Exercise & Sports Biomechanics
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Week 05
Lecture 24
Angular Kinematics

[Hello, everyone! Welcome back to this course. Today, we are going to discuss angular kinematics. So, we will start with the introduction to **angular kinematics**. We have already studied angular motion].

Angular motion:

Angular motion is nothing but movement around an axis. And angular motion will always move around an axis. Angular motion is always measured in degrees because most human movements, particularly human joints, undergo angular motion. That is why we measure joint angles in degrees. A **goniometer** is the equipment we use to measure joint angles, and we have already discussed absolute and relative angles.

 We have provided two pictures. The first picture represents the absolute angle, and the second picture represents the relative angle. We have a goniometer where the two arms are fixed on two segments, and the joint center is fixed. The joint center is the axis of rotation, and based on one segment being fixed, the other segment will move. We measure the joint angle using a goniometer, and some joints have an instant center of rotation, meaning for every degree of movement, the center of rotation changes. This can particularly be seen in the knee joint, and you can observe it in this picture where the knee joint moves from a specific zone.

Rotation to the other, you can see the center of rotation changing. And here we have the example of angular motion. The angular motion will have 360 degrees of rotation, and the maximum angular motion is 360 degrees. Half the circle is 180 degrees, and a quarter of the circle is 90 degrees. Here you see a picture of a person who is holding a dumbbell. The dumbbell biceps curl exercise can be done from 90 degrees to 140-150 degrees, they will flex the arm, particularly the elbow, and then extend. One flexion and one extension is equal to one repetition. So, if any one person wants to perform biceps curls, say for example, 10 repetitions per set. So, he will undergo one flexion and extension particularly similarly, and he will undergo 10 repetitions.

The next picture shows the **clockwise and anti-clockwise movement** using the golf club. So, when the golfer goes for a backswing, which is a clockwise movement. In angular motion, the clockwise movement is called negative movement or movement in the negative direction. And the counterclockwise movement is when the golf club hits the ball and goes for a follow-through. And this is called counterclockwise movement, which is positive angular motion. So, the angular motion can be categorized into negative angular motion, which is happening in a clockwise direction. And counterclockwise movement is called **positive angular motion.**

In angular motion, the **unit of measurement for angular motion is degrees**, and apart from this, when the angular motion is happening over a larger distance, that means when it is moving in circles, say for example 90 degrees, 180 degrees, 360 degrees, and the product of a particular rotation.

In angular kinematics, we have the fundamental metrics, namely angular distance, angular displacement, and time. So, these are the fundamental basic metrics whenever we go for the analysis of angular motion.

The units of measurement for angular motion are degrees. And when the distance increases, the next unit of measurement is radians. So, 57.3 degrees is equal to 1 radian. And a complete movement is 360 degrees in angular motion. So, one complete circle is equal to one revolution.

 If you see in this picture, the one radian value is given, which is 57.3 degrees. And the pendulum is moving from one position to the other. So, when it moves from one position to the other, the distance covered by the pendulum is measured in degrees. And the displacement is also the same. But when the pendulum comes back to the original position, the displacement is zero.

Because the difference between the initial position and final position should be the angular displacement. But here... If you see the angular distance, if the pendulum is moving forward and backward, the total distance covered by the pendulum is calculated in degrees or in radians. And the angular distance is denoted by the Greek letter π. And the angular displacement is denoted by the Greek letter θ, which we will see in the next slides.

And If you see the comparison of degrees to revolution, 90 degrees is 1/4th of the revolution, half the circle is 180 degrees, which is 1/2th of the revolution, 270 degrees is equal to 3/4th of the revolution, and a complete 360 degrees, one circle, is equal to one revolution. And this is how we measure the angular distance. And when it comes to angular speed and velocity, angular speed is measured by the formula,

$$\text{angular speed } (\sigma) = \frac{\text{angular distance } (\phi)}{\text{change in time } (\Delta t)}$$

And angular velocity $(\omega) = = \dfrac{\text{angular displacement } (\theta)}{\text{change in time } (\Delta t)}$

Whereas the angular speed is a scalar quantity and angular velocity is a vector quantity, and similar to this, the angular distance is a scalar quantity, whereas the angular displacement is a vector quantity, and we have an example to explain how the angular speed is calculated and how the angular velocity is also calculated. If a person flexes his elbow joint 120 degrees in 2 seconds, what will be the angular velocity, here we use the formula angular displacement divided by the change in time to find out the angular velocity. So, the angular displacement is 120 degrees and the change in time is 2 seconds.

$$\omega = \frac{\theta}{\Delta t}$$

$= 120/2$, angular velocity $(\omega) = 60°/s$

So, we use the formula and we find out that the angular velocity is equal to 60 degrees per second. And at the same time, if we want to find out the angular distance, we need to find out the measure the angular distance in degrees and time in seconds, where we can find out the angular speed, which will be measured in degrees per second, radians per second, or revolutions per second. The same unit of measurement is applicable to angular velocity; the only thing is we need to mention the direction.

[next comes angular acceleration.]

Angular acceleration:

angular acceleration $(\alpha) = \frac{\text{change in angular velocity } (\Delta\omega)}{\text{change in time } (\Delta t)}$

The angular acceleration is nothing but the rate of change of angular velocity with respect to time is angular acceleration. The change in angular velocity and change in time determine the angular acceleration, and we have an example where the elbow is moving, that is, it is undergoing flexion movement, and if a person extends his elbow with the change in angular velocity from 30 degrees per second to 60 degrees per second in 0.5 seconds, what will be the angular acceleration? For this we use the **formula:**

$a = \frac{\omega(\text{final}) - \omega(\text{initial})}{\Delta t}$

$a = \frac{60°/s - 30°/s}{0.5 \text{ s}}$

$a = 60°/s^2$

What is the final angular velocity?

It is 60 degrees per second. The initial angular velocity is 30 degrees per second. So, we divide the change in angular velocity by change in time. And hence, the angular acceleration is 60 degrees per second squared, and we can find out the knee angle duration during running, and a person is running on a treadmill. You can find the instantaneous angular motion, which means we can see the knee joint is undergoing flexion and extension movement. We have instantaneous values which are coming through the video analysis, and the graph is given here, and you can find out the graph angle and time. So, these are the two pieces of information with regard to the knee angle joint, and knee flexion and extension degrees are mentioned against the time. So, through the video analysis, we find out the angle against the time in the graph.

[Here, we are going to see the relationship between linear and angular kinematics].

When we discuss linear and angular displacement, you can use the formula that states curvilinear distance is equal to the radius of rotation and angular displacement; this is represented by theta. S = rθ

This is represented by theta. So, two common pieces of information we have to keep in mind when converting angular distance into linear. So, one is that radians are the common factor used to convert angular displacement into linear.

Whenever we measure or convert angular displacement, the angular displacement should be mentioned in radians. And the second one is that the radius of rotation should be in the same unit of measurement as the curvilinear distance. So, the curvilinear distance and radius of rotation should be in the same unit of measurement. And the same is applicable to linear and angular velocity. And linear and angular velocity have tangential velocity and radius of rotation. $V = r\omega$

So, V represents the tangential velocity, which is linear. R represents the radius of rotation, and omega represents angular velocity. The radius of rotation and angular velocity determine the linear velocity. And as we discussed already, the angular displacement and angular velocity should be measured in radians. At the same time, the radius of rotation is measured in meters.

And you can see here, linear and angular velocity. The linear and angular velocity can be seen in this picture. The first picture shows three different radii of rotation. So, the farther or greater the radius of rotation, the farther the distance the point will move. So, we come to understand that the greater the radius of rotation, the longer the distance an object can move. And in the second picture, the greater the velocity of the ball or the greater the velocity of the bat, the farther the ball will travel. The greater the angular velocity of the baseball bat, the farther the struck ball will travel when other conditions are equal. Here is an example of two baseballs being consecutively hit by a bat. The first ball is hit 20 cm from the bat's axis of rotation, and the second ball is hit 40 cm from the bat's axis of rotation.

If the angular velocity of the bat was 30 degrees per second at the instant that both balls were contacted, what was the linear velocity of the bat at the two contact points? Here we use the formula to find out from the known factors, that is, the radius of rotation,

$R_1 = 20$ cm

$R_2 = 40$ cm

$\omega_1 = \omega_2 = 30$ rad/s

When we find the solution, the formula to be used is the equation relating linear and angular velocity. $V = r\omega$

And $V_1 = (0.20$ m$) (30$ rad/s

$V_1 = 6$ m/s

$V_2 = (0.40$ m$) (30$ rad/s$)$

$V_2 = 12$m/s

That is, the first struck ball will travel with the velocity of 6 meters per second, and the second one will travel at the velocity of 12 meters per second. Here we come to an understanding that the point of contact, so the greater the radius of rotation, the higher the angular velocity of the ball when the ball is struck at a better radius of rotation. So, we see the linear and angular acceleration relationship.

So, whenever we have tried to solve the linear and angular relationship, there are two perpendicular linear acceleration components in this chapter. So, one is tangential acceleration. So, which is acting along the path of the angular motion. So, the second one is perpendicular to the path of angular motion, that is radial acceleration.

So, when you want to find out the tangential acceleration, so you need to use the formula $a = \frac{V2-V1}{t}$. So, V represents tangential velocity. And similarly, when you want to find out the radial acceleration. So, you need to use the tangential velocity by the radius of rotation. The formula to find out radial acceleration, ar = v^2/r

Here, we are going to discuss the unit of measurement. So, the unit of measurement is the international system of measurement we use today. It was introduced in 1875, and by the Treaty of the Meter, which was agreed upon by 17 countries to follow the international system of units of measurement. That includes the meter for length, the kilogram for mass, and time in seconds.

LINEAR KINEMATICS	FORMULA	UNIT	ANGULAR KINEMATICS	FORMULA	UNIT
Distance(d)	d	Meter (m)	Angular Distance(θ)	θ	Radians (rad)
Displacement (ds)	S= Final position − initial position	Meter (m)	Angular Displacement (θ)	θ	Radians (rad)
Speed (s)	s = d/t.	meters per second (m/s)	Angular Speed(σ)	σ = θ/t.	Radian per second (rad/s)
Linear Velocity (v)	v = s/t.	meters per second (m/s)	Angular Velocity(ω)	ω = θ/t.	Radian per second (rad/s)
Linear Acceleration (a)	a=Δv/Δt	meters per second squared (m/s²)	Angular Acceleration(ω)	α=Δω/Δt	Radian per second squared (rad/s²)

If you look at the international system of units of measurement, which is the metric system, you can subdivide any number by a fraction of 10. Except for time. So, time is measured in seconds, which are based on 60. There are also other measurements, which are called the English system of measurement. So, mass is measured in pounds, distance is measured in yards, and smaller distances are measured in inches.

Although it was introduced by the British, it is now followed by the Americans. But throughout the sports science community and the scientific community, they practice and use the international system of units of measurement, which is the metric system. So, here we will learn about linear and angular kinematics, their formulas, and their units of measurement. So, here we have linear kinematics: distance, displacement, speed, angular velocity, and angular acceleration. And we have already seen the formulas. The units of measurement are meters for distance and displacement in linear kinematics.

And speed is measured in meters per second, and angular velocity is measured in radians per second. So, the unit of measurement of kinematics and its formula is given here for both linear and angular kinematics. So, the linear kinematics formula has already been seen in the previous chapters. Just we are going to see the unit of measurement. So, distance and displacement in linear kinematics are measured in meters, and speed and linear velocity are measured in meters per second, and linear acceleration is measured in meters per second squared.

Similarly, in angular kinematics, the angular distance and displacement are measured in radians, degrees, or revolutions, and the angular speed and angular velocity are measured in degrees per second, radians per second, or revolutions per second. Whereas the angular acceleration is measured in degrees per second squared, radians per second squared, or revolutions per second squared. To easily remember how to calculate the speed, we have the triangle method, which is DST. So, if you want to find out the speed, we already know that it is S = D/T, and we have two metrics: the distance, which is 100 meters, and the time taken to complete the distance is 10 seconds, and here we use the formula and find out the speed, which is 10 meters per second.

With the same formula, if we want to find out the distance, we use D= S*T, and the speed is already known, which is 10 meters per second, and the time is 10 seconds. When you multiply these two, you will get the distance, which is 100 meters, and here we want to find out the time. So, to find out the time, you need to use T = D/S. So, the distance is already known, which is 100 meters.

Speed is 10 meters per second. And when you substitute these two, you will get the time, which is 10 seconds. This is how you can easily remember and calculate the kinematic variables quantitatively.

[Next comes the graph and interpretation].

How to interpret the graph?
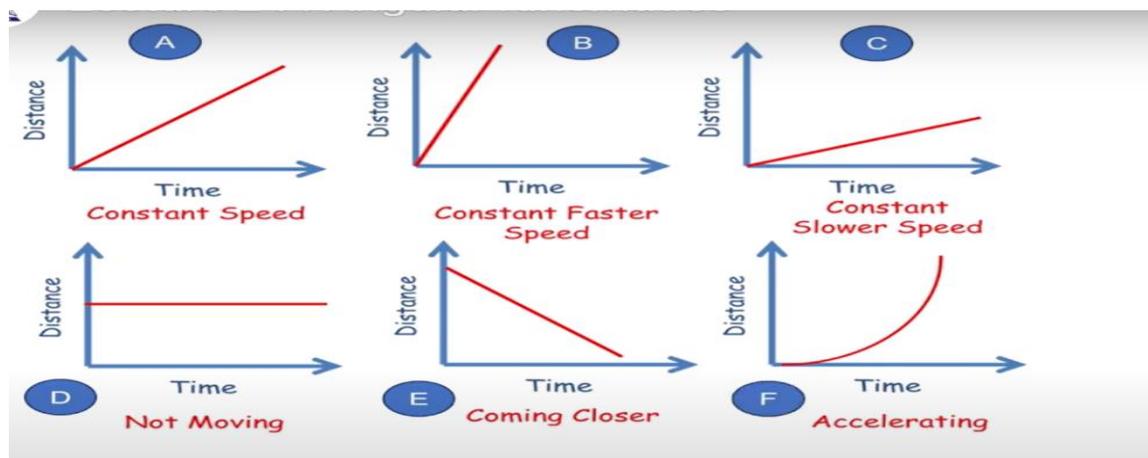
In linear kinematics, particularly the basics of the distance-time graph are given here. So, the distance is given on the Y-axis, and the time is given on the X-axis. So, when the distance-time graph has a diagonal line, that shows that the object is moving at a constant speed, and when the distance-time graph is steeper, right? And as shown in picture B, that means the object is moving at a faster speed.

And graph C shows that in the distance-time graph, the object is moving at a slower speed. And when there is a horizontal line in the distance-time graph, that means the object is not moving, as far as graph D is concerned. And the next graph, E, shows that there is an inclined line, which indicates that the object is coming closer to the original position. And the next picture, F, shows that the object is moving with acceleration.

That means instantly it is accelerating. This is how we can interpret the distance and time graph. Next, we have given the reaction time and time information of Usain Bolt's 100-meter performance in the 2008 Beijing Olympics. He clocked 9.69 seconds. And this data, this table gives you the data of the time.

That is each 10 meters' time and cumulative time, and the third one is each 10 meters and what is the speed of Usain Bolt in each 10 meters and what the velocity is. Because since 100 meters is a linear rectilinear motion, both speed and velocity are the same. And when it comes to acceleration, the acceleration is given for each 10 meters. And from this table, we have calculated a distance-time graph which shows a linear increase in the velocity. From this table, you can find out the distance-time graph and velocity-distance graph as well as the velocity and acceleration graph.

So, when it comes to the velocity-time graph, so, you can see the velocity on the y-axis and time on the x-axis. So, which is given in seconds. And initially, the velocity keeps on increasing. After the 6th second, you can find the constant velocity that is 0 acceleration.

And there is a decline in velocity in the final 2 seconds. That means, maybe in 80 to 100 meters, athletes obviously decelerate. That is a decrease in velocity. This is how we can interpret the graph through the velocity-time graph. As we discussed in the previous slide about Usain Bolt's performance with regard to the velocity-time graph.

So, here the distance and velocity are given. So, for each distance, you can find how the velocity increases from 10 meters, 20 meters. You can find out there is an instant increase, an exponential increase in velocity from 5.4 to 9.8 the moment he runs the first 20 meters. And thereafter, you can find a gradual increase in the velocity, but by the time he reaches 60 meters, you can find out 12.2 meters per second, 12.2 meters per second.

From 60 to 80, he is having constant velocity, which is zero acceleration. Further, from 80 to 100, there is a decline in velocity, or he is decelerating. So, this is how we can interpret the graph, and we can find out the performance of any player by the velocity-distance graph or velocity-time graph. And, you can find out the speed in kilometers per hour and distance in meters. And,

This is another way of expressing or interpreting the performance of Usain Bolt. So, here from the first 10 meters to 20 meters, you can see the exponential increase in the speed of Usain Bolt. Then, there is a gradual increase. Similarly, from 60 to 80 meters, you can find the speed at which he is running. That is 43.9 meters per second speed.

Kilometers per hour is the speed of Usain Bolt. Then, after 80 meters, between 80 and 100, there is a decrease in speed. So, this is how we can find out the performance of any player through a speed-distance graph, speed-time graph, velocity-distance graph, or velocity-time graph. So, this is how we can measure. And

Another method of expressing is instantaneous speed versus average speed. So, the instantaneous speed is given in a zigzag pattern, and the average speed is given in the black line. So, this graph shows how the average speed moves and how the instantaneous speed moves. So, similar to the previous graphs, we are going to see the angular velocity versus time graph. So, in the first graph, the angular velocity and time graph is given.

So, angular velocity is given on the y-axis, and time is given on the x-axis. And the flat line shows that the object is moving at a constant angular velocity. That means the flat line shows zero acceleration. And then the second graph shows that there is a diagonal line against the angular velocity, which is mentioned on the y-axis, and time is on the x-axis.

So, this shows that there is a constant increase in angular acceleration. And similarly, in the third graph, right? So, there is a steep increase in the angular velocity, which indicates there is an increase in angular acceleration. And next comes the angular acceleration versus time graph. So, in this graph, angular acceleration is given on the y-axis, and time is given on the x-axis.

And the flat line shows that there is a constant angular acceleration. And the second graph shows a diagonal line, which indicates that there is an increase in angular acceleration. So, this is how we have to read and interpret the graph with regard to linear and angular kinematics.

[Thank you, and meet you in the next video].