Managerial Economics
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Lecture - 35
Theory of Production (Contd…) - I

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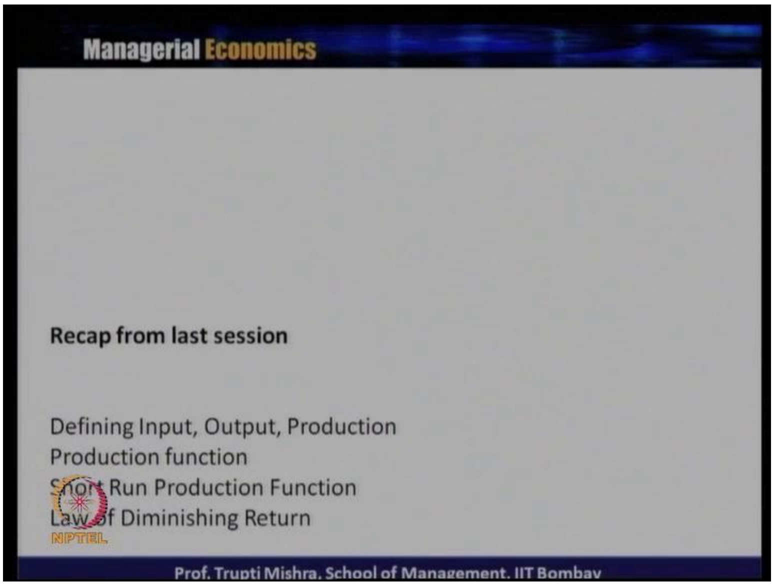

In today's session we will start few more topics on theory of production and cost. So, if you remember in the last session when we are introducing the different concept of production theory, we defined the input output and production. Then we defined what is a production function; which are the dependent variable, which are the independent variable; and then we segregated the production analysis into two way: one is the short run, other is the long run.

Then in case of short run production analysis we understood the law of diminishing return, generally how the total product decreases, when you are keeping one input fixed, when you are going on adding or increasing the other inputs. So, after a certain threshold point, generally the total product decreases, average product decreases and also marginal product leads to a negative segment.

In today's class we are going to discuss about the long run analysis of production, and through specifically the return to scale. So, if you remember in the last class we discuss essential difference between the short run and long run. Apart from the time dimension, it is about the usage of the inputs. In case of short run at least there is one input has to be fixed, typically if the production function is consisting of two productions, two inputs that is labour and capital. But in case of long run analysis, whenever there is a need to increase the output, whenever there is need to increase the production, generally the producer has to change the input combination, and in that way he has to change both the usage of inputs like labour and capital in order to increase the output.

And when the increase in the inputs takes place, whether the change in the output is proportional, more than that, less than that, that we will understand through the return to scale. So, mainly the long run analysis of production function will be explains through the return to scale.

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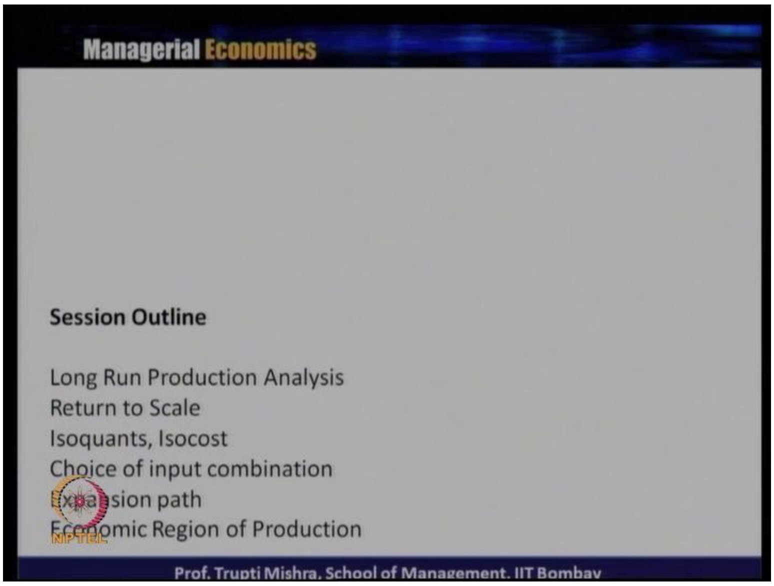

So, in today's class we will talk about the long run production analysis, mainly about the return to scale, then we will introduce the concept of isoquant and isocost, mainly to understand that how the choice of input combinations are being made. Then we will talk about the expansion path or typical if you call it different point of producers equilibrium. And then we discuss the economic region of production, what is the feasible region of production for the producer, given the number of isoquants or may be given the level of isocost line, what should be the economic region of production.

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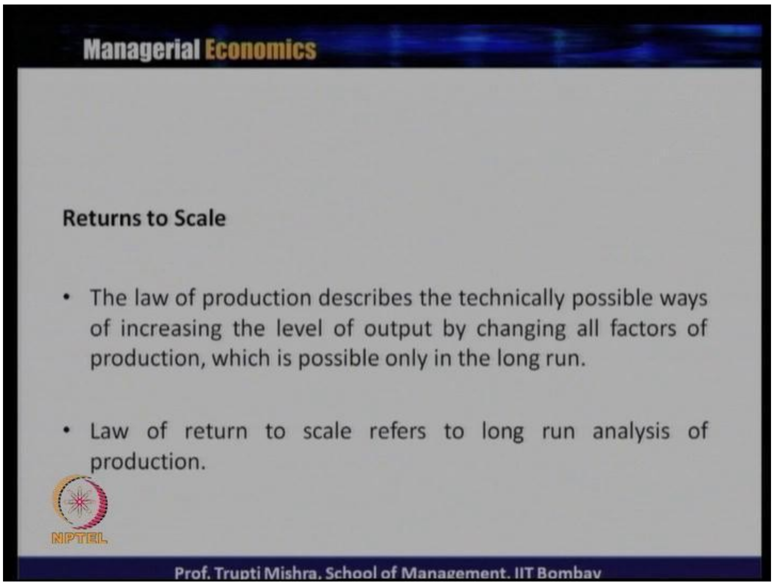

So, to start with, we will discuss the long run production analysis through the return to scale. And if you remember the law of production describes technically possible ways of increasing the level of output, by changing all factor of production which is possible in the long run. So, basically law of production focuses, the technically possible ways of increasing the level of output, by changing all factor of production basically by changing the input combination, and which is possible only in the long run because in the short run if you look at, you cannot do many combination of input because one input has to be fixed.

But in the long run since the output can be changed by changing all the inputs, in that scale number of factor combination can be developed. And that is the reason the law of production describes the technically possible way of increasing the level of output, by doing a ideal mix of both the input that is labour and capital.

So, law of return to scale refers to the long run analysis of production. And we talk about the scale relationship here, because all the inputs are changing and it brings either a proportionate change in the output less than proportionate change in the output, or more than proportionate change in the output.

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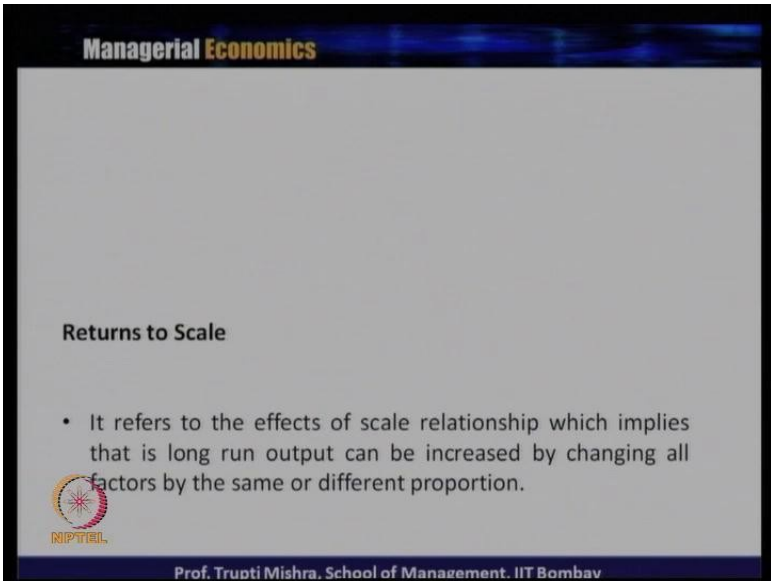

It refers to the effect of scale relationship which implies that the long run output can be increased by changing all the factors by the same proportion or different proportion. So, if there is one unit increase in the output, either that can be changed by half-half unit change in both the inputs, or may be one-one unit change in the output, that is one way to understand this. And the other way to understand this that if both the inputs increases by the same proportion, what is the proportionate change in the output? And when both the inputs change in different proportion, what is the different, what is the different outcome on the output? So, basically this is scale relationship which implies that along run output can be increased by changing all the factors by the same, or the different proportion.

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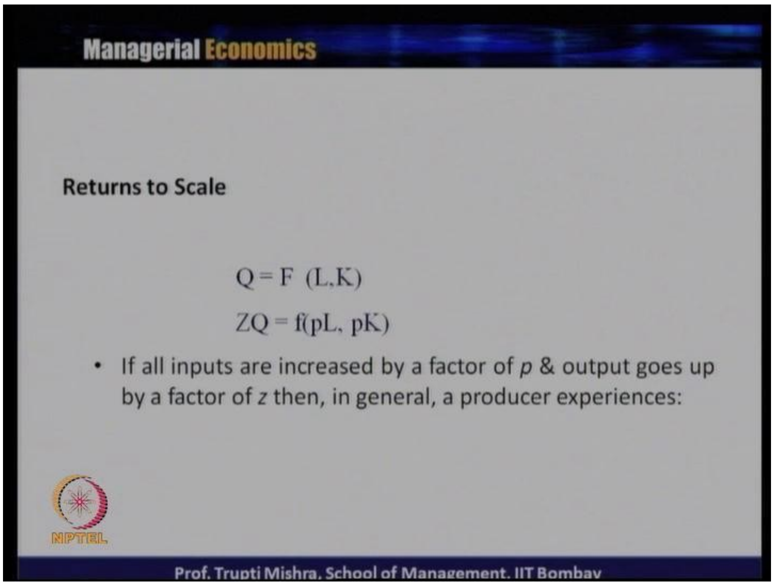

So, last class if you remember we took a production function which is, where Q is the dependent variable, where L and K is the independent variable. So, Q is output here, L and K is the, L is the labour and K is the capital. We simplified the production function by taking just only two variables, although there are number of other inputs are there which influence the output. Like if you remember, the technology, the time, the entrepreneurship, then we have land, then we have the other variable like technology, but here we have considered only two variables, two inputs that is labour and capital, to understand the relationship with the output.

Suppose if the, all the output increase by p and the output by factor Z then in general the producer experience like, now in order to understand scale relationship, let us understand that whenever we need to increase the Q we need to increase the input combination or we need to increase the inputs labour and capital. So, suppose in order to increase the output Q, if L is changed by p amount, and K is changed by p amount it means both the input that is change in same proportion, and if the output goes off by factor Z, then in general a producer experience three type of scale relationship. So, input getting changed in the proportion of p, and output is getting changed, with respect to change in the input that is in the proportion Z.

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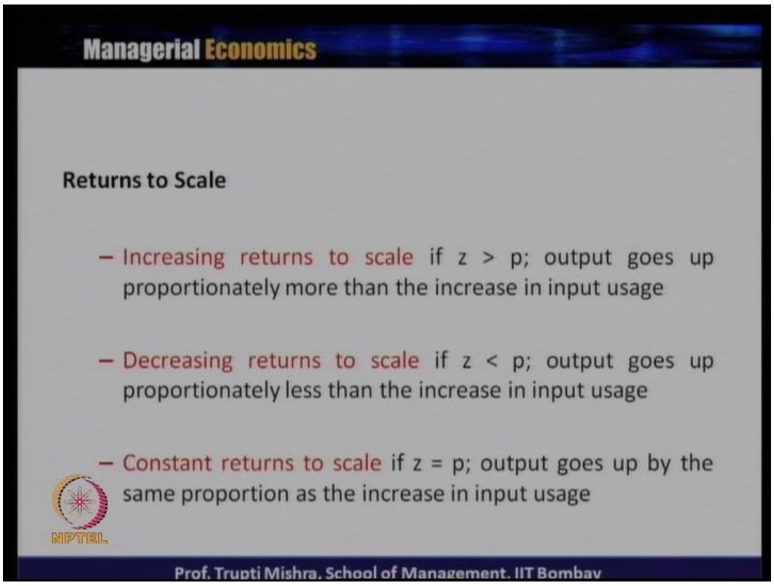

So, in this case the producer experience three types of scale relationships. One, increasing return to scale; two, decreasing return to scale, and last it is the constant return to scale. Depends on the value of z and p generally the return the different type of return to scale is defined. Like in case of increasing return to scale, the value of z will greater than the value of p. So, output goes up proportionately more than the increase in the input usage. So, p is the increase in the input usage, z is the increase in output.

So, in case of increasing return to scale how it happens? In case of increasing return to scale, if z is greater than p, then output goes proportionately more than the increase in the input usage. And in case of decreasing return to scale, if z less than p, output goes up proportionately less than the increase in the input usage.

And in case of constant return to scale, if z is equal to p, then output goes up by the same proportion as the increase in the input usage. So, numerically just to give a number, if input usage increases by 2 percent, and the output increases by 4 percent, this is the case of increasing return to scale, if input usage goes up by 2 percent, and output just increases by 1 percent that is decreasing return to scale; and in case of constant return to scale 2 percent increase in the input usage lead to 2 percent increase in the output and that is the reason this is the constant return to scale.

So, if you look at, the value of change in the input usage and the value of change in the output due to change in the input usage, that decides the scale relationship. If the increase in the output is more than increase in the input then this is the case of the increasing return to scale. If the increase in the output is less than increase in the input usage, this is decreasing return to scale; and if both the changes are proportional, both the changes are equal then this is the case of the constant return to scale.

Then let us understand, since we are going on adding a particular term over here that whether input goes by fixed proportion, or whether input goes by the different proportion both the things it is having some effect on the output. So, let us understand the concept of homogeneity over here that which one is the homogeneity homogenous production function.

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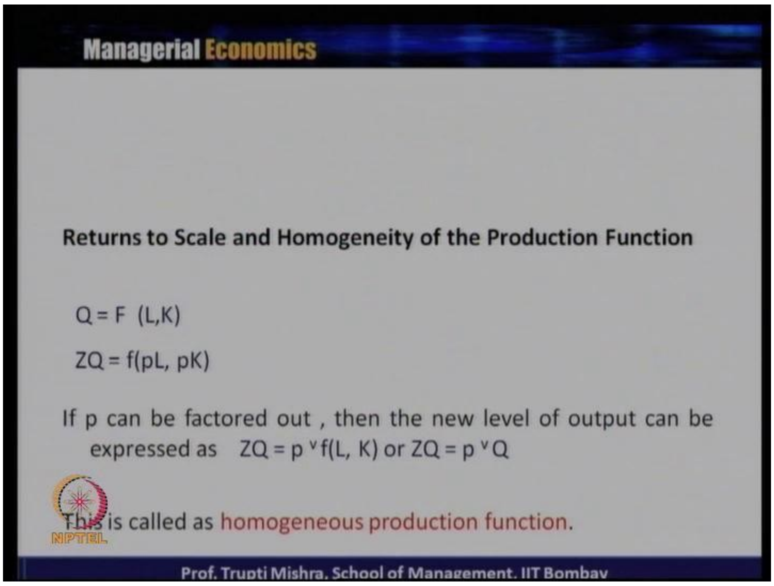

So, taking the same production function where Q is the function of labour and capital, and both the inputs are getting changed by the proportion p, and p, with respect to both L and K, and the output goes by the change by the proportion Z then in this case the value of Z and p that decides the return to scale. But here we will understand that on this basis how to find out a homogenous production function. So, if p can be factored out then the new level of output can be expressed as Z Q that is p to the power v which is a function of labor and capital, and Z Q that is p to the power v and that is to the Q which is the output.

Now to understand simply what is a homogenous production function? If mathematically we can take out the proportionate change in the input, then this is the case of your homogenous production function. So, if it is labour is changing by 2, capital is changing by 2, we can factor out; labour is changing by 2, capital is changing by 4, we can factor out; labour is changing by 3, capital is changing by 3, we can factor out; for example, labour is changing by 3 capital is factored by 6 we can factor out.

So, if the value of p can be factor out mathematically, then the new level of output will be p to the power v Q and this is a homogenous production function. So, degree of homogeneity or homogenous production function is 1, where the parameter associated with the variable is having a, having a, value which can be factor out. So, in this case if you remember, when the both the inputs are getting changed at the rate of p, and the final outcome that is Z Q which can also be also represented as p to the power v Q, in this case the value of v will decide that what is the degree of the homogeneity.

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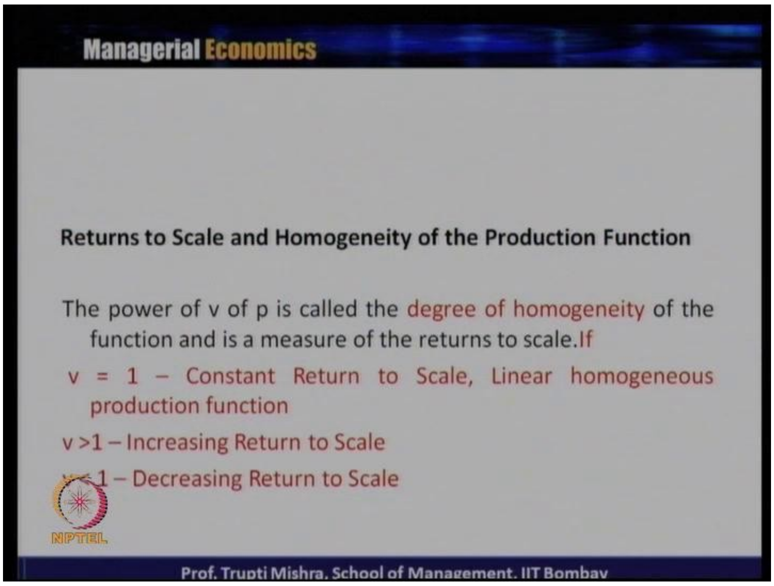

So, the power of v is generally called the degree of homogeneity of the function, and it is a measure of return to scale if v is taking a value of one constant return to scale or generally we call it a linear homogenous production function, if v is greater than 1 this is the case of the, increasing return to scale and if v is less than 1 then this is the case of a decreasing return to scale. So, we have reached to the output, which is like p to the power v and Q and the input changes by the proportion p.

Now this power of v, through the value of v, we can find out what is the degree of homogeneity, in the production function. So, if v is equal to one, this is the case of constant return to scale, and we also call it as linear homogenous production function. If it is greater than one increasing return to scale because the proportionate change in the output is more than the proportionate change in the input, and v is less than one this is the case of a decreasing return to scale.

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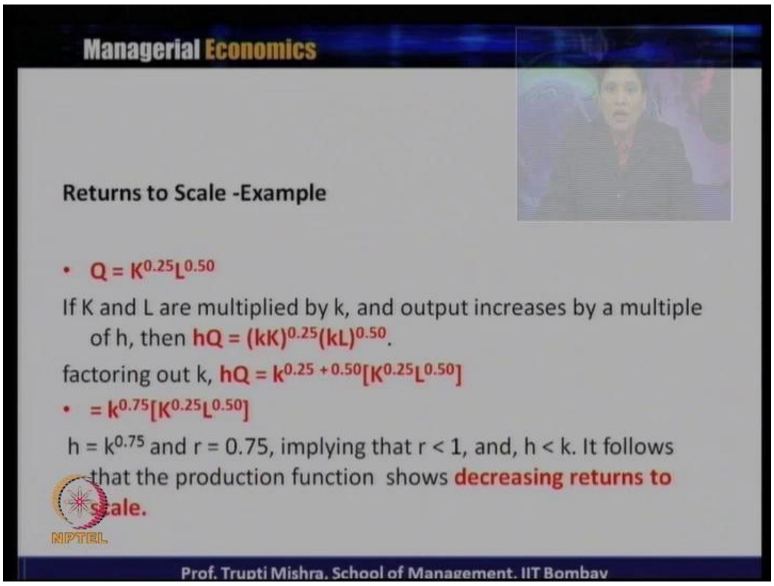

Now we will just take a example to understand this constant return to scale, increasing return to scale, and decreasing return to scale. Given a production function, how to identify whether it is following a constant return to scale, increasing return to scale or a decreasing return to scale.

Suppose the production function is Q which is K to the power 0.25, L to the power 0.50, now how to identify whether the production function is showing, which return to scale to understand. This let assume that K and L are multiplied by the factor K, and output increases by multiple of h; then the Q will be h q whereas the, both the input that is capital and labour, they will change by proportion k. So, that leads to K k to power 0.25 small k L to the power 0.5. Now if you are taking out, factoring out K because if you remember in case of homogenous production function also if you can take out the factor then it is the case of the homogenous production function. So, in this case when you are factoring out K then it comes as h Q which is equal to K to power 0.25 plus 0.50 and again this K to the power 0.25 and 0.50 L to the power 0.50.

So, that again if you simplify this then K to power 0.75 and K to power 0.25 and L to the power 0.50 within the bracket. So, here what is the value of h or what is the value of may be, the, by which proportion the output is changing, h is equal to K 0.75 and r is equal to 0.75, implying that r is less than 1 and h is less than k. So, it follows that the production function shows decreasing return to scale, if you remember, if v is talking a value less than 1 which is here actually the r, we are considering this as r, this case if r is taking a value less than equal to less than 1 then in this case it is the case of a decreasing return to scale. When r will take the value which is equal to 1 then this is the case of the constant return to scale; and when r which is taking a value greater than 1 this will take as the increasing return to scale.

So, how the entire problem has been solved to understand the return to scale, initially the production function which is the power 0.25 and L to the power 0.5, in order to understand return to scale relationship we have multiplied, we have changed the input, input proportion by the amount K which leads to change in the output proportion by h. If you simplify this then the K has been factored out K takes the power K to the power of 0.75, and here if you look at then h is equal to K 0.75 which is a power of 0.75 so there is calls about the degree of production function which is 0.75. So, since 0.5 is less than 1 then in this case we can call it this is a decreasing return to scale.

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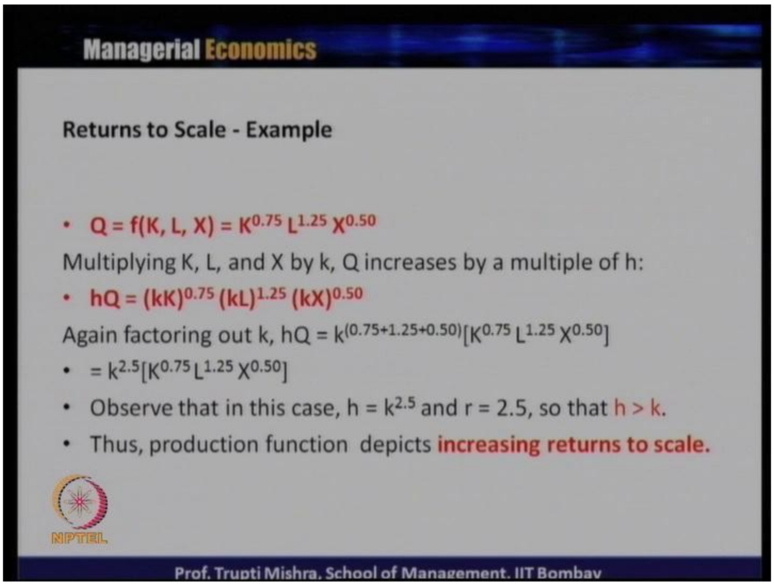

Now, similarly we will take one more example to understand what is the return to scale? Here if you look at, this production function is different from the previous production function. Now, what is the difference over here? The difference over here is, here the production function the Q is not only the function of capital and labour, rather this is a function of three inputs that is capital, labour and X.

So, there are three inputs which decide the output. So, Q is a function of capital, labour and x or we can say that Q is a factor of K, L and x. Now Q is equal to K to the power of 0.75, L to the power 1.25, and X to the power 0.5. In order to understand the scale relationship, we will multiply K, L and X by the amount small k, and Q increases by the multiple of h.

So, when change the input proportion by the small K that is K k to the power 0.75, small k L to the power 1.25, small k X to power 0.5. So, when all the inputs changes in the proportion of K, then the output gets change in the proportion of h. If you can factoring out K then h Q is become K to the power 0.75 plus 1.25 plus 0.5 and in bracket; again this is the same production function that is K is equal to 0.75 L is equal to 1.25 and x is equal to 0.25. Simplify this, we get a value of K to the power 2.5 and the production function. So, in this case h is equal to K 0.25, r is equal to 2.5. Since r takes a value greater than 1, because r is having a value of 2.5; and h takes a value which is greater than K, then the production function depicts increasing return to scale.

Because if you remember the power of v, the v if it is equal to one it is constant return to scale, if it is greater than 1 it is a increasing return to scale, and if it is less than 1 this is a decreasing return to scale. Since in this case it is taking a value which is greater than 1, this is the case of the increasing return to scale.

So, if you look at whenever in through the return to production function, if you want to know that what it is showing, what kind of scale relationship it is showing, generally the best way to find out is to change the input proportion; and the similar way, what is the effect on the output; then factoring out the change in the input proportion and finding out what is the degree of homogeneity. If the degree of homogeneity is equal to 1 then it is a case of constant return to scale, if degree of homogeneity is greater than 1 it is increasing return to scale, and if the degree of homogeneity is less than 1 it is a case of decreasing return to scale.

So, in the short run analysis we generally understand the relationship between the input and output through the law of diminishing return; and in case of long run we understand the relationship between the input and output, in case of the, by taking the help of return to scale.

Now we will understand, or now we will get into the optimum, optimization problem of the producer, where the producer is always wish to optimize the output, with the minimum cost or the minimum input combination. And to understand the optimum input combination or to understand the maximization of output we need the help of the isoquant and the isomap. So, first we will introduce the concept of isoquant, isomap and then we will see how to achieve the lowest input combination, or how to achieve the lowest possible cost in order to maximize the output.

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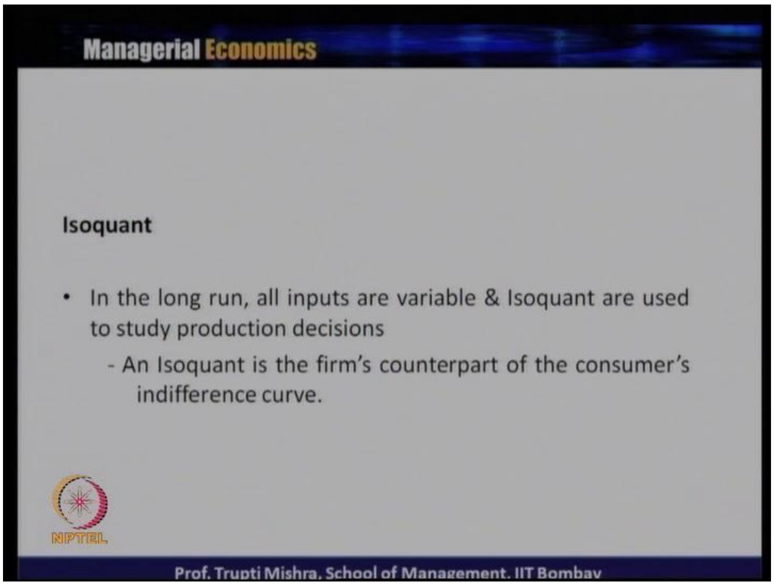

So, if you remember your indifference curve what we discuss in the case of a consumer theory. So, in case of production analysis, it is just like the indifference curve what we discussed in the consumer theory, the isoquant serve the same kind of utility in case of the production analysis. So, in the long run, all inputs are variable and isoquant are used to study production decisions. Now, what is an isoquant? An isoquant is the firms counterpart of the consumer indifference curve. So, if you remember the indifference curve, it is nothing but the producer indifference curve, in case of theory of production or in case of the production analysis.

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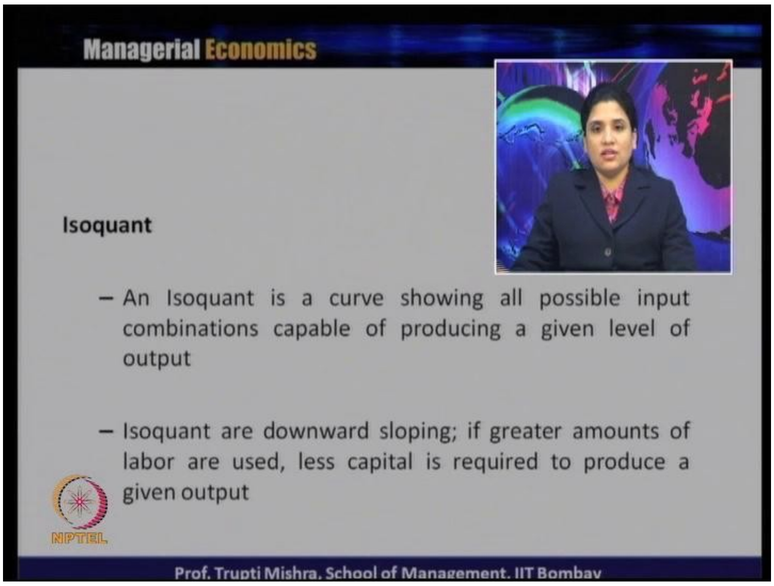

So, isoquant is a curve, showing all possible input combinations capable of producing a given level of output. And isoquant are downward sloping, if greater amounts of labor are used less capital is required to produce the given output.

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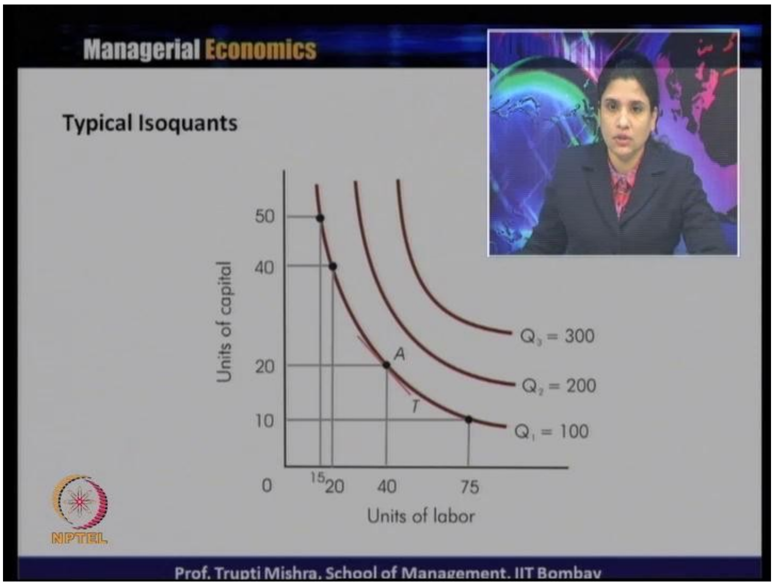

So, now let's find out how the isoquant is being developed. So, suppose our production function is Q which is a function of capital and labour.

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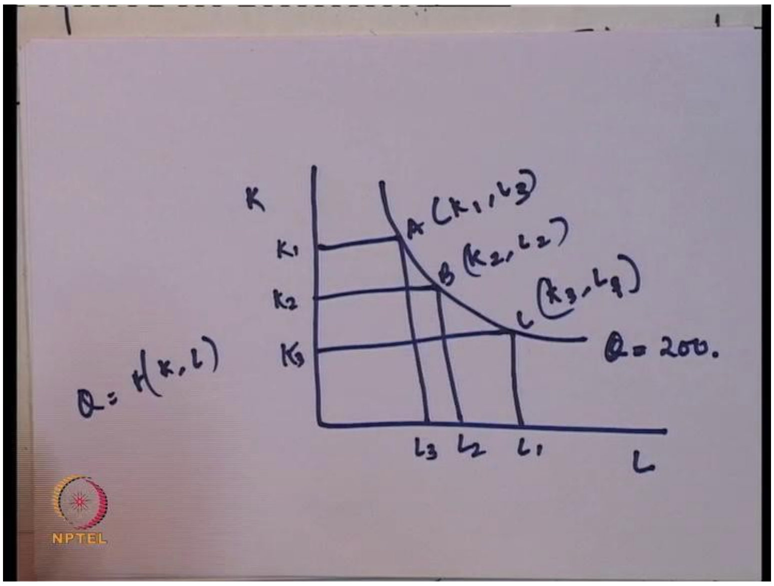

Let us take labour here, and capital here. Now what is a isoquant? Isoquant is the level of, or isoquant is the locus of different combination of capital and labour, which keeps the same level of output. So, if Q is equal to 200, and in this case isoquant represents the different combinations of capital and labour, that will gives the output which is equal to 200 unit. So, suppose this is K 1, this is K 2, this is K 3, then this is L 3, this is L 2, this is L 1. We have 3 points A, B and C. So, point A gives a combination of K 1, L 1, point B gives us a combination of K 2, L 2, point C gives the combination of K 3 and L 1. This one is L 3 K 1, this is K 2 L 2 and this is K 3 and L 1. So, irrespective of the combination whether it is point A, point B, or point C they gives the, they produce the same level of output.

So, A is gives a combination of more of capital, less of labour; B gives a moderate amount of capital and labour; and C gives more of labour and less of capital. So, you can say that A is the capital-intensive production process because if it uses more of capital and less of labour. And C is the labour-intensive production process because it uses more of labour and less of capital. So, isoquant is one, where the different combination of the input that is capital and labour, they gives the equal level of output.

So, irrespective of the capital and labour combination, they give the, producer produce the same level of output. So, one thing we need to assume here is, since the different combination of capital and labour gives the same level of output, it means the capital and labour are closely substitute to each other. Both the inputs they are closely substitute to each other, and that is the reason if you look at, irrespective of changing the output level they are just changing the input combination still they are producing the same level of output. So, whether a producer chooses a production process A, chooses a production process B, chooses a production process C only the input combinations are getting changed, otherwise the output produced become same.

So, isoquant is the locus of combination of different quantities of capital and labour which gives the same level of output. Now if you look at the graph over here, Q 1 is equal to 100 units of output, Q 2 is equal to 200 units of output, and Q 3 is equal to 300 units of output. In the y axis we are taking capital, in the x axis we are taking labour. So, what is the difference between Q 1, Q 2 and Q 3 over here.

When we use combination of more of labour and more of capital then you produce a higher level of output. And if it is a higher level of output, it is a higher level of isoquant. Similarly, if you still uses more of capital more of labour then more than 200 units of output, then it leads to again more level of output, the producer produces more output because they are using more of capital and more of labour, and that is the reason the output level is 300 units. So, Q 1 is 100 units, Q 2 is 200 units and Q 3 is 300 units, and all these different levels of output represent different isoquants.

So, Q 1 is 1 isoquant, Q 2 is the other isoquant, and Q 3 is the third isoquant; and the essential difference between these 3 isoquants is that, in case of higher isoquant, higher amount of capital labour being used to produce the output. So, higher isoquant always gives a higher level of production, and lower isoquant always give a lower level of production. Now what is marginal rate of technical substitution.