to L 3. The movement from, may be, E 1 to E 2 is budget defect because the producer is trying to keep the income level, the real income level of the producer at the same level that is the reason, we got a compensated budget line which is may be K dash and L dash which compensate or which may be the reduce the, real income of the producer in term of change in the input prices, and that leads to a different combination that is E 2.

So, E 1 to E 2 E 1 the movement from E to E 1 is the price effect, which in term of the labour input consist of use becomes L 1 and L 3. Movement from E 1 to E 2 is that is L 2 to L 3 is because of the income effect, because the real income is changing. And movement from may be E to E 2 is the substitution effect because of the change in the real income. So, if you look at the price effect is the combination of the substitution effect and the income effect.

So, if you remember your price effect, your substitution effect, and income effect in case of the consumer theory this is nothing but the counter part of the, counter part of the, counterpart of that in the production theory which talks about the change in the input prices. If there is a change in the input prices, generally the producer try to substitute that with a cheaper input, as compared to expensive input, that is the reason they go on using more of that input. So, in this case also the same thing has happened, the producer is, once the price of labour has gone down, the producer has tried to optimize it, and use more of the labour as compared to the capital.

And that leads to the combination of the change in inputs combination or also the change in the level of output. Next we will see the input combination or may be the input combination, how it changes or how to find out numerically when the production function is given price of inputs is given like w and r. And if the production function is given and to produce a specific level of output, how to optimize the cost of production and what should be the minimum cost of production. For that we will just take a example of, we will take a numerical example like Q is equal to 100 K that is to the power 0.5. And L to the power 0.5. This production function is in the form of a Cobb-Douglas production function, where w is 30 and r is 40. So, w is the price of labour that is 30 rupees, r is the price of capital that is 40 rupees, and what we need to do here? We need to find the quantity of labour and capital, that the firm should use in order to minimize the cost of producing a cost of 144 units of output. So, if you look at, this is the minimization case like the, second case where the unit of output is given, we need to minimize the cost, and what is the minimum cost? We need to find out that.

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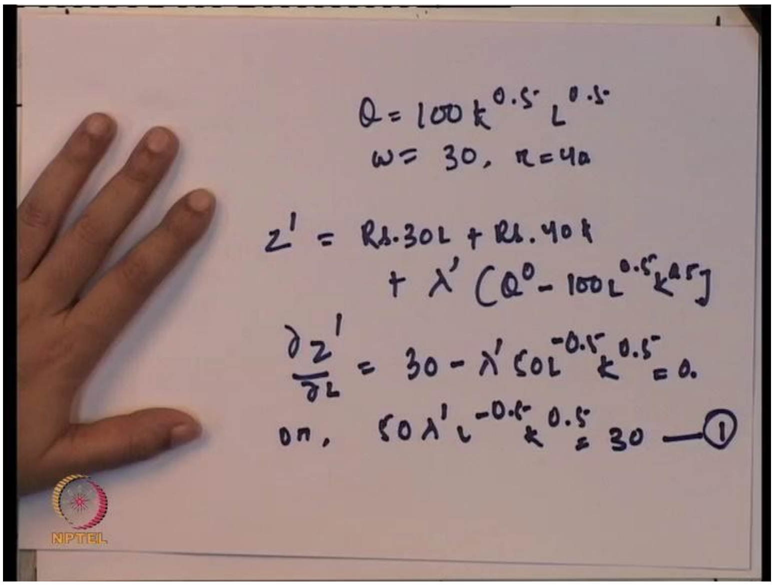

Then how we will go for this, we will use we will take the help of the Lagrangian multiplier to solve this. So, Q is equal to 100, K 0.5 and L 0.5. And here if you look at, then we have w is equal to 30 and r is equal to 40. So, first we will try to find out the composite function. Now composite function is rupees 30 L plus rupees 40 K plus lambda dash that is Q 0 minus 100 L to the power 0.5 K to the power 0.5.

Now, what the, what we need to do? We need to find out the first order condition. we need to find del z by del L that is 3 minus lambda dash fifty L minus 0.5 K 0.5 has to equal to 0 or 50 lambda dash L minus 0.5 K 0.5 has to be equal to 30, suppose this is our equation one.

Now we will need to look at the first order partial derivative or the first order derivative or the partial derivative with respect to the other input. In the first case we have checked it for L, now we will check it for K.

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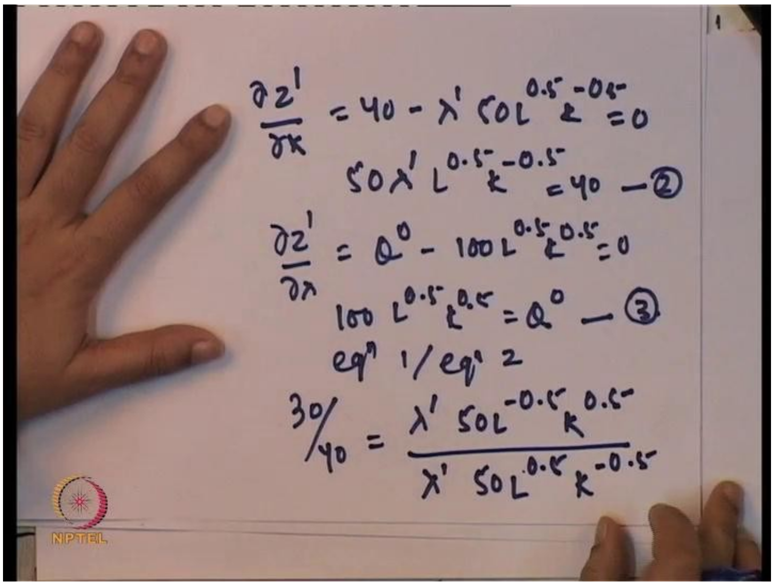

So, here we need to find out the del partial derivative with respect to K, that gives us 40 minus lambda dash 50 L 0.5 K minus 0.5 which is to be equal to 0; or 50 lambda dash L to the power 0.5 K to the power minus 0.5 which is equal to 40. Let us call it equation 2. Now to find out the partial derivative with respect to the lambda that is Q 0 minus 100 L 0.5 K 0.5 which is equal to 0. So, you can call it 100 L to the power 0.5, K to power 0.5 which is equal to Q 0. That is equation 3.

So, now, if you divide equation 1 by equation 2, suppose equation 1 by equation 2, then this 30 by 40 equal to lambda dash that is 50 L minus 0.5 K 0.5, then lambda dash 50 L 0.5 K minus 0.5.

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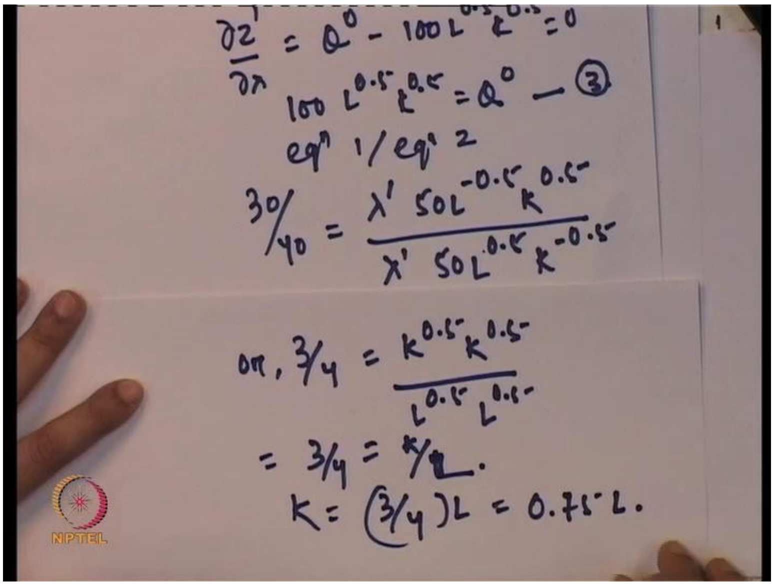

Simplifying this again, so if you simplify this, then this is 3 by 4 equal to K 0.5 K 0.5 L 0.5 and L 0.5 that comes to 3 by 4 by K by L. So, K is equal to we can say 3 by 4 L or 0.75 L. So, in order to take the first order partial derivative, once we get the partial derivative the first order derivative with respect to L, then the first order derivative with respect to K, and then the first order derivative with respect to lambda. And then we solve for the value K in term of L.

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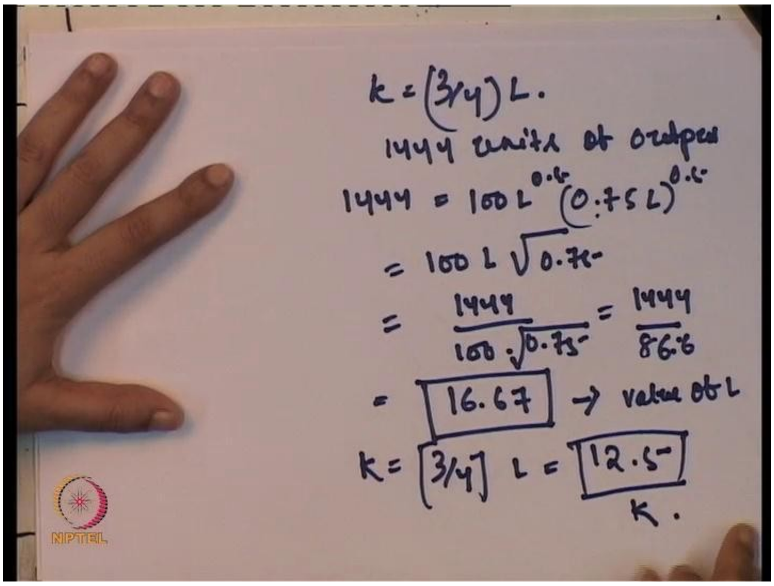

Now, what we have got from all this calculation, that is K is equal to 3 by 4 L. Now we will see how we can substitute the value of K into this into the equation, and find out the value of K and L. because ultimately, what we need to find out? We need to find out ultimately, to produce 144 units of output, what is the minimum cost because the minimum cost, the because the firm has to incur or what should be the minimum cost or on which isocost, they have to plan.

So, if you substitute the value of K into the given production function for 144 units of output that is 144 that is 100 L 0.5 and 0.75 L because K is equal to 0.75 L. So, that comes to 100 L 0.75 which comes to 144 100 0.75. Then it is comes to 144 86.6 that comes to 16.67 67. So, 16.67 is the value of L.

Now we need to find out K is equal to 3 by 4 L. So, that comes to 12.5. So, 12.5 is the capital, and 16.67 is the labour. So, capital and labour we [nee/need] got the value of capital and labour. Next we need to find out, what is the cost value, when the capital is labour is 16.67 and labour is capital is 12.5. Because ultimately, again let me remind it, ultimately we need to find out what is the minimum cost of producing this given level of output.

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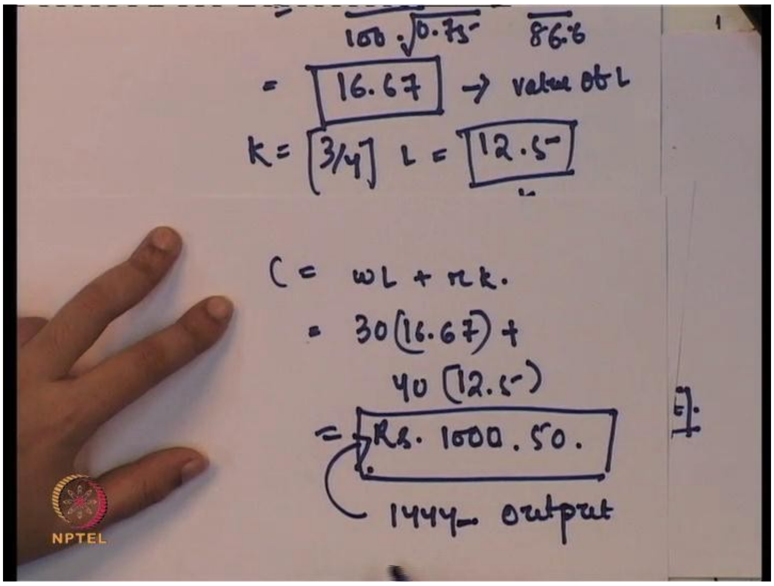

So, C is equal to w L plus r k. So, w is 30, L is 16.67 plus r is 40 and K is 12.5. So, that comes to rupees 1000.50. So, in order to produce 1444 output, this is the minimum cost. So, this is the given level of output and this is the minimum cost. So, whether it is a minimization case or maximization case, how generally we solve it, numerically, we solve it numerically, with the help of this Lagrangian multiplier method, where we take into the, we take the constant in the form of the Lagrangian multiplier.

We formulate a composite function then we take the first order partial derivative with respect to 0. Simplifying this that gives us the value of capital and labour, we put the value of capital and labour in the production function equation, we get the exact value of capital and labour. Use this in the cost function and that gives us the minimum cost of producing the given level of output.

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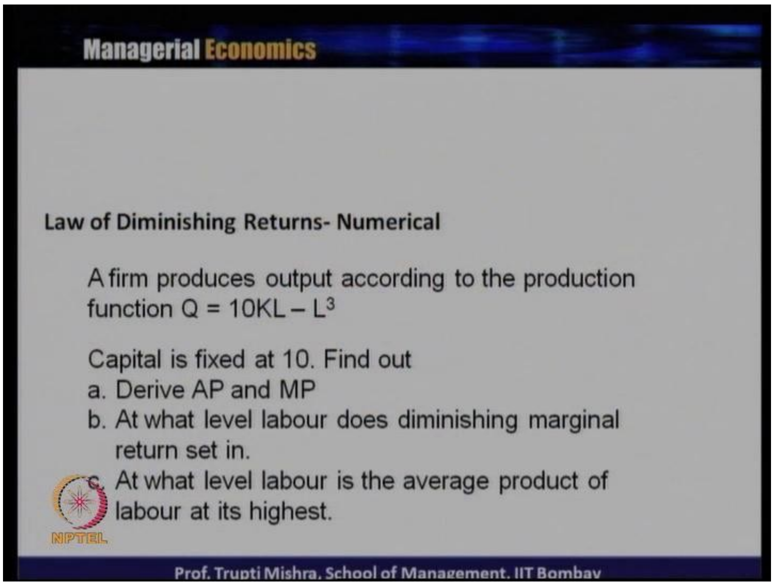

Now let us see, if you remember in the last class we talk about the short run production function that is in term of the law of diminishing return. Next we will see that numerically how we get the value of, the three stages, the different stages of production; and how we find out the value of average product, marginal product; and at what level generally the diminishing marginal set in; and at what level is the average product of the labour is the highest. So, the firm produces the output according to the production function that is Q is equal to100 K minus L q.

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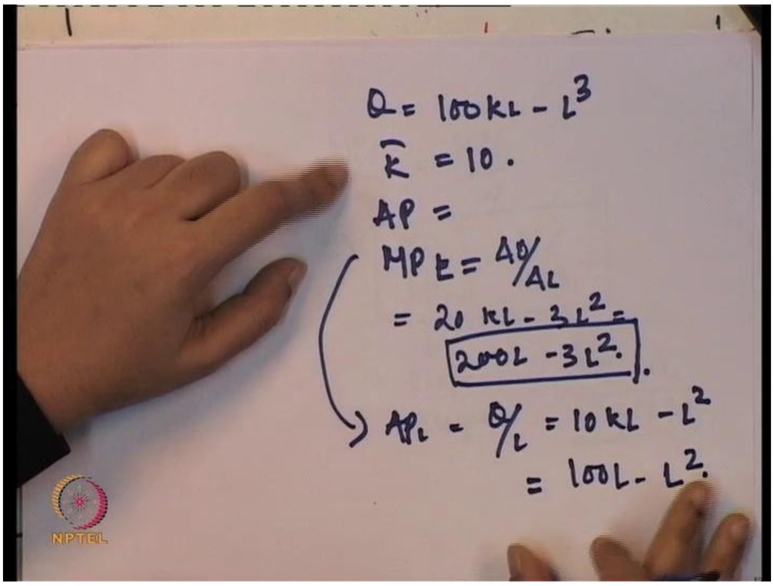

So, the production function is 100 K L minus L to the power cube. Capital is fixed at ten because this is a short run production function. Now what we need to do? We need to find out the value of average product; and we need to find out the value of marginal product. And then maybe, we can find out what is the different stages or what is the level of output or level of labour where the firm, where the producer has achieved the different level of output.

So, first we will find out the marginal product of labour. Now, marginal product of labour is del Q by del l. So, that comes to 20, that comes to there is a 20 K L minus 3 L square. So, that comes to 200 L minus three L square. Now to, so this is the marginal product of labour. Now we will find out the average product of labour. Average product of labour is Q by l. So, that comes to 10 K L minus L square which is equal to, since K is equal to 10, so this is 100 L minus L square.

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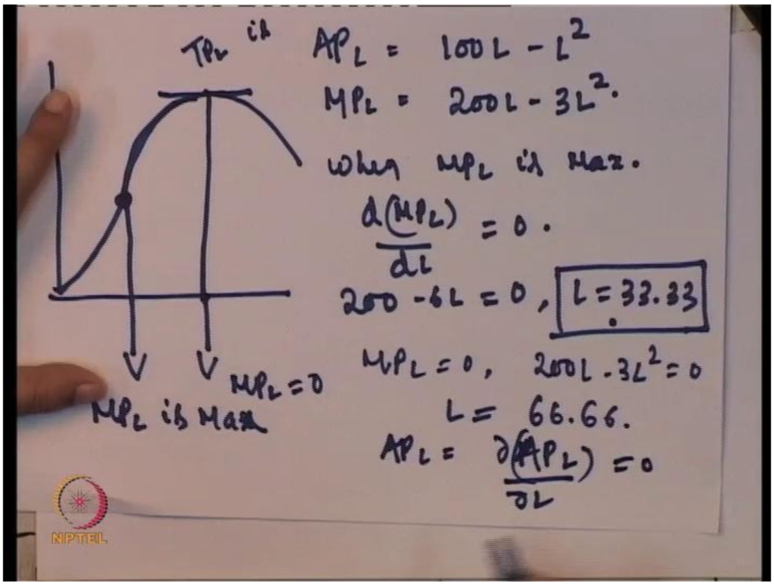

So, average product for, average product is 100 L minus L square. Marginal product for labour is 200 L minus three L square. Now, second one is, we need to find out when m P L is maximum. M P L is maximum where the first order partial derivative with respect to L is equal to 0. So, that comes to 200 minus 6 L which is equal to 0 L is equal to 33.33.

Now, what is the significance of this level of labour? The significance of this level of labour is that, at this level since the marginal product of labour is maximum, beyond this generally, the law of diminishing set in. If you remember you three stages of production function, like how the total product curve, so initially it is convex, then it is convex, then it is decreasing. So, this corresponding to this our marginal product of labour is maximum.

Because after this the total product of labour is increasing at a decreasing rate; and beyond this, this M P L is maximum and beyond this if you look at then the total product is increasing at the decreasing rate and marginal product is decreasing. And we can say, so corresponding to this point N P L is maximum and this is the point where the law of diminishing return set in. Then the second point we will look at is, when m p L is equal to 0; m p L is equal to 0, when this 200 minus three L square is equal to 0. So, in this case may be L is equal to, if you find out this comes to 66.66.

And where this value of L comes, the value of L comes at this point, because corresponding to this N P L is 0, and T p L is maximum. So, if you remember from, till this point, this is your point, this is your point; beyond which there is the law of diminishing return set in. then when the average product of labour is highest, average product of labour is highest when D A

P L with respect to d L is 0, that comes to L is equal to 50. So, this comes somehow here, the average product of labour is maximum when it is intersecting the marginal product of labour.

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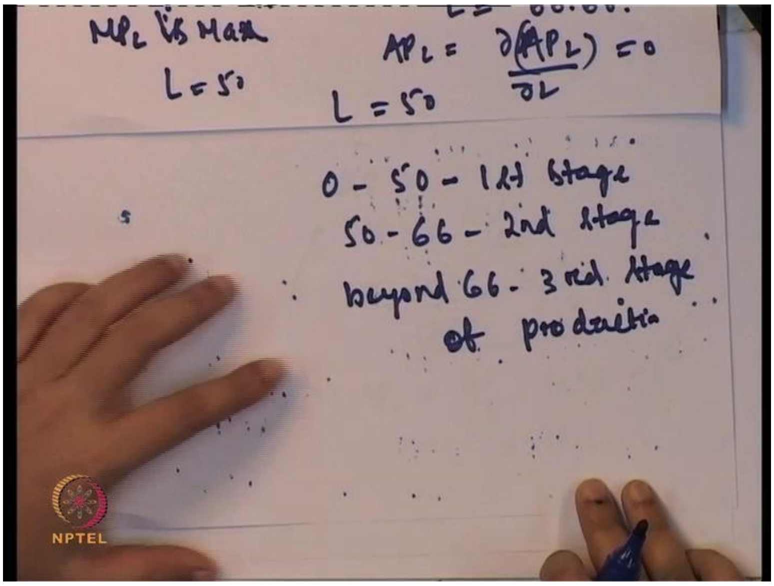

So, corresponding to this, L is equal to50. So, now we can say that from 0 to 50; the first stage of production from 50 to 66 unit of labour; second stage of production, and beyond 66 unit of labour we have third stage of production. And how we have identified this first stage, second stage, and third stage.

First stage ends, till the point the marginal product is equal to the average product. And the average product is maximum at that point where marginal product is equal to the average product. Where the second stage ends; second stage ends, when the total product level is maximum or the marginal product is 0. So, that achieve at the labour unit at 66; beyond 66 we have the third stages of production. So, depends up on the labour unit, we can find out at which level generally the stages of production are decided; whether it is the first stage, whether it is the second stage and the third stage.

So, first stage is up to a point where the marginal product of labour is equal to the average product of labour. Second stage is the point where total product of the labour is maximum, marginal product of labour is 0 and third point is beyond this. So, once we find out the labour where marginal product of labour is equal to 0 that is the definition of the second stage. Once we find out the maximum level of a p L that gives us the, may be beginning of the second stage. And when we find the marginal product of labour is maximum that gives us the point beyond which the law of diminishing return set in. So, in the next session we will talk about the cost of production, different types of cost. How the cost function is formulated; and what is the logic of different shape of the cost function, in the short run and in the long run. So, these are the session references. Generally, these are the references that are being used for the preparation of this particular sets.