

Non-conventional Energy Resources
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Lecture – 21
Wind Energy: Energy considerations

Hello, in these classes we are now discussing wind energy and we are basically interested in trying to figure out what is the best that we can do with it. We have seen some of the statistics associated with it, we saw the countries that are significantly pushing towards wind energy, we saw even within India you know the various states the kind of effort that they are putting in towards capturing wind energy. And all this is because you know it is sort of an investment that once you put up it keeps collecting energy for you, relatively clean and the technology is also relatively mature at this point, people seem to understand what is happening and at least I mean the manner in which it is being captured is quite well established and people are able to do it quite effectively.

So, therefore, its being perceived quite actively, it can be set up at various locations as long as you know you are getting the wind there sufficiently and so there is a wide variety of advantages for trying out this technology and that is why you so many countries are pushing hard towards it. And not to mention the fact that it is completely clean, I mean in many ways there is nothing coming out of it I mean it is just air that is already anyway moving you just tapping it for the purpose of capturing electricity. It has also been around as an age old you know activity or in you know technology that is been around for a very long time, but in the middle its sort of lost its popularity because lot of you know other gadgets came in and now it is sort of being rediscovered.

But in the process we have also understood a lot more about the science of how the wind turbines or windmills work and so the modern windmills are vastly more efficient than the sort of the ancient ones that we tend to see in you know magazines or pictures from you know maybe many places in Europe where they had these old-style windmills. So, we will look at the difference between those windmills as we go along.

So, in today's class we are really going to look at some specific energy related considerations when it comes to capturing wind energy. So, some calculations relative to, related to this energy process, what is involved, what is likely that we can do with it and

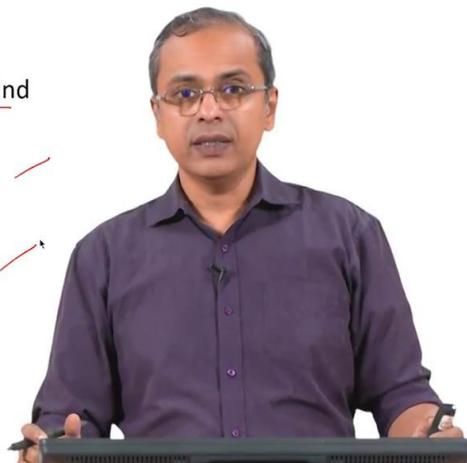
we will also increasingly get a sense for you know why is this structure; that the structure that you see in your picture here such a long structure here right. Why do not we just have it on top of our rooftop, why do not we just have it in a garden etcetera and these blades are also so long, so many things are there today's modern windmill has this kind of a design as what you see here.

You can see you know how tall it is relative to the trees here these are all trees out here. So, the tree level is somewhere out here and this is significantly taller than that that you see there. So, it is interesting to understand what is the reason for all this and why are they setting it up this way and how significant is the reason. We may have some general idea that you know; oh there is wind there and therefore, you want a tall structure, but how significant is it what is the technical rationale behind it that is something that we would like to look at.

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Learning objectives:

- 1) To determine the relationship between wind speed and power
- 2) To understand typical performance characteristic and performance limits of windmills
- 3) To become aware of theoretical limits associated with capture of wind energy



So, our learning objectives for today's class are to determine the relationship between wind, speed and power, so wind speed and power. So, at least intuitively we understand that you know if the wind is very slow there is not much energy in it there is not much power in it and therefore, if you have a windmill that is trying to capture it, it is going to correspondingly capture less energy or less power and we also know that you know if the wind speed is higher correspondingly all these things are higher. But what is that

relationship, how significant that relationship is that is something that we will try to get a understanding of.

We will also try to understand or at least become familiar with typical performance characteristics associated with windmills and something about some at least an initial feel for what are the limits that we can expect or anticipate when you put this windmill out there and you are trying to capture energy from flowing wind.

And we will also try to become aware of the theoretical limits associated the capture of wind energy. So, that is the other thing that we will try to do. So, in I mean sort of the objective 2, we are sort of looking more in the practical aspects associated with it and here we are sort of looking at the more theoretical aspects associated with it. We will do brief discussions on those, we can do more elaborate discussions on those as well, but for the moment in this class we will keep it to you know discussion that sort of sits within this framework of these topics that we are putting together and will have a corresponding level of detail right. So, this is what we are trying to do alright.

So, we will begin by first doing some calculations.

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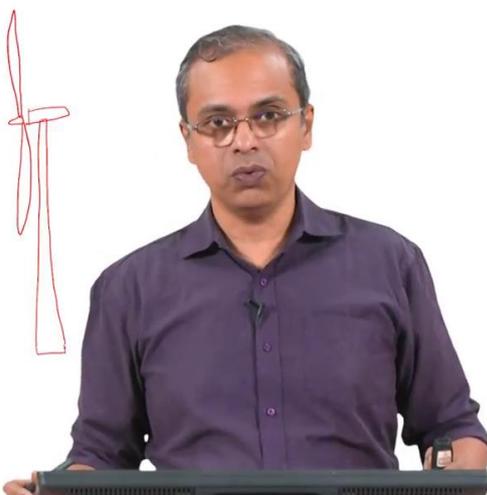
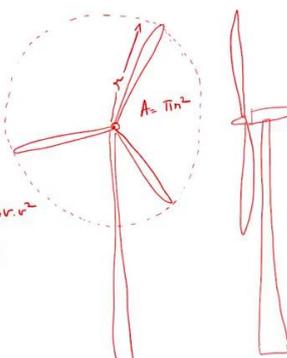
Energy calculations:

velocity v
 \therefore Kinetic Energy = $\frac{1}{2} m v^2$

Air has density ρ Volume V has gone through the region corresponding to windmill

$$KE = \frac{1}{2} \rho V v^2 = \frac{1}{2} \rho A l v^2 = E$$
$$\text{Power} = P = \frac{dE}{dt} = \frac{1}{2} \rho A \frac{dl}{dt} v^2 = \frac{1}{2} \rho A v^3$$
$$\text{Power} = \frac{1}{2} \rho A v^3 = \frac{1}{2} \rho A v^3$$

Power $\propto v^3$



So, we will start some energy calculations. So, now, let us just see here we have on this right extreme I will just draw something here. So, we have this tall tower and on that we have a windmill right. So, this you have a windmill, now this is rotating. So, if you see it

from the front the blade would look something like that, let us say I said I mean if there is 3 blades then you will typically see something like that, that is rotating right. So, now so that would be that would be a tall tower there again. Now, we would like to understand what is the energy that we are captured.

So, there is a lot of wind that is moving out there right. So, you have this entire you know the entire landscape has wind that is moving I mean you have move have been moving the entire landscape. So, on what basis should we understand what is it that we have captured. So, what we are trying to do in this calculation is to actually see what is the energy or power that is available in the wind that is within the framework of this windmill. So, if I sort of draw a circle here and I mean, this is the breeze or wind that the windmill is accessing right. So, there is breeze all around so that is not what is of relevance here, this is the breeze that the windmill is accessing. So, the breeze that goes through this circle that I have drawn here this I mean approximate circle that I have drawn here that is the breeze that the windmill is accessing.

Now, we want to understand that given, first thing we want to figure out is what is the energy or power that is available in this wind. So, in the wind that has gone through this region that is the first point that we want to understand. So, we have to make some assumptions, we would make the assumption that first of all the wind is like you know perpendicular to this windmill. So, you have the windmill vertically present and then the breeze is going perpendicular to it and we would like to look at what is the power that is there this in the wind that has gone through this area this cross sectional area the section of area.

So, to do that, first we accept or we start with the assumption that the wind is having a velocity v , there is a velocity v associated with the wind and it is going perpendicular to this windmill and therefore, its kinetic energy is simply half mv square. So, half mv square is the kinetic energy associated with any you know object of mass m that is moving with a velocity v . So, the kinetic energy is half mv square.

Now, in our case it is not a solid object it is a gas basically air which is you know nitrogen oxygen everything which is moving through this place. So, we have to estimate the mass. So, the mass here is basically we can say that the air has a density ρ , ρ and therefore, mass would be if you understand, if you can figure out what is the volume of

air that has gone through a particular region, the mass would simply be the density times the volume right. So, if a certain volume of air has gone through this region that volume times the density is the mass of air that has gone through this region. So, we do not know what that volume is for the moment we just assume it is V , volume V has gone through that region corresponding to the windmill blades which is basically the circle. So, that is what we have.

So, now, the kinetic energy I just write that as KE is simply half instead of m we will have ρ and V . So, ρV is the mass, v square small v square. So, this is the kinetic energy. Now if this area is A , so we can say that the area associated with this circle that is a fixed area because we know that the radius of this blade is r right. So, we can say the radius of this blade is r and therefore, the area is known πr square. So, area would be, so area is πr square and we will assume that in whatever is the time frame that we are considering certain length of air. So, the area times the you know amount of air that has gone through is corresponds to this length of air this height of that air that goes through that the wind turbine is the amount of this is the volume. So, you have a circular area, a circular area and certain you know depth of air corresponding to that circle has moved through that region corresponding to the windmill. So, we will assume that that depth is air in some time t . So, we do not worry about it some depth l .

So, kinetic energy whatever is that value l depending on the value of then the kinetic energy is correspondingly higher. So, it is simply half ρ , cross sectional area is A and some length which can be varying based on how long your you know waiting there and figuring out how much wind has gone by $\rho A l v$ square. So, this is what we have.

Now, so this is the energy kinetic energy we will just call it as E which is the energy associated with the and all this energy is essentially kinetic energy at this point this is the amount of kinetic energy that was there in the air that went through that wind mill right. So, the power is simply the amount of energy that the rate at which that energy is you know being consumed or being delivered. So, it is basically dE by dt , power is equal to P it is equal to dE by dt it is energy in unit time right. How much energy in unit time is power. So, Joules per second is watts that is basically what this thing is. So, if you differentiate your expression for E with respect to t you find, half is a constant, ρ is a constant that is the density of air, the area corresponding to the windmill is also a constant because the diameter or the radius of those blades is fixed, the radius of those

windmill blades is fixed and we are using a horizontal axis windmill, the axis is perpendicular to this picture that we have drawn.

So, the blades are you now sitting like that and the axis is perpendicular to it and so that is the area corresponding to that circle is fixed. So, the l is not fixed because that depends on how much time we are you know counting there. So, that is why when we differentiate with respect to time we have a dl by dt time v square the velocity is fixed, that is the velocity we will assume that it is a constant velocity of that is flowing at that point in time. So, that is also constant.

Now, the dl by dt is also velocity that is the rate at which the breeze is moving the wind is moving. So, the l was the distance the wind had moved during some time t which we did not really I mean which we can which we could have kept track off, but more importantly now when we do differentiate we find that it is dl by dt that is the rate at which the wind is moving. So, that is basically v . So, this is equal to half $\rho A v$ into v square. So, this is essentially power is equal to half $\rho A v$ cube. So, half $\rho A v$ cube that is the power that is available in wind which is traveling with a velocity v . So, that is the power that is available in the wind.

So, we are not discussing right now on what the windmill is capturing. So, that is a different question, or that is also an important, that is an important aspect in the end that is what is really valuable to us. But first of all what is available that is what we are calculating. So, what is available is this $\rho A v$ cube. Let us write it more clearly here half $\rho A v$ cube. So, this is what is available for us in the breeze. So, now, if you see here the ρ is a constant because that is the density of air, the A is a constant that is decided by the size of the windmill right. So, you can have windmill of different with blades of different sizes.

So, whatever you select as the size if r is the size that is fixed, the area is fixed. So, only thing that can vary is the velocity. So, one day you can have some low velocity wind going, another day you can have very high velocity wind going and so on. So, once the windmill is set up the only parameter that can change as a function of time or day or whatever it is the velocity. So, we find very importantly that power that is there in the wind is proportional to velocity cubed. So, therefore, it is a very strong function of velocity. So, therefore, when you are trying to capture wind energy it is very important to

locate the windmill in a situation, either a sight, or you know based on its height whatever it is all factors taken into account you should try to locate the windmill in such a manner that it gets a very high velocity of wind. So, we spoke in fact, about these location of Muppandal which is the you know largest wind energy farm in India is set up, there because of the mountain passes that are there the breeze picks up a lot of speed along be between the mountain passes and therefore, the wind velocity is very high.

So, if the wind velocity is high the power that is available in the wind is the cube of that velocity. So, it is not a linear function it is a cubic function. So, it is a very strong dependence on the wind velocity. So, you get a very large amount of power that is available in the wind if the velocity is high right. And some fraction of this is what the windmill will capture. So, the windmill may not capture this efficiently with 100 percent efficiency. In fact, we are going to see that there are some theoretical limits to it, but it is so, it is never going to capture this with 100 percent efficiency. So, there is going to be some limits some a fraction of this power it is going to capture. But regardless whatever fraction of the power it captures if the total power is high, naturally the fraction of that power is also going to be a large number right. So, therefore, putting the windmill in a situation where it can get access to high power in the blowing breeze is a very important aspect of the entire wind energy technology activity right.

So, that is why lot of work goes on and they do not just plant windmills wherever they wish they will they do a lot of homework, figure out if that site is appropriate for the windmill and then set it up. And even there you have you know the fact that you know you have this really tall windmills, this really tall windmill that you see here it is also got to do with the fact that the higher you keep this windmill it gets access to more you know steady and stronger breeze or wind that is available there. So, this is the calculation that I just wanted you to take a look at. So, now, the same things I am showing you here.

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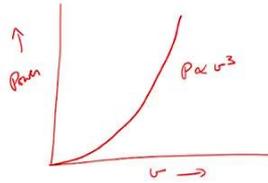
Energy calculations:

$$\text{Kinetic Energy (KE)} = \frac{1}{2}mv^2$$

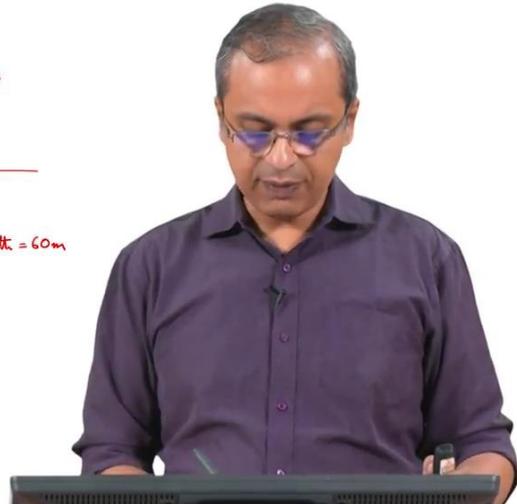
$$KE = \frac{1}{2}\rho Vv^2 = \frac{1}{2}\rho Av^3$$

$$\text{Power} = \frac{dE}{dt} = \frac{1}{2}\rho A \frac{dl}{dt} v^2 = \frac{1}{2}\rho Av^3$$

$$P = \frac{1}{2} \times 1.225 \times 3.14 \times 60^2 \times \left(\frac{v \times 1000}{3600}\right)^3$$



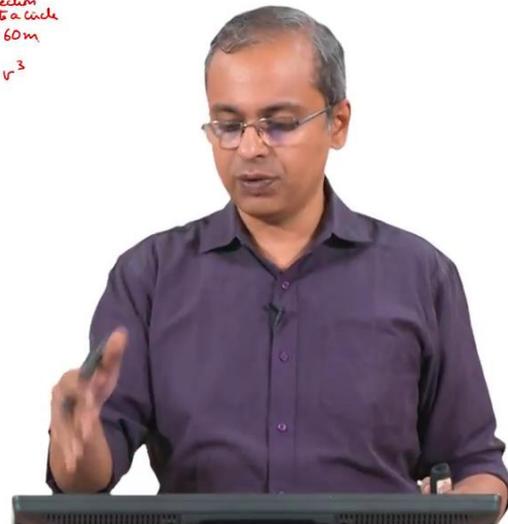
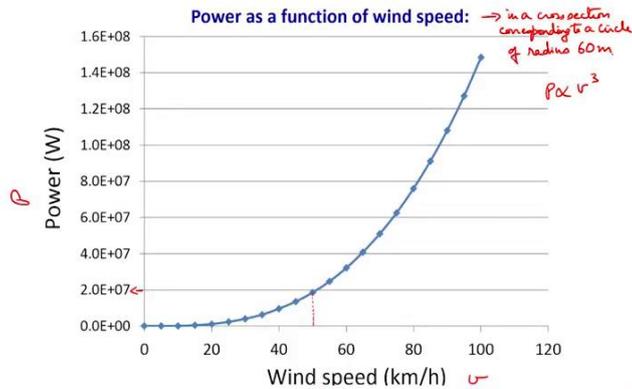
$\rho = 1.225 \text{ kg m}^{-3}$
 $A \Rightarrow$ Based on Hubcap length = 60m
 $v \Rightarrow$ km/h



I have just you know put these down as equations you can take a quick look at. So, that is what we did kinetic energy is half mv square, that is what we did. So, the same kinetic energy once you write that as using the area and some length corresponding to how much of breeze has gone past the windmill, then that is the amount of kinetic energy that was there in that wind that went past the windmill, went through the windmill effectively pushing the I mean doing something with the interacting with the windmill blades.

And power is dE by dt which is what we did and so once you do that dE by dt you will get rho A dl by dt v square and that is rho A v cube right. So, if you actually plot power versus v the velocity you will get a function which is basically cubic function, something like that right. So, it is strongly dependent on, p is proportional to v cube this is basically what we get. So, that is a very important thing to remember. This curve is very I mean important curve to keep in mind when you think of a windmill. So, that is the point, so in fact, lot of work that goes on with respect to windmills. At some point I mean fundamentally acknowledges this curve and then you know accounts for this in the design. So, that is the point that we have to keep in mind right.

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So, the same parameter that we just saw I am plotting here. So, for example, if you just look go back here we can take for some values to try out. So, for example, rho for air is about 1.225 kilograms per meter cube. So, that is a value you can use.

To calculate A, the area we need to make some assumption on the radius of the wind turbine blade right. So, some assumption we have to make, modern wind turbine blades that you see including the photograph that I am showing you, that I showed you and what you see dotted you know around in various locations in various places are and you know 50 60 70 meter kind of range in terms of radii. So, we can assume that is, we will assume that the length of the blade is 60 meters, so, and then we can have a velocity in of course, in some unit we can select and I have just plotted kilometers per hour because that is often the that is a value that we can more easily you know understand or visualize.

So, what we are really looking at is power equals half rho is 1.225 into area will be pi which is 3.14, r square, 60 square, so pi r square into so velocity we will we will take some velocity in kilometers per hour. So, kilometers into 1000 will give it to us in meters by 3600, will give it to us in meters per second, this cubed. So, this is the; if you do this calculation for every velocity v in kilometers per hour you will get power in watts. So, this is in watts and this v here is kilometers per hour right. So, that is half the factor half that is out here we have put in a value for row 1.225 kilograms per meter cube, A we

have calculated as πr^2 , where r we have assumed is 60 meters and v we have assumed is some velocity in kilometers per hour. So, to convert that into meters per second we have multiplied by 1000 and divided by 3600 and the whole thing is cubed.

So, this is what is the you know actually equation that we need to use for power. So, for various values of v we can now get various values of P right. So, that is the plot that I am I am showing you here the power W , I mean power in watts. So, this is the P that I just spoke about and wind speed in kilometers per hour which is the v that we just discussed. So, power verses wind speed.

So, you can see again as I said you know this is proportional P is proportional to v^3 and that is the function that you are seeing on your screen you see this plot which is you know steadily climbing up. So, for example, if you look at it at say roughly around 50, 50 kilometers per hour, the power that is available in the wind is about 20 megawatts. So, this is 10^7 , 20 megawatts is what is available in the wind at 50 kilometers an hour right.

So, this curve actually I have gone to a very wide range of velocities from 0 to about you know nearly 100 kilometers per hour is what we have drawn. But that is something we will discuss about in just a moment. So, please remember this is not the power that has actually been captured by the windmill, this is not yet at that point. We are simply discussing what is the power that is available in the wind. So, this is what is available in the wind. So, the wind is blowing at 50 kilometers an hour then the power that is available in that wind is about 20 megawatts right. So, as faced by a windmill of 60, I mean 60 meter radius right. So, this is as a power, as a function of wind speed, so this is a power as a function of wind speed in a cross section corresponding to a circle of radius 60 meters. So, that is also something that we have to remember, because 50 kilometers an hour if you have a huge amount of wind going at kilometers an hour, correspondingly power is more right. So, this is you have to limit the size of it and that size is limited by this circle of radius 60 meters.

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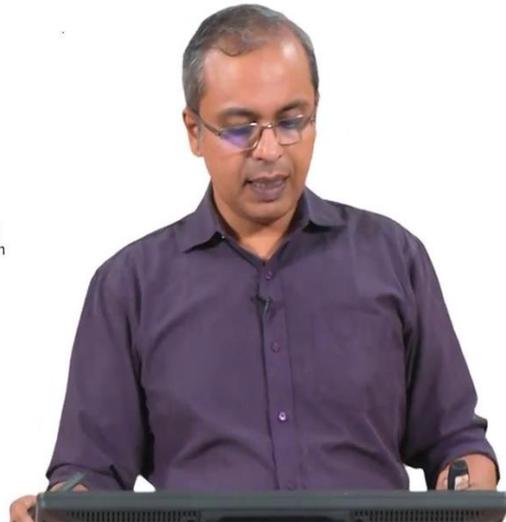
Performance Characteristics:

Tip speed ratio: Ratio of rotational speed of blade to wind speed.
Maximum of 10 for lift type blades

Cut in speed: Minimum wind speed at which the blades will turn.
10 km/h to 16 km/h

Rated speed: The wind speed at which the windmill generates its rated power. Usually it levels off in power beyond this speed. Around 40 km/h

Cut out speed: Usually at wind speeds above 70 km/h, the windmill is stopped to prevent damage



So, now we will, I need to discuss that curve a little bit more we will come back to that curve in just a moment. Before that I just wanted to discuss a few things about performance characteristics. There are some terms that are used in the context of windmills. So, we will you know briefly look at those terms, and on the basis we will again revisit that curve that we just saw. So, the first thing is there is this thing called a tip speed ratio, tip speed ratio. It is the ratio of the rotational speed of the blade to the wind speed and usually it is a maximum of about 10 for blades that are considered lift type blades.

So, we will see a in a little bit later on what we mean by lift type blades. So, there is a certain type of blade which is called a lift type blade and if that kind of a blade is used then you will get a tip speed of, tip speed ratio of 10 which is and we will see that in greater detail, but fundamentally it conveys the idea of how quickly the windmill is rotating. So, it is a rotational speed of the blade to the wind speed, for the same wind speed if the blade is rotating faster and faster and faster you have a higher tip speed right. And based on the design of the windmill some windmills will have the tip speed that is low and some will have it high, so in some cases it is only 1, it is a factor of you know it is just the same as the wind speed some cases it is much higher. And there is some

consequences of to this based on what you are trying to do with the windmill, so that we will see in a little bit.

Second thing is there is something called a cut in speed. This is the minimum speed that should be there in the wind before which the blades will turn. So, if the wind speed is too low, it does not have enough power there is not enough energy in the wind to start pushing those blades. So, the blades look, appear stationary. So, even though there is energy in the wind, even though there is power in the wind it is insufficient to get the blade to get past whatever frictional aspects are there, whatever you know loads are there on the blade which prevent it from rotating because it is going to be attached to a gear that gear is going to be attached to a generator. So, there is a lot of you know in the back there is some load that is there mechanical load which it has to overcome before it starts rotating. So, you need a certain amount of wind speed before typical windmill start rotating till then it will not it will not respond to the wind it will look like there is no wind, but there is wind it is just not enough to get the windmill to move and that is usually somewhere between 10 to 16 kilometers an hour.

So, based on various aspects associated with the windmill it will take you at least 10 to 16 kilometers per hour wind speed before the windmill begins to rotate. So, that is quality cut in speed. Then there is something called the rated speed. The rated speed is the wind speed at which the windmill generates its rated power. So, rated power it will have when they say that you know this is windmill is a 1 megawatt windmill or you know half a megawatt windmill whatever it is some rating they give for that windmill right. So, you once they give you that rating for the twin mill. it usually it will require at least a certain level of wind speed before it starts delivering at that rated capacity right. So, sometimes the rated capacity may be a nice number, but you will not be getting that rated capacity most of the time because wind speeds may be less than whatever is that optimum speed that is required for the treated capacity.

So, you do not always get that energy from the windmill, but there is that rating is there so you have to be aware that there is such a thing as the rated speed, and only when you reach that speed you will start getting that power. Generally what happens is the profile of the power generated by the windmill is such that till you reach that speed once you cross the cutting speed as the wind speed keeps increasing the power generated by the windmill will keep climbing up. Then it will reach this rated speed, once it reaches the

rated speed at which its delivering its rated power which is usually the best power, best condition for it, it is sort of level off it will even though you may have a range of wind velocity is higher than that for various other aspects associated with the design of that windmill it will sort of level off at that value and it will only deliver that amount of power. And then and, therefore, it is sort of levels off, pass the point. And this is usually around 40 kilometers per hour speed is where you will get this leveling off and after that it stays level, it does not really do much.

Then there is a cut out speed and this is usually at wind speeds above for 70 kilometers per hour. So, this is mostly a safety issue. So, the wind windmill or the wind turbine is made of certain types of materials, they can handle certain types of you know certain level of forces on them, torque on them or all these things there will be some rated capacity to for these turbines for those long blades that are present there etcetera. And if you if the wind speed is too high, this let us say there is a storm and you can easily have you know storms which come which have 100 kilometers per hour wind speeds right. So, those are significantly higher than this 70 that I am talking about. So, 40, at 40 it is already delivering its rated power. So, they will allow it leave way up to about 70 kilometers an hour beyond 70 and there will be there will be days in the year when there is a storm or cyclone or whatever you want to call it where the wind speed will be higher than this and at that point it is not safe for the windmill to operate. So, they usually have mechanism by which they stop the windmill, the windmill is basically stopped to prevent damage. That is basically you know very practical issue, some of the things that we are talking of here are practical issues. A cut in speed is a practical issue.

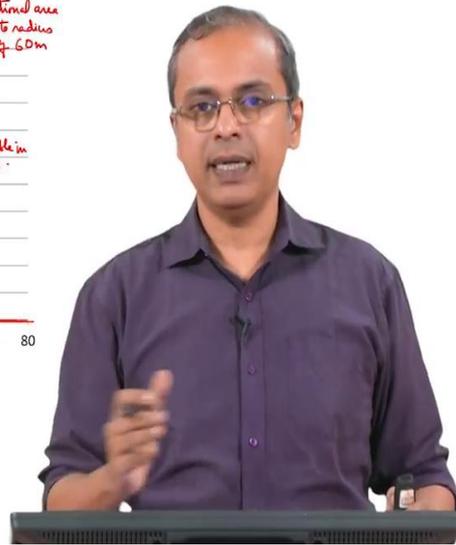
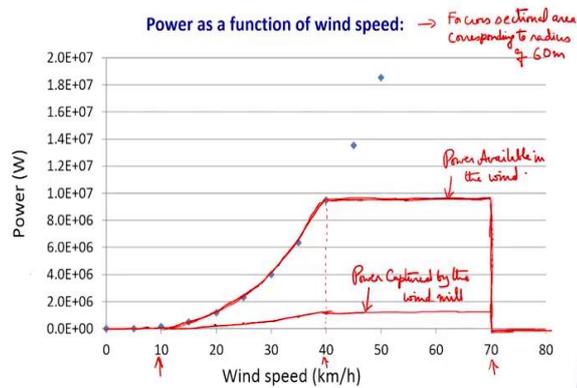
In a hypothetical perfect you know you know friction free situation this should just start running even the slowest wind it should be running, and it should get you the you know once you even will go past the rated speed it should continue to give you higher and higher power. And even cut out speed in principle you should never have a cut or speed higher the wind you should be actually you know very happy and generating you know at the cube of the velocity is what we are looking at. So, you should be able to generate a significantly high amount of power. So, these restrictions that I am showing you here are all practical restrictions. So, there is a cut out speed; and this cut out speed is about 70 kilometers per hour after which the windmill is stopped. So, for this there are various

mechanisms inside there is usually sometimes they have a break in the system which will physically you know act like a break which will stop the rotation of the windmill.

They also have a way in which they can rotate the windmill so that you know normally the working very effectively when it is facing the wind. So, they can turn the windmill away from the wind and make it now face perpendicular to the wind, so it is seeing less of the you know action from the wind so that also enables you to slow it down they can add spoilers to it to slow down the process and various other things can be done. So, there is a standard set of mechanisms they have by which they will shut down the windmill if the wind conditions are very severe. And in many cases they can even automate it so that it automatically senses that the wind speed has crossed some limit it will get into the shutdown mode and then when once the wind speed drops below this you know condition this high speed condition, once it drops below that threshold value of 70 kilometers an hour and stay stable there for some period of time if the wind will again start operating. So, these are some practical aspects that we have to keep in mind. So, just to keep in mind, these are 3 numbers here about 10 kilometers an hour about 40 kilometers an hour for rated speed and about 70 kilometers per hour for the cut out speed right.

So, the same plot that we just saw here where I had power versus wind speed where we went up to 100 kilometers an hour. I am going to actually show you another plot which is exactly the same thing in a you know magnified way where we are sort of magnifying the lower end of this graph here so that we can know look at this cut-in speed, rated speed and cut out speed and so that is basically what we have here.

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So, this is what we will look at here. We will come to that theoretical limit in a moment. So, you can see here the same thing this is power as a function of wind speed for cross sectional area, for cross sectional area corresponding to radius of 60 meters. So, now, if you see those values I have wind speed here I have power here and power you can see here since we are taken a smaller section of that diagram that we previously saw, we have you know some values here in megawatts, 2 megawatt here, 4 megawatt, 6 megawatt, 10 megawatts and then it continues up to about 20 megawatts. So, that is basically what we have here.

So, at 10 kilometers per hour which this is your cut in speed, so up to this the windmill does not even move you just sort of stays stationary. So, that is our cutting speed and then about 40 kilometers an hour it is where it reaches it is at a rated capacity right and then finally, at about 70 kilometers an hour we have the cut out speed. So, in practice what we actually see if I just mark this up here say that is the value there. So, what we will see is this is the actual curve that we will be using as the windmill operates. So, what you see here is that till 10 nothing is happening to the windmill, till 10 kilometers per hour there is nothing happening in the windmill no power is being generated it is just basically stationary. Once you cross about 10 kilometers an hour the power it starts generating power then that power, this is the power available in the wind, this is not yet

the power that the windmill is actually generating. But this is the kind of power that is there available in the windmill.

There is some factor that is that needs to be put into this which we will just see in a moment. This is the power available in the wind and then so, it will reach some rated power and at that point at 40 kilometers an hour this is the power available in the wind, and that is the power that the windmill is experiencing and then at 70 kilometers an hour we shut it down. So, that is what is happening here, shut it down and so, even if the speed is higher than that the windmill stays shut down. So, this is the power in the wind corresponding to those speeds that we just spoke about. It is not yet the power that the windmill is capturing.

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Theoretical Limit:

Betz law (1920)

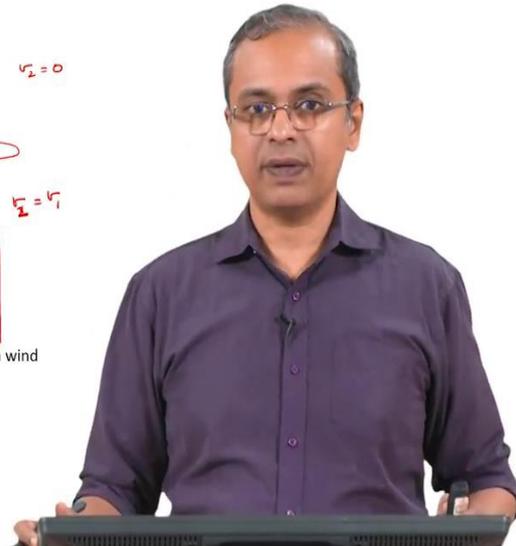
- Wind fully stopped by windmill → The windmill also stops
- Wind unaffected by wind mill → No energy transfer to wind mill

$\frac{1}{2} \rho A v_i^2$ → Theoretical limit

$\frac{16}{27} = 0.59$

$0.59 \times \frac{1}{2} \rho A v^3$ → Theoretical limit

Practical efficiencies obtained: 10%-30% of energy originally available in wind



So, there is some theoretical limit. So, this has come under a law that is referred to as the Betz law, it was proposed in 1920 that is what you see here. But actually historically if you go and see the records at that point there were actually 2 3 people who came up with this law, at the roughly the same time there is one person I think who is credited to have even indicated this around 1915 and then there were 2 people who indicated this in 1920 and somehow it got associated with Betz and so, it is called the Betz law.

And where an analysis has been done on the idea that there is wind that is blowing it has some energy what is the maximum energy from that energy that is available in the wind that we can capture. So, if energy, it comes with an energy e , if it has an energy e what fraction of that e can be captured; can we capture 100 percent of it or can we can only capture 50 percent of it; can we only capture 30 percent of it, what is the theoretical limit of this. Is there a theoretical limit and if so what is the theoretical limit.

So, a very nice analysis was done. So, just to understand why there are limits to this process we can just consider the idea that you know this wind is coming with some energy and for the windmill to capture the energy the wind has to interact with the windmill right. So, you have a windmill. So, you have wind incident on this windmill. So, now, we are trying to capture this energy. So, what are the options? So, this is the wind is arriving at some energy. Let us assume that the windmill somehow you know completely captures the energy of that of the wind that is coming towards it. So, let us say some we just assume hypothetically we will consider this situation that wind arrives with some energy e which is basically what, its kinetic energy. Its kinetic energy, it is arriving its arriving with the kinetic energy half mv square and it arrives with that energy arrives incident on the windmill.

And let us say we have managed to capture 100 percent of it. What is the implication of it? The implication of it is that the wind immediately after the windmill. So, I will just say it has come with a velocity v_1 . So, half mv_1 square right, so that is the energy it has come with. So, if you say v_2 , is the velocity of the wind immediately after the windmill if all the energy of the wind has been captured then v_2 equals 0 right. So, if all the energy that is available in the wind has been captured successfully then the wind comes to a complete halt because all there it has lost all its energy, everything has been captured by the windmill and therefore, v_2 is equal to 0.

Now, in principle that is possible, but the issue is that if you have brought the wind to a complete halt then there is no further wind flowing through the windmill right. So, it is like hitting out, it is like hitting a wall, you brought the wind and the wind of the first a first little bit of wind that crossed it basically reached 0 velocity. So, that wind is parked there it is not going anywhere and it is just building up there. Any other wind that is going to come cannot go further past the windmill because this wind is just parked there,

it is not moving anywhere right. So, that is the implication of saying that you have captured all the energy present in the wind.

You have basically stopped the wind. So, the moment you stop the wind there is no further energy to capture. So, if you ever manage to capture all the energy in the wind you can do so, only momentarily. The moment you do it all the energy you will no longer have a wind that is flowing because you stopped the wind and therefore, there is no further interaction of the wind with the windmill and therefore, there is no further energy that is captured, right. So, when you go to this condition that you have captured the entire energy the windmill stops. The wind is fully stopped by the windmill the windmill stops, because the wind stops.

Another option is you have wind arriving with same thing half mv_1 square and let us say it is completely unaffected by the windmill right. So, completely unaffected by the windmill which means it comes out of the side out of the windmill also with velocity v_1 , v_2 is equal to v_1 . The velocity with which it comes past the windmill is the same as a velocity with which it arrived with the windmill. What is the implication of that? The energy that it had before it reached the windmill is completely preserved as it comes past the windmill; that means, no energy was delivered to the windmill. So, both when the wind is unaffected by the windmill. So, no energy transferred to the windmill. So, both when the velocity is completely stopped and when there is you know where the velocity is completely protected in both these instances there is no energy transfer to the windmill. So, these then therefore, set the limits. So, therefore, if you think that you know you can capture 100 percent of the energy you cannot, we have just seen that you cannot capture 100 percent of them.

So, we also see that if we capture nothing, of course you have captured nothing. So, there is 0 on this side and there is 0 on that side, based on what is the velocity of the wind that is that is there, what is the velocity that is possessed by the wind that has just cross the windmill right. So, as you can see there must be some optimum value of velocity in the middle where a fair bit of energy has transfer has been transferred to the windmill and still the wind is moving and therefore, more breeze can keep coming here right. So, that is the idea. So, it turns out that if you do the calculation and there is some, I mean basically you write all the equations associated with this flow of wind and put in the parameters are appropriate to capture all these ideas together when you do that you will

arrive at a factor of $\frac{16}{27}$ or 0.59 almost 60 percent, that is the best that you can capture from the wind.

So, we had the half rho A v cube right as the power that is available in the wind if you go back here, half rho A v cube that is the power that is available in the wind. So, if you take that half rho A v cube that is the power that is available named wind. We can actually capture only 0.59 into this thing theoretical limit. This is a theoretical limit of how much energy or how much power in this case that you can capture from the power that is available in the wind that is coming to the wind turbine. And in reality actually what you capture as you can expect is actually less than this theoretical limit, so often we are actually capturing only 10 to 30 percent of the energy that is available at the wind.

So, only 10 to 30 percent of the energy that is available in the wind is actually being captured which is less than the theoretical limit which is what you will expect. Some nice designs will enable you to capture you know, this itself at the at 30 percent, this is 50 percent of the what is possible in the theoretical limit, 50 percent of what is the theoretical limit, you can actually get to 60 or 70 percent of what is possible of the theoretical limit. So, instead of 0.6 you will have 0.6 into 0.7. So, correspondingly you will have you know 40 percent of the energy that is available in the wind. So, something like this can be done, this is the limit and so, this is what we have to keep in mind.

So, if we go back here, this is the power that is available in the wind that has arrived. So, as we have seen the theoretical limit actually is about 60 percent of this. So, it is distinctly less than this and what is actually being captured is only about 10 percent or so, of this. So, only about 10 percent of the graph that you see here is actually being captured at any given point in time. So, for example, if you take, for example, at 40 kilometers an hour this is almost about 10 megawatts. So, actually what you are going to capture is only about around 1 megawatt right. So, in reality this is what you will capture. So, if you plot those points correspondingly the actual curve that you will see is this one right. So, we will get about 1 megawatt from this plant, after taking into account the theoretical limit that is there and all the practical inefficiencies that are there. So, the curve on top is what is there in the wind, what is available in the wind, and what you see below is the power captured by the, power captured by the windmill is this value that the lower curve that you see.

So, this is what I would say as the you know performance characteristic of that windmill which sort of nicely captures the main aspects of how the you know power available in the wind varies as a function of velocity and the fact that there are some limits on cut in speed rated speed and cut out speed and within that framework is what the windmill actually operates. And so, this plot here shows you what is the framework within which the windmill is operating.

So, I will just add a couple more comments here as we you know sort of wind up this discussion on the windmill and what energy is associated with it.

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Blade types:

Drag type: Greater torque, lower rotational speed. Better suited for mechanical work

Lift type: Higher rotational speed. Better suited for power generation



We spoke about this you know the tip speed ratio here, tip speed ratio here and it is a ratio of the rotational speed of the blade to the wind speed and I told you that it has some significance. So in fact, what you will see is that if you look at windmill designs in olden days there was a certain type of design which was referred to as the drag type design and usually that would have many more blades. So, it would have many more blades and the phenomenon that was occurring there was the breeze or the wind would physically push the blade out of the way. So, that is how the original wind mills operated, the wind would flow in one direction and it pushed the blade away, physically push the blade away.

Generally those kinds of windmills have much lower rotational speed, but they have much greater torque and it was therefore, very convenient for the use that it was being

put to at that point in time which was primarily is you know, there for you know grinding grains or you know pumping water, where torque was a major requirement. The speed was not such a major requirement, but the torque was required because it had to push against the grains or lift up water from a deep well etcetera and so that you now torque was required to enable that to happen. So, it is better suited for mechanical work of that nature where you are you know grinding something, putting all water from deep well etcetera.

The modern type of windmills are of the lift type. So, they actually use something that is referred to as the aerofoil design which is similar to the design of the wings of aircraft etcetera and there it is got to do, how the blade moves has got to do with the velocity of the wind velocity and pressure of the wind on one side of the blade relative to the velocity and pressure on the other side of the blade. So, that is how that is designed and it creates what is called a lift. And when it does that you actually get much higher rotational speeds. And as a result the power generation part when you want to do power generation the rate of rotation of that turbine is a very critical component which decides how effective your power generation process is and therefore, the tip speed has to be high, it cannot be slow, it has to rotate rapidly, if it keeps rotating rapidly you are in a better position to generate power from it.

So, the modern wind windmills are sort of optimized towards the power generation aspect of it. The old windmills by just by chance they were designed in such a way based on whatever knowledge was available at that point in time that they actually generated a lot of torque which was very useful for the activity that they were being used for which was primarily for grinding or pumping water. So, there are some difference there in the design and so that is something that is worth you know keeping in mind or you know even examining in greater detail, but that is something that we should be aware of.

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Conclusions:

- 1) The power available in Wind is proportional to the third power of wind velocity
- 2) There are practical aspects that limit the range of wind velocities that can be effectively tapped
- 3) There is a theoretical limit to the extent to which energy available in the wind, can be captured



So, to conclude we have we have noted through this class that there is power available in the wind and it is proportional to the third power of wind velocity. So, it is not linearly proportional to the velocity of the wind, so if you actually double the wind velocity, the power that is available in the wind is goes up by a factor of 8, right. So, what is the velocity goes up by a factor of 2, the power and the wind goes up by a factor of 8. So, that is a huge difference, factor of you know I mean cube, v^3 is, the power is proportional to v^3 . So, it is a very important result to keep in mind with respect to wind energy capture and usage.

We also noted that there are practical aspects that limit the range of wind velocities that can be effectively tapped. So, just because we have P is proportional to v^3 , at least we cannot just assume that you know it would be great to you know capture the energy that is available in a storm, but really there are practical aspects that prevent us from capturing the energy that is available in a storm because that is very strong winds and it may not even be steady winds. So, you will have gust of winds, sudden stop, sudden gust of a wind etcetera. So, it is not a very convenient way to capture power and that may be based on that limits, the material limits of that windmill structure because it should not you know break up.

So, therefore, there are some practical aspects that limit the range of wind velocities that can effectively be tapped for you know energy generation. And also we noted that there

is a theoretical limit where we took into account the possibilities that neither the wind does not interact at all with the windmill and the other extreme where it is interacted to the point with the wind has come to a complete halt. So, we saw that on both I mean both extremes the you know power being captured by the windmill is 0, and therefore in between you have a range of velocities where naturally you expect that the power will keep increasing, it will reach some kind of an optimal value and then it will start decreasing some high value and then start decreasing, a maximum will be reached and then it will start decreasing. Some optimal value of velocity at which the maximum will be reached at which; this is the velocity that at the exit point after the windmill.

So, it turns out that the best that we will get is about 60 percent, 59 point some percent and that is a limit, the theoretical limit to what can be accomplished. Practically, we actually get less than that, maybe half of that maybe two-thirds of that and so on. So, there is a theoretical limit and where which we need to be aware of, which limits the extent to which energy can be energy that is available in the wind can actually be captured.

So, these were the major points that I wanted to cover in today's class, what is the power available in the wind as a function of velocity, what are some practical aspects that limit how you can use the windmill and what are some theoretical aspects that limit what you can get from a windmill.

So, with this we will halt today we will take up for more details associated with this in our subsequent classes.

Thank you.

Lecture: In this lecture, the relationship between power and wind speed of a horizontal axis wind turbine is discussed. The theoretical and practical limits with respect to the energy that can be captured from the wind are explained.

Keywords: Horizontal axis wind turbine, Relation between power and wind speed, Betz law, theoretical limitations and practical limitations.