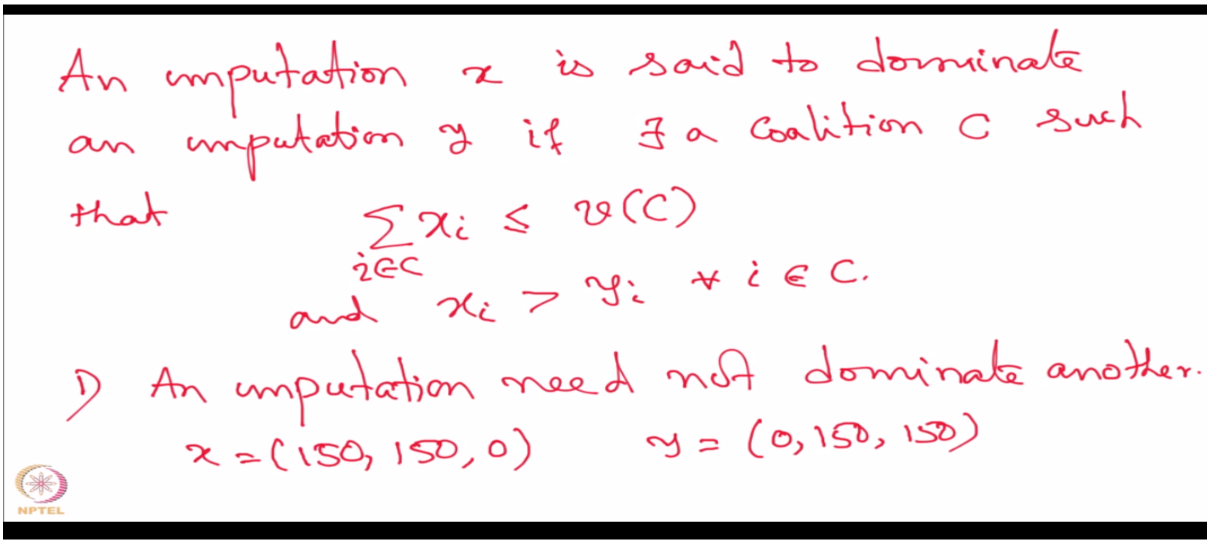
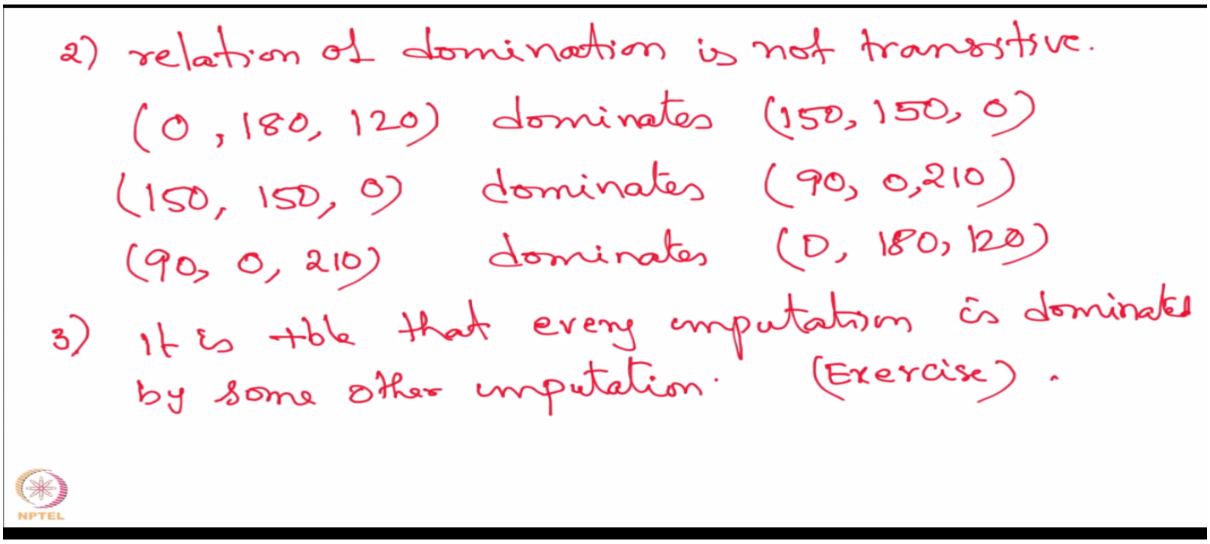
Game Theory
Prof. K. S. Mallikarjuna Rao
Department of Industrial Engineering & Operations Research
Indian Institute of Technology - Bombay

Lecture - 36
Cooperative Games: The Core

In the previous lecture, we have seen that an imputation is an allocation of payoffs to the individuals in such a way that the allocation satisfies individual and collective rationality.

Domination of imputation





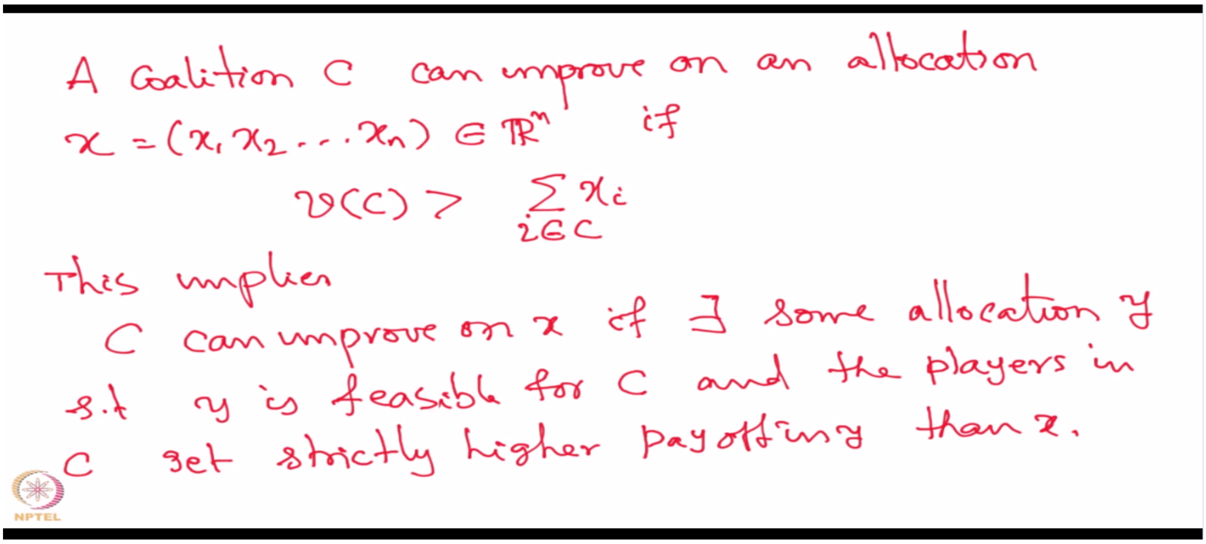
So, now that we have recalled the definition of imputation and all. Now, we will go to introduce what is called the core.

Core

Given TU game (N, v), where N is the set of players and v is the worth of coalitions. Core is the set of all imputations which are coalitionally rational, i.e.,

$$\text{Core}(N, v) = \left\{ x \in \mathcal{R}^n | \sum_{i \in N} x_i = v(N), \sum_{i \in C} x_i \geq v(C) \; \forall C \subseteq N \right\}$$

Consequences of the definition of Core



From above we can say that,

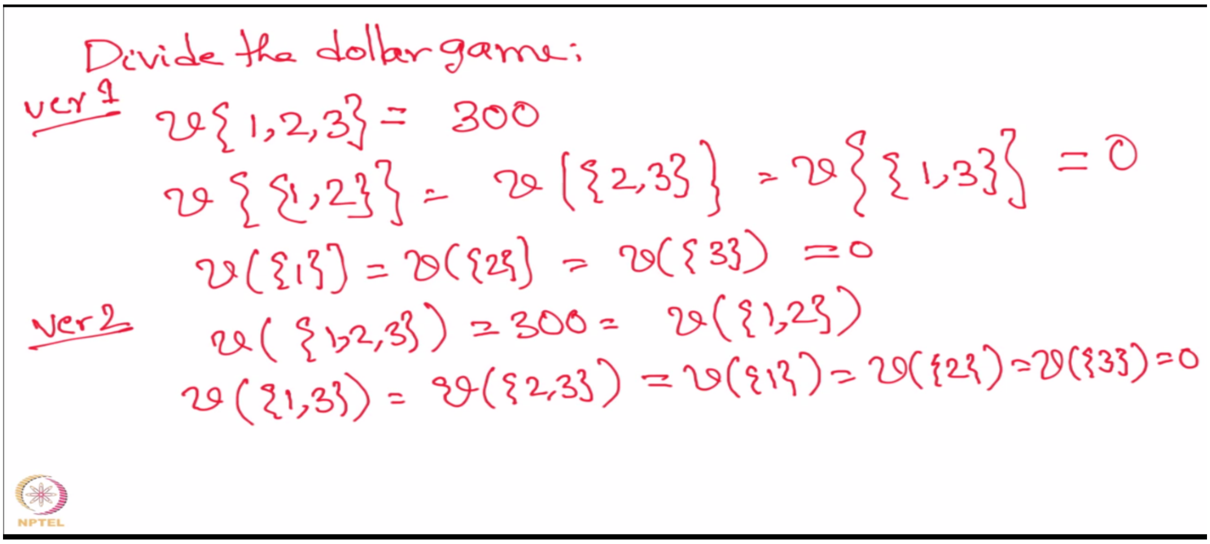
An allocation x is said to be in Core of (N, v) iff x is feasible for N and no coalition can improve upon it, i.e.,

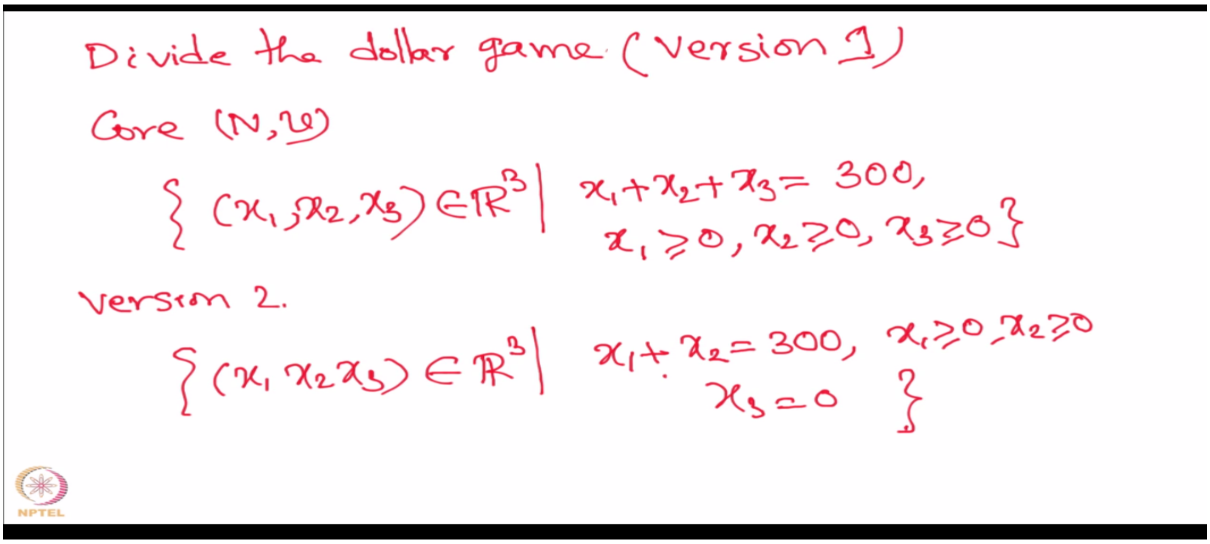
$$\sum_{i \in N} x_i = v(N)$$
$$\sum_{i \in C} x_i \geq v(C) \; \forall C \subseteq N$$

An allocation x is said to be not in core, if there exists some coalition C such that all players in C would do strictly better in some other allocation.

Examples

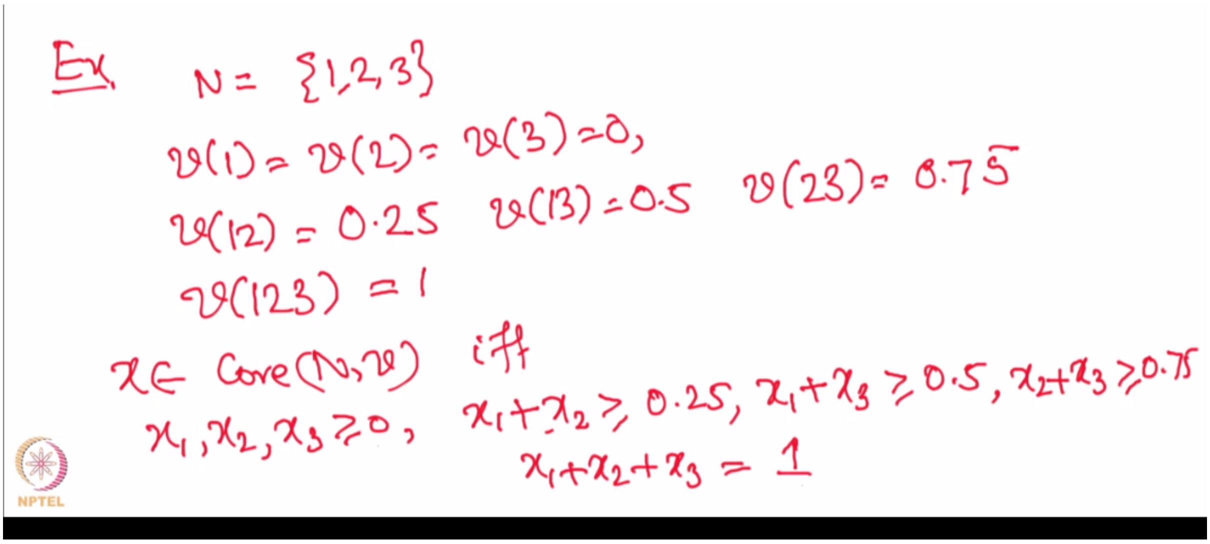
Example 1: Divide the dollar game

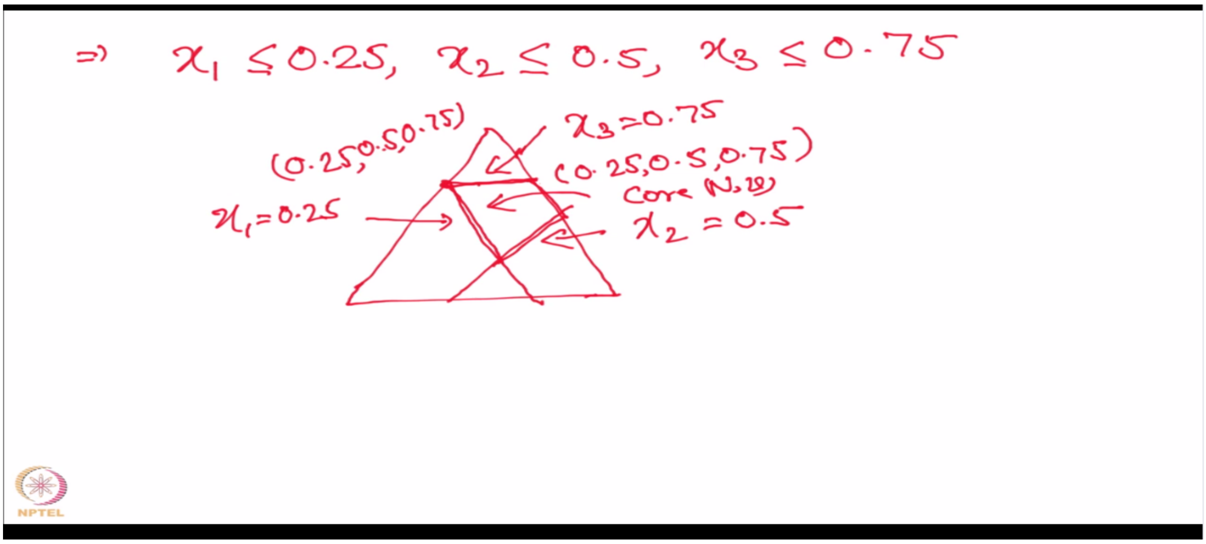




The core in above two versions of divide the dollar game is turn out to be infinite set.

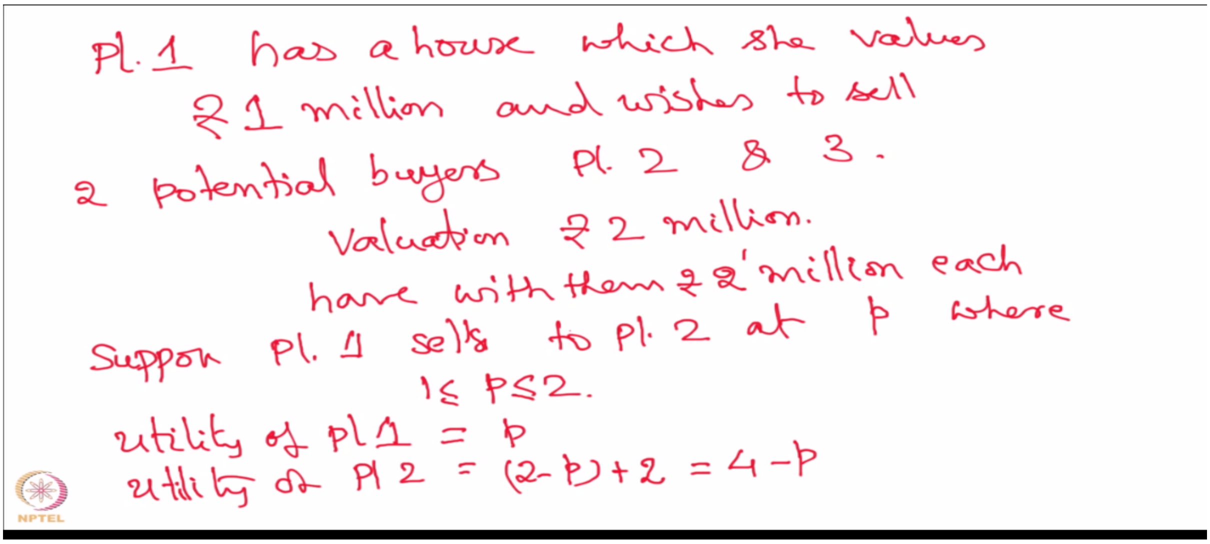
Example 2:

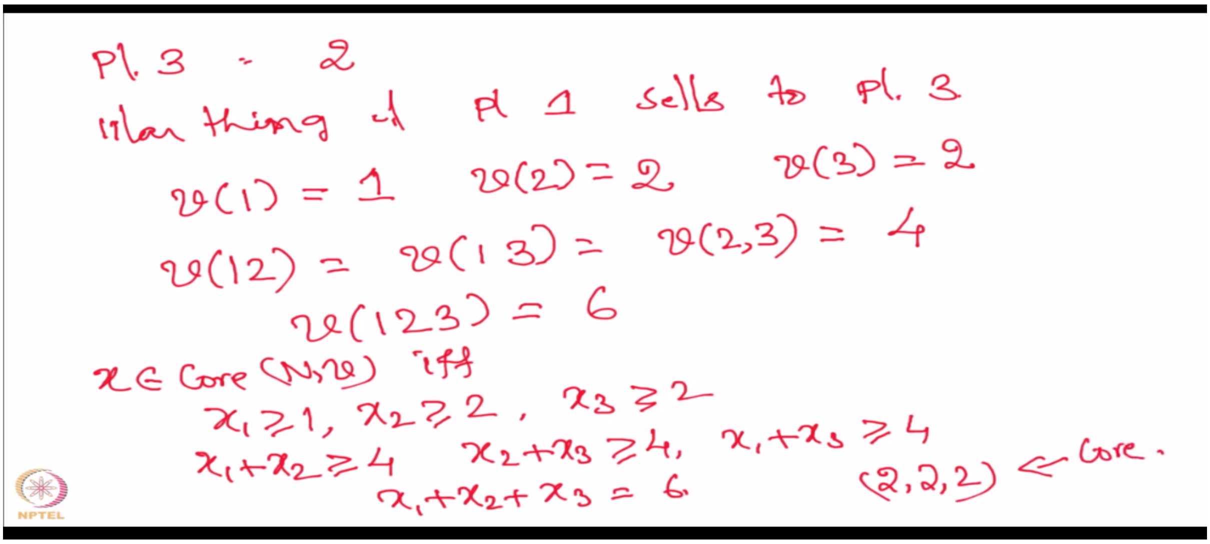




The core of this game is the middle region of the above figure. All the points in this region constitutes core.

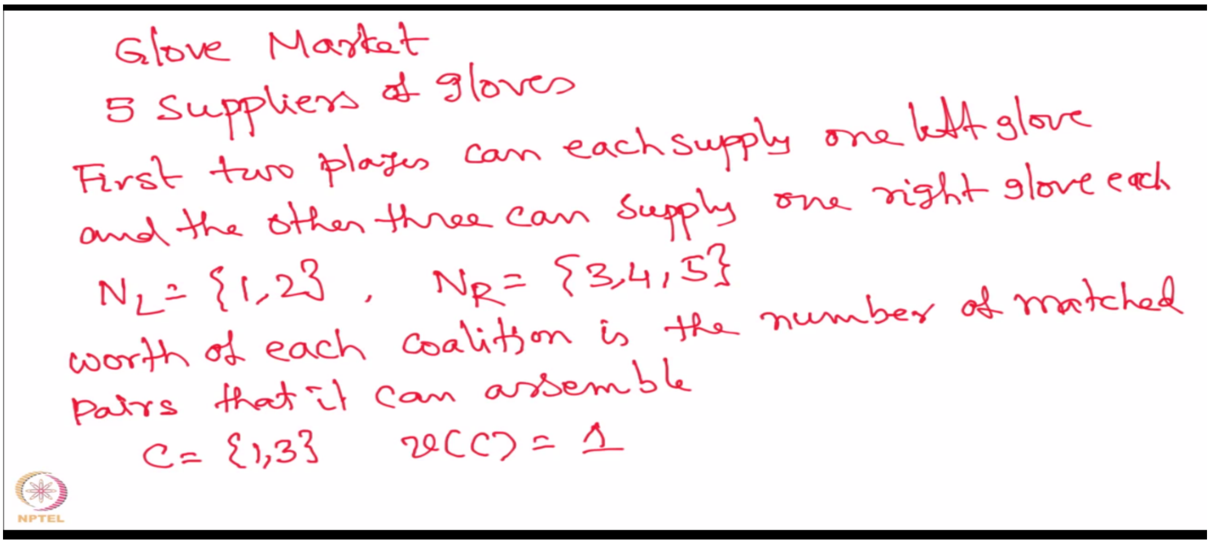
Example 3:

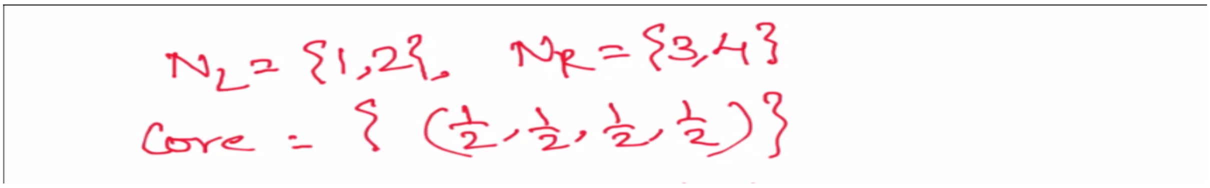




$(2, 2, 2)$ is unique core of the above game.

Example 4:





$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ is the core of above game. So with these examples, we conclude this lecture.