Symbolic Logic
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Lecture – 29
More on Conditional Proof
Strengthened Version
Its Formats and Procedure

Hello, how are you today? We are going to start the lesson for today, this is our module number 29 for the Symbolic Logic course the NOC course that we have being doing. You may remember that, we were talking about the limited scope assumption proofs and we have already looked in details at one of the procedures called the Indirect Proofs.

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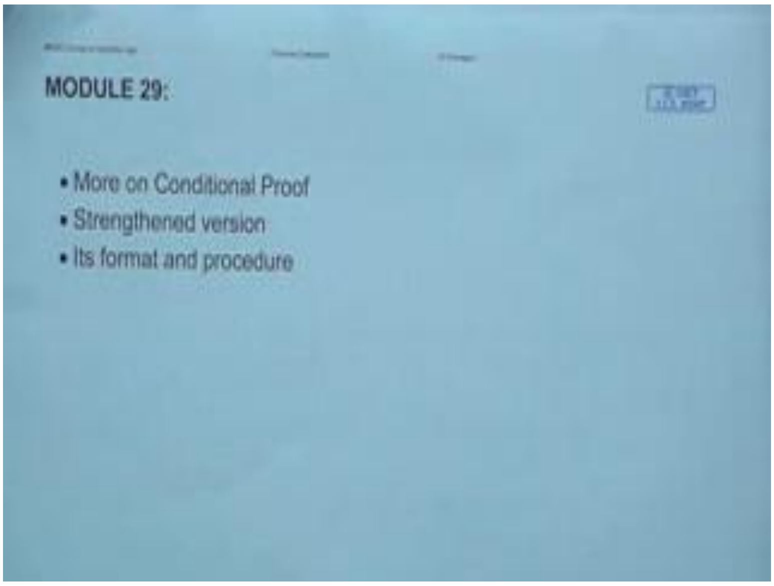

So today we are going to look in to what is known as the Conditional Proof. We have already done this preliminary version any way. So you have some idea about what the conditional proof is like how it moves and what are it is assumptions, but there is still more to learn specially because we have said that there is a a preliminary version therefore, there is the anticipation for what we call the advanced or the strengthened version of the conditional proof.

So that is what we are going to look it today and the strengthened version you will see you I will try to highlight the differences and the also the power of this procedure, there is a reason why we and we have not stopped with the preliminary version, so all that will be discussed and; obviously, the new format and the procedure and so on. So this is on our agenda for today and module 29.

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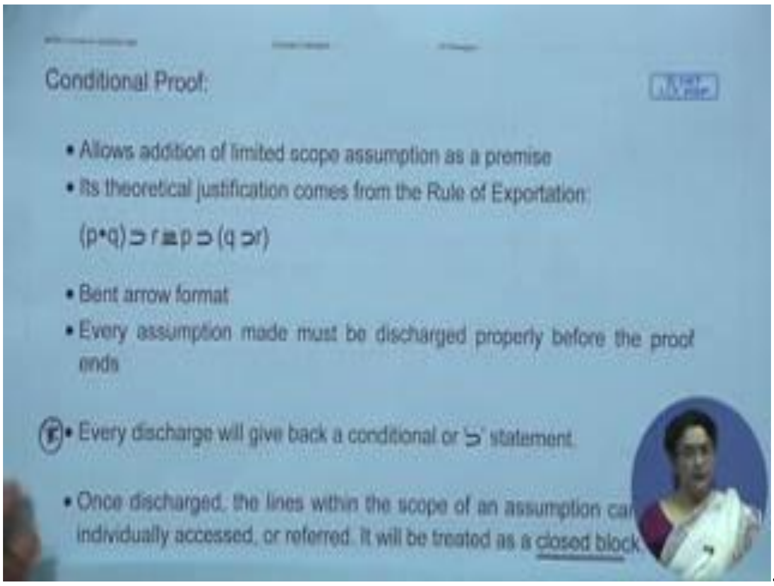

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Let me let us remind ourselves what is that we have learned. So for that in conditional proof we have learnt that this is one of the limited scope assumption procedure, which means that it allows insertion of a premise insertion of a limited scope assumption as a premise that we know. Now the theoretical justification if you are thinking in in that direction then let me just point at this that, the theoretical justification comes in a nutshell from the rule of exportation. Which you have encountered earlier when you were discussing the 19 rules of inference and if you recall now then this is the nature of exportation that we say that if p then q then r is equivalent to same if p and q then r.

So this is the more as operand of the conditional proof, that you are already given some some premises and we add to that an assumption called q. If together they show that r,then what you have shown is that if p than if q than r. So the r derivation depends upon conditionally upon q along with p. So that is the in a very brief manner what would be the justification for this conditional proof procedure. As always the limited scope assumption means that you are going to follow the bent arrow format, we already seen how it works so I am not going to spend time on that and you are going to see examples of that today also. All you need to remember is that the assumptions are limited scope which means they have a specific purpose and they have a specific beginning point and an end point. If within the proof you have to show that every single assumption made has been discharged.

So no assumption should be unclosed or undischarged. So when the proof ends and proof stands on it is own feet, but what we have learnt is that somehow in the conditional proof, every assumption that you make and then when you close it you going to get back a conditional or horse shoe type statement and I have said this earlier I will repeat it again is that you better know what to do with this conditional statement in the proof right there is a there is a reason why you are taking up the conditional proof and that objective should match with what you are going to obtain after the discharge and just reminding you that once the assumption block is closed, there is no way you can access the individual lines in it, nor can you refer nor can you utilize any of the lines inside. So this closed block situation is what we need to remember.

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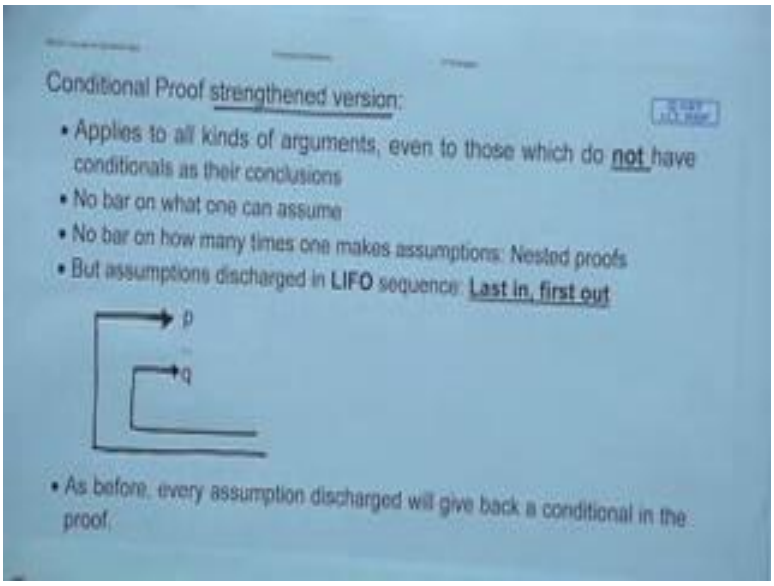

Let us now. So that was our just recap of what we already knew namely how the conditional proof works, but now it is time to come to the conditional proof of strengthened version. So how is it strengthened and what are it is strong points. This is what we are going to pick up for. First notice that the strength comes from it is wider application. The conditional proof in this version applies to every kind of argument. So irrespective of whether the conclusion is the conditional statement or not it does not matter. Remember preliminary version has limited application; it applies only if you are lucky enough to have the conclusion as a conditional, but look at this. This says that it does not matter what argument you are dealing with and whether the conclusion is a conditional statement or not it does not matter you can still apply this how you are going to learn. So that is the first strong point about this procedure.

Second note that there is no restriction on what you can assume. Anything that you find necessary for your proof, you are entirely to assume right. So earlier in the preliminary version what did we know, that only the antecedent of the conclusion can be assumed then you solve for the consequent, but here you have being given a lot of freedom not a freedom no bar on what you can assume and I mean it. So you will see soon that this means that how strong your strategically you need to take a good look at you proof to decide what is it that you do not have and therefore, whether you can assume that right no bar.

Also there is no bar on how many times you want to make an assumption. So there can be number of assumptions you can start, one of the other not simultaneously, but one after the other. So you are going to see nest in proofs meaning that they are the first assumption starts and then later on you feel you want to add a second assumption and that starts and so on. So you will see a structure that is more complex than the preliminary version. So the nesting of the sub derivation within the derivation and. So on and that is how it is going to be. When you are making several assumptions, that is one after the other please note, but there is a certain order in which the assumption are to be discharged and the sequence is the LIFO that is last in first out.

So the latest assumption that you have made is the first one to go and the earliest assumption, that you have met the first assumption is will be the last one to go will have the major scope, the maximum scope, will be the first assumptions. So remember that. So that when you are drawing these lines, the assumption bent arrow lines the wires are not going to cross right. So this sequence is something to remember when you are doing this when you specially handling more than one assumption. So in a way if we do it pictorially, then this is how it is going to sort of look like, this is where this is an arbitrary random example of a proof, where the first assumption has been made on this line and this is p and then you have done the proof and you felt the need for q, look this is been started and therefore, q is the first one to be discharge after that you are entirely to discharge p not before that.

So this is the sequence in which we are going to work and as before what is to be remember and this I say with full indication for what is to come is that. As you can see in the strengthen version, you have been given a lot of freedom. You can assume anything you want as many times you want and so on, but with freedom comes responsibility. So the responsibility is here, that you are entirely to assume anything but remember that you have to discharge each and every one of this assumptions and when you renew that you are going to get back a conditional in the proof and that conditional you should be able to utilize any of proofs.

So this is the croaks of the of the strengthen version there is a lot of freedom, but there is also a lot of strategy as you can see till the end you need to know what is it that you going to do. So have this, in front of you as you we are going to try this strengthen version of conditional proof.

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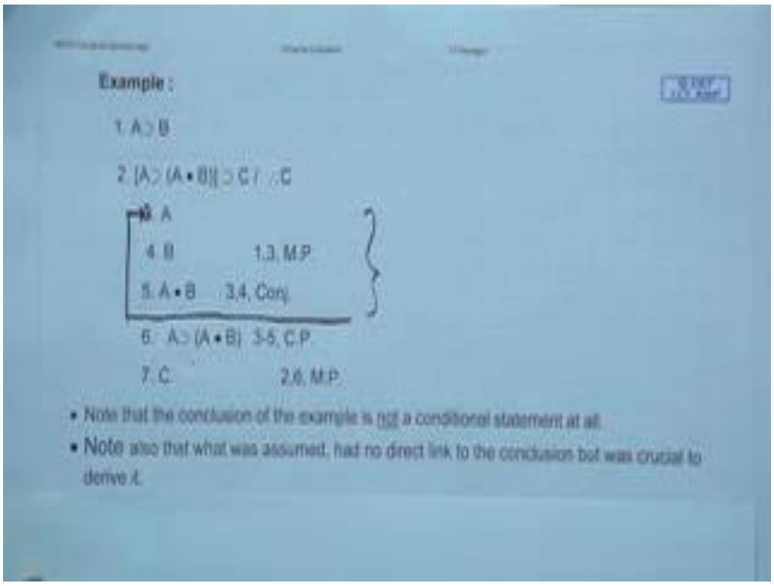

Will start by looking at some examples, so here we are, remember this is one small proof that we are going to try, where there are two premises and here is C all right. The question is we are now applying the strengthen version thank god because in preliminary version there is no way you can apply the preliminary version of conditional proof on this. Why? Because C is just stand alone C, it is not a conditional C, but in strengthen version, there is no restriction. So we can start thinking what is it that we need and what is it that if we have it we can come to C now your best bet from these two premises is that if we can have this then we need to have C. Then we can easily get the more respondents and you see.

But then, what is that we have to assume, should we assume this whole thing or should we work a little different. This is where your strategy comes. What is it that we are going to work on and how I am going to know utilize that. Remember whatever you assume you are going to soon have horseshoe in your proof and that horseshoe should be used for it. So if you for example, start with this whole thing and then you solve C what will you get back this whole thing, but is that your conclusion. Conclusion is C and that is what we need. So when it you think a little what is that we can have?

Another hand and this is where you need to be orient yourself that there is no need that we solve immediately for this, but it is good enough if by the procedure we can simply get this bar then we apply that to line 2 and we get C, out the question is what is it that we need to assume in order to get. This take a look at here this is A horseshoe A dot B.

So we need to work on that. So we can assume A and then we have to solve for A dot B right only then you are going to get back A horseshoe A dot B can we do that. So let us start slowly, this is my beginning point and there is a reason why we have chosen this. The moment you assume a, the bent arrow starts. Now what can we do with A, well we can easily plug in with 1 and get B, why? Because then we can put them to gather to get A dot B right. So let us try that. So here is B from 1 and 3 more respondents.

Now we put 3 and 4 together and we get A dot B and if we close our assumption here because this is what I needed we are going to get back, what a horseshoe A dot B, by what, by 3 through 5 CP. Once more reminding you this is a entirely an assumption block. So we have to refer to it as a block. There is no way we can refer to the individual lines, is this where we stop to proof no, there is a further line 2 and 6 will give us what we are looking for namely C. Please note that line C is no longer depended up on your A right the assumption has been closed.

So this is a way to appreciate what the strengthen version of CP can do for here did you understand this. So even when you are looking at this ask yourself, what is it that we can assume and because we are all beginners with better to be a little cautious and try out on the margin to. See if I assume this and if I solve for this remember, when I close it I am going to get a horseshoe back and why know what to do with that horse shoe you, know sometimes we are too eager to jump in to the proof without thinking it thoroughly. So here is a point of caution, that instead of doing that this, you know work it out a little bit in the rough work and then come back into the proof.

So this is one example of how it is done. Please note that apparently that we have done conditional proof on an argument which does not have a conditional as a conclusion point taken. So now, our scope the possibility of application has really grown, increased. Second is that what was assumed, apparently is no part of the conclusion, it is a purely convenient assumption with very pragmatic objective that we need to have A so that we can have A dot B. This thinking is something that you need to get used to now. So what is it that I need in order to get C out, that is the kind of operational pragmatic consideration that should guide you in the choice of your assumption all right and you are not going to get any clue from the premise, like in preliminary version you know there is a definite starting point namely the (Refer Time: 16:08) , but there is a lot of strategy and somewhat thinking involved in it and it will take a look in to another example as we go along with this.

So and this is when you should also start realizing, that you are now almost in a serve a more advanced and mature way of doing proofs right because CP has given you so much possibility. Now open to you the proofs that you that was. So difficult to do earlier then now probably with the CP you will be able to it in much faster way and more efficiently. So I will show you another example and then again I will ask you to do this, what this have on your own. So that you get some habit or some practice with it

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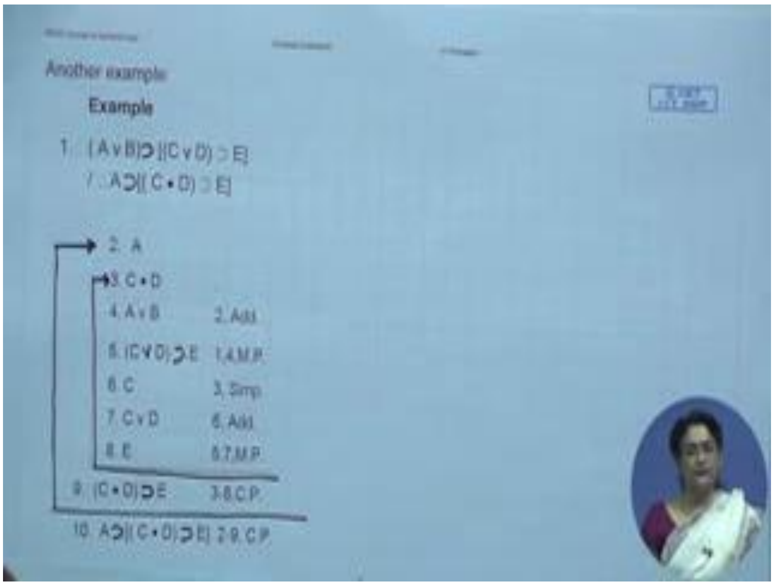

So here is the proof, one premise and that is this and here is the conclusion. Now this conclusion is a conditional, one if you want you can choose to have the preliminary version also. That is you start with A and you solve for E. The strengthened version does not limit to you to just A, what you need to think is what is more efficient to assume here or what will give me the result quickly and correctly.

So take a good look have a strategy first of all and the place to work with the strategy is not in the scope proof space, always do it in the margin. Draw a line and think a little about the proof how to go about it, what is it that you need and I am sure most of you are already thinking that I will start with A. The question is why? You need to work not from the conclusion, but from the premises that you have. A will give you A wedge B right. Why do you need that? You need because so that you can derive C wedge horseshoe E, but my point is that what you need here there is a slide difference C wedge D and this C dot D. So you need to work on that a little bit before you can get the e and remember this is conditional proof.

So if you let go of the assumption you need to remember that you are going to get a horse shoe statement back in the proof right. So have you figured it out where to start, will start with A, if you wish will start with A and the second step is obvious or not when you are a little bit experienced in this I will say that you need to think a little before. This is what I am saying that you do think on your margin and all the assumptions that you need you will learn soon that you can start them right at the beginning or you assume them only when you absolutely have to. So if you want you can do this line for here instead of line 3 and another assumption you can do A wedge B here, also but I choose to do it before because I knew sort of what I need to do. So this is actually the line that after 2, you can do this is why you can needed A wedge B, that A wedge B will give you as I said C wedge D horse shoe E right.

So how do I come from that, C wedge D horseshoe C to C dot d think in a different way. Why do you have to come from C wedge D to C dot D. Rather can we not go from C dot D to C wedge D that is easier. So this is the reason why the second assumption has made C dot D. So that once you have obtained this is 1 and 4 of course no problem, this is 1 and 4 got you C wedge D horseshoe E, but now you also have 3 C dot D, from which we can do a simplification on that and then add this. This is the most interesting line here that on line 6 we can add this D, C wedge D appears once C wedge D appears E just a matter of more respondents and now you see what happens. Now you see the power of this proof procedure, once you have this we close what shall we get back well the last in first out. So we are going to get back C wedge D horseshoe E and here is 3 through 8 conditional proof. Are we done no there is one unclosed or undischarge assumption, now we are going to discharge that and if you discharge that this is how it is going to show up, but this is exactly what we need as the conclusion got it.

So this is how the conditional proof moves. If you were troubled and if you if you still wondering that why I could not see that C dot D can be assume that is because the procedure is very new and you still have not realized the kind of power that you can, while using this procedure right. So this is why I said that you know you take it in an it is a different kind of thinking process, that you have to get used to, but you know there is always a beginning, there is always first and so on, but this is what the strengthen version of conditional proof is like.

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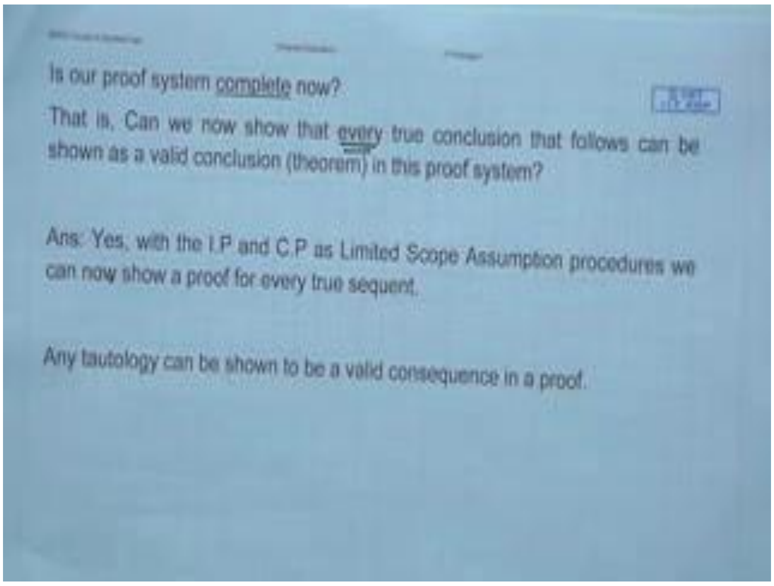

So we come back to the final point that, the reason that why we included IP and CP was with the question is our proof system complete and we found out earlier that it was not with the 19 rules it was not and that is the reason why we added this proof procedures IP and CP.

So with IP and CP, now we are complete. So our claim is now give us any true conclusion including all tautologies we should be able to give you a proof and that is what the whole enterprise was all about. This is why we came here. So that remains to be shown, that give us any tautology, we can show it as a valid consequence in a proof, but the will be discussed in our next module right now.

You concentrate on mastering this conditional proof and it is strengthen version all right. So with that I will close this module thank you very much and thanks for your time.