**Climate Change Science**
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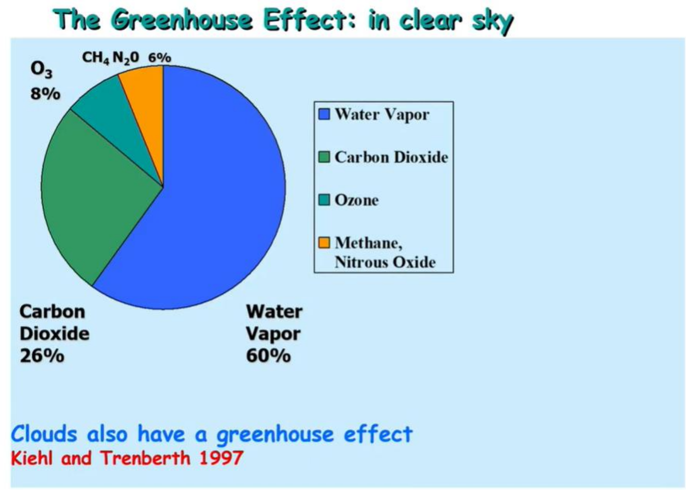
# Lecture 06
# Simple Energy Balance Model

In the previous lecture, the greenhouse effect was examined by comparing the atmospheres of Venus, Mars, and Earth. It was shown that Venus exhibits a strong greenhouse effect due to its extremely dense atmosphere and high concentrations of carbon dioxide, while Mars, having a very thin atmosphere, experiences virtually no greenhouse effect. Earth's greenhouse effect lies between these two extremes.




Among the greenhouse gases present on Earth, water vapour contributes the most to the greenhouse effect, followed by carbon dioxide and other minor gases. However, a key point emphasized is that humans cannot directly control the amount of water vapour in the atmosphere, as it is largely governed by temperature. In contrast, carbon dioxide acts as the primary driver of Earth's temperature, and water vapour responds to changes initiated by $CO_2$. This dynamic makes carbon dioxide the critical gas in regulating Earth's climate, despite water vapour being the most potent greenhouse gas in terms of effect.

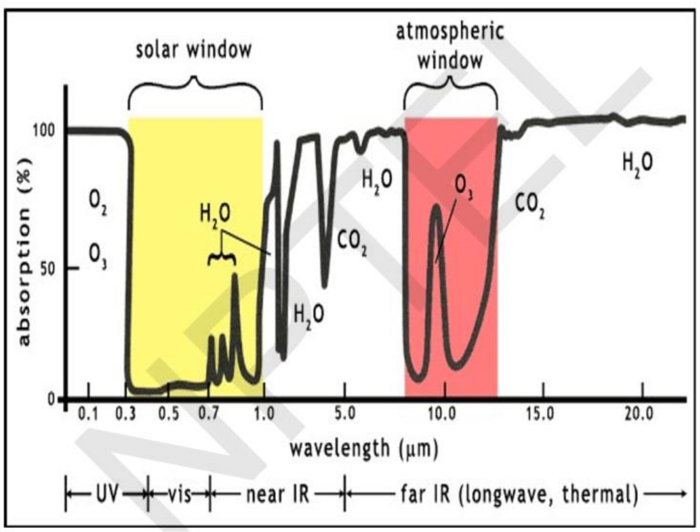
To study Earth's mean climate, researchers often begin with a simple energy balance model.

$$\frac{S}{4}(1-\rho) = f\sigma {T_g}^4$$

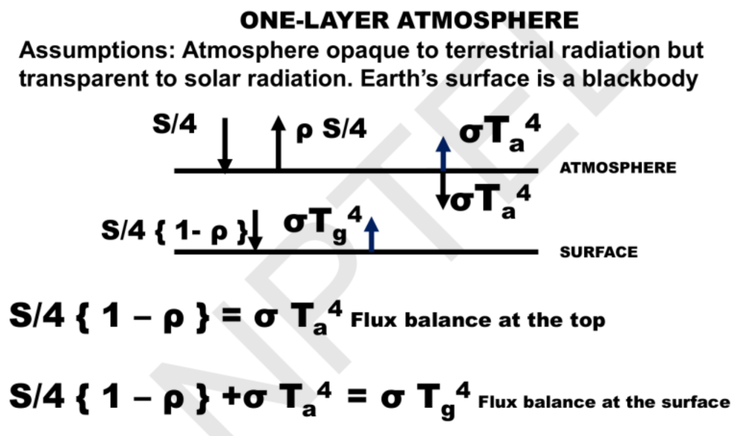
On the left-hand side of the balance, the energy absorbed from the Sun is represented. The solar constant $S$ is divided by 4 to account for the spherical geometry of the Earth, giving the average incoming solar radiation per unit area. A portion of this radiation is reflected by the Earth-atmosphere system due to its albedo $\rho$, and the remaining fraction, $(1-\rho)$, is absorbed. On the right-hand side, the energy leaving the Earth system and going to space is considered. This is commonly modelled by assuming the Earth's surface behaves like a blackbody emitter, with a factor $f$ representing the fraction of emitted radiation that escapes to space. This fraction $f$ is influenced by the greenhouse effect.

Although many books and papers refer to $f$ as the "effective emissivity," this terminology is misleading. We stress that $f$ is not solely determined by the emissivity of the atmosphere. Rather, it also depends on additional factors, such as Earth's albedo in the solar spectrum and the atmosphere's ability to absorb solar radiation. Therefore, while this simplified model is widely used, it is important to recognize its limitations and the complexity underlying $f$.

Satellite measurements show that the value of the solar constant $S$ is approximately 1364 W/m². Observations of Earth's albedo across different months reveal an average annual value of about 0.3. If the parameter $f$ in the simple energy balance model were equal to 1, implying a planet with no atmosphere, the resulting surface temperature calculated from the model would be 255 K. However, this is inconsistent with the observed global average temperature, which is around 288 K. This discrepancy clearly indicates that $f$ is not equal to 1, meaning not all surface radiation escapes to space.




The reason for this lies in the nature of Earth's atmosphere: it is almost transparent to incoming solar (shortwave) radiation but highly opaque to outgoing longwave (infrared) radiation, with an absorptivity close to 100% in the longwave region. There is, however, a narrow atmospheric window around 10 microns where the atmosphere is relatively transparent. This unique spectral property of Earth's atmosphere is what leads to the greenhouse effect, preventing some of the surface-emitted radiation from reaching space and thereby warming the planet.




We now construct a simple model of the Earth's atmosphere. As before, the incoming solar radiation per unit area is $\left(\frac{S}{4}\right)$, and the planetary albedo is denoted by $\rho$. The portion reflected back to space is $\left(\frac{S}{4}\right)\rho$, while the absorbed portion is $\left(\frac{S}{4}\right)(1-\rho)$. For simplicity, we neglect atmospheric absorption of solar radiation, so the absorbed solar energy is assumed to reach the surface directly.

$$\frac{S}{4}(1-\rho) = \sigma {T_a}^4 \ (Flux\ balance\ at\ TOA)$$

The Earth's surface emits radiation as a blackbody, according to the Stefan-Boltzmann law. In this simple model, we also assume that the atmosphere behaves like a blackbody, emitting radiation equally upward and downward, although this is not physically accurate. The fundamental assumption here is that at equilibrium, the solar energy absorbed by the Earth-atmosphere system (as seen from space) is balanced by the longwave radiation emitted to space by the atmosphere.

$$\frac{S}{4}(1-\rho) + \sigma {T_a}^4 = \sigma {T_g}^4 \ (Flux\ balance\ at\ the\ surface)$$

At the surface, two sources of radiation are considered: the absorbed solar radiation $\left(\frac{S}{4}\right)(1-\rho)$, and the downward longwave emission from the atmosphere, given by $\sigma {T_a}^4$, where $T_a$ is the atmospheric temperature. Their sum equals the longwave emission by the surface, $\sigma {T_g}^4$, where $T_g$ is the ground temperature. For this model, we neglect other heat transfer mechanisms like evaporation and turbulent exchange, which will be included in future refinements.

By eliminating $T_a$ from the energy balance equations, we obtain a simplified result:

$$\sigma {T_g}^4 = \frac{S}{4} \times 2(1-\rho)$$

Substituting the observed mean albedo value of $\rho = 0.3$, this equation yields a surface temperature of approximately 303 K, demonstrating a greenhouse warming compared to the 255 K calculated without an atmosphere.

So, this very simple model we are starting with is giving better estimate than 255 K, but it is still too high. We know that the surface temperature of the Earth is around 288 K. This model says it to be 303 K. So, we are off by around 15 K, and that is because we are making two mistakes. One, we have neglected the absorption of solar radiation by the atmosphere; and two, we have also assumed the atmosphere to be a blackbody. So, we will now have to relax these assumptions to achieve better results.

Although this very simple model gives a better estimate of Earth's surface temperature than the earlier value of 255 K (which assumed no atmosphere), it still overestimates the actual observed temperature of 288 K, predicting 303 K instead. This discrepancy of about 15 K arises due to two main oversimplifications in the model. First, the model neglects the absorption of solar radiation by the atmosphere, assuming that all non-reflected solar energy reaches the surface. Second, it treats the atmosphere as a perfect blackbody, which is not physically accurate.

To improve the accuracy of the model and bring the estimated surface temperature closer to observed values, these assumptions will need to be relaxed in the following steps—by accounting for atmospheric absorption of solar radiation and by treating the atmosphere as a graybody rather than a blackbody.

In the next version of the model, atmospheric absorption of solar radiation is incorporated. A new parameter, 'A', is introduced to represent the fraction of incoming solar radiation absorbed by the atmosphere. The rest of the model structure remains similar to the previous one, but now the Earth's surface receives less solar radiation, as a fraction 'A' is absorbed before reaching it.

$$\frac{S}{4}(1-\rho) = \sigma {T_a}^4 \ (Flux\ balance\ at\ TOA)$$

$$\frac{S}{4}(1-\rho-A) + \sigma {T_a}^4 = \sigma {T_g}^4 \ (Flux\ balance\ at\ the\ surface)$$

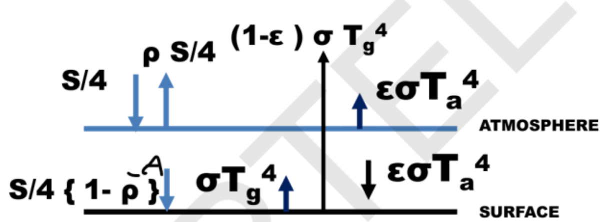
Once again, the temperature of the atmosphere $T_a$ is eliminated to derive a simplified result for the surface temperature.

$$\sigma {T_g}^4 = \frac{S}{4}[2(1-\rho) - A]$$

Using an atmospheric solar absorptivity of $A = 0.2$ (based on model estimates), along with a measured albedo of 0.3, the resulting surface temperature is 291.5 K. This is a much-improved estimate, only 3.5 K higher than the observed global mean temperature of 288 K, demonstrating that including atmospheric solar absorption significantly improves the model's accuracy.

The model developed so far is based on just three parameters: the incoming solar radiation (S), the fraction of solar radiation absorbed by the atmosphere (A), and the Earth's albedo (ρ). With only these three quantities, the model estimates the Earth's surface temperature to within 3.5 K of the observed value, which is already quite impressive. However, this level of accuracy is still not considered sufficient. To improve the model further, another important physical factor is introduced: the atmosphere is not completely opaque to the Earth's longwave radiation. In reality, a small fraction of the radiation emitted by the surface, roughly 5 percent, can escape directly to space through a region in the spectrum known as the atmospheric window, centered around 10 microns. This escaping radiation needs to be included in the model to enhance its realism.

**When the atmosphere is not opaque to terrestrial radiation**




**Note that we have assumed that the atmospheric emission is the same both upwards and downwards**
**This is not true because the temperature of the atmosphere decreases as we go up. We have neglected the energy transfer by evaporation and turbulent processes. These can be included but we cannot get an analytical solution**

Therefore, the model is now refined to account for the partial transparency of the atmosphere to longwave radiation. In this improved representation, radiation from the Sun is partially reflected by the Earth-atmosphere system, while the rest is absorbed either by the atmosphere or by the surface. The emissivity of the atmosphere (ε) is introduced as a parameter to characterize its non-blackbody behavior in the longwave (infrared) spectrum. This emissivity controls how much radiation the atmosphere emits both upward to space and downward toward the surface. Additionally, the model includes the absorption of solar radiation by the atmosphere, which had been neglected in the previous step but is now properly incorporated. With all these factors in place, the revised energy balance equations are established and used to solve for the Earth's surface temperature more accurately.

$$\frac{S}{4}(1-\rho) = \varepsilon\sigma {T_a}^4 + (1-\varepsilon)\sigma {T_g}^4 \; (Flux\ balance\ at\ TOA)$$

$$\frac{S}{4}(1-\rho-A) + \varepsilon\sigma {T_a}^4 = \sigma {T_g}^4 \; (Flux\ balance\ at\ the\ surface)$$

In the refined energy balance model, the radiation absorbed from the Sun at the top of the atmosphere is shown on the left-hand side of the top-of-atmosphere equation. The first term on the right-hand side of the top-of-atmosphere equation represents the radiation emitted by the atmosphere into space, governed by its emissivity. The last term is a newly introduced component: it accounts for the radiation emitted by the surface that is transmitted through the atmosphere without being absorbed. Since the atmosphere is not a perfect absorber, a fraction of the surface-emitted radiation escapes directly to space. This fraction is represented by $(1 - \varepsilon)$, where $\varepsilon$ is the emissivity of the atmosphere. Thus, $(1 - \varepsilon)$ effectively models the transmissivity of the atmosphere in the longwave spectrum, especially relevant in the atmospheric window around 10 microns.

At this point, the model includes four key parameters: the solar constant $S$, the atmospheric emissivity $(\varepsilon)$, the absorptivity $(A)$ of solar radiation by the atmosphere, and the albedo $(\rho)$ of the Earth. By eliminating the atmospheric temperature $T_a$ from the surface and top-of-atmosphere energy balance equations, a simplified expression for the surface temperature is obtained.

$$\sigma {T_g}^4 = \frac{S}{4}\frac{[2(1-\rho) - A]}{[2-\varepsilon]}$$

When this model is evaluated using observed and estimated values, $S \approx 1364\ W/m^2$, $\rho \approx 0.3,\ A \approx 0.2,\ and\ \varepsilon \approx 0.95$, the result closely matches the observed global mean surface temperature of 288 K.

This match is somewhat fortuitous, as it relies on simplifying assumptions and approximations that do not significantly distort the result. Nevertheless, the value of this model lies in its ability to capture the essential physics of Earth's climate system using only four parameters, without the need for complex numerical simulations. It demonstrates that the observed surface temperature can be explained by a basic radiative transfer model incorporating solar input, surface reflection, atmospheric absorption, and emissivity.

However, the model has important limitations. It neglects non-radiative energy transfers such as latent heat flux from evaporation and condensation of water, as well as sensible heat flux due to turbulent mixing by winds. These processes are significant in the Earth's energy budget and must be considered in more comprehensive models. Still, this simple radiative model serves as a foundational tool for understanding the Earth's mean temperature.

Another key simplification in the model is the assumption of an isothermal atmosphere, which leads to the idea that the atmosphere emits radiation equally in both upward and downward directions. In reality, this assumption is not valid. The atmosphere is not isothermal—its temperature decreases with altitude. This is a well-known observation:

the temperature is highest at the surface (sea level) and decreases as one ascends, for example, up a mountain like Mount Everest. This vertical temperature gradient occurs because most of the solar radiation is absorbed by the Earth's surface, which then heats up and transfers energy to the atmosphere through various processes. As a result, the atmosphere is generally colder than the surface, especially at higher altitudes.

This temperature stratification is a significant factor in radiative transfer and should ideally be incorporated into models. The assumption of a uniform atmospheric temperature leads to inaccuracies in estimating radiative fluxes. However, the errors introduced by this simplification, along with the neglect of non-radiative fluxes like evaporation and sensible heat transfer, happen to approximately cancel each other out in this case. This fortunate cancellation leads to a surface temperature estimate that is quite close to observed values. While this outcome is encouraging, it must be acknowledged that the accuracy arises from compensating errors, and the result should be interpreted with caution.

While it is possible to incorporate the effects of non-radiative heat transfer—such as evaporation and sensible heat fluxes—as well as the variation of atmospheric temperature with altitude into the model, doing so introduces complexity. The key difficulty lies in the fact that, once these two important aspects are included, a simple analytical solution is no longer possible. Instead, the problem requires numerical methods for a solution. This marks a departure from the previous model, which offered a neat closed-form expression for the Earth's surface temperature.

$$\sigma T_g^4 = \frac{S}{4}\frac{[2(1-\rho)-A]}{2-\varepsilon}$$

**For $\rho=0.3$, $A=0.2$, $\varepsilon = 0.95$, $T_g = 288$ K close to the observations**
**If there are no clouds $\rho= 0.15$, then surface temperature will be 305 K. This shows the important role played by clouds in modulating the earth's temeprature**

**Limiting Cases :**

***No atmosphere***: $A = 0$ & $\varepsilon = 0$

$$T_g{}^4 = \frac{S}{4}(1-\rho), \; for\; \rho = 0.3, Tg = 255\ K$$

***No solar absorption by atmosphere***: $A = 0$

$$T_g{}^4 = \frac{S}{4}[2(1-\rho)], \; for\; \rho = 0.3, T_g = 303\ K$$



To summarise, the simple radiative balance model that uses only a few key measured parameters is able to accurately reproduce the Earth's global mean surface temperature. However, an important insight from this model is the sensitivity of Earth's climate to cloud cover. Clouds contribute significantly to Earth's albedo, which is currently measured to be about 0.3. If clouds were completely absent, the albedo would drop to around 0.15, leading to a substantial rise in surface temperature to 305 K, which is 17°C higher than the current value. This indicates that clouds play a critical cooling role in Earth's climate system.

As a result, any future changes in cloud cover—potentially driven by human-induced climate change—can have significant implications. An increase in cloudiness might offset some of the warming from greenhouse gases, while a decrease in clouds would amplify the warming. Unfortunately, cloud response to climate change remains one of the major uncertainties in climate modelling. Different climate models project different outcomes—some predict an increase in clouds, others a decrease. Moreover, clouds are not uniform; there are various types of clouds with diverse radiative effects, making it difficult to determine their net influence on climate. This complexity is hidden in simple models, which rely on a fixed albedo value, even though albedo can change dynamically with cloud cover.

It is important to remember that the simple radiative balance model achieves accurate results primarily because it uses observed values for its key parameters. However, while it works well for the current climate, it cannot predict future changes in temperature unless we know how these parameters, especially cloud cover, will evolve. Since clouds greatly influence Earth’s albedo, any uncertainty in predicting cloud behavior translates directly into uncertainty in future climate projections. The model also demonstrates how critical the atmosphere is in maintaining current temperatures. In the hypothetical absence of an atmosphere, with both atmospheric absorption (A) and emissivity ($\varepsilon$) set to zero, the surface temperature would be just 255 K. On the other hand, if only the solar absorption by the atmosphere is neglected, as done in an earlier simplified version of the model, the estimated temperature rises to 303 K. This comparison highlights how different atmospheric processes play crucial roles in determining Earth’s surface temperature.

All aspects of the simple climate model are now summarized. It is essential to understand how the four key parameters—incoming solar radiation, planetary albedo, solar absorptivity, and infrared emissivity of the atmosphere—control Earth’s climate. This model, originally proposed by Professor K. N. Liou of the University of Utah, successfully predicts the global mean surface temperature using only these four parameters. However, it is important to recognize that each of these parameters is influenced by many complex factors, such as cloud type and coverage, infrared characteristics of trace gases, surface ice extent, and vegetation cover. By assigning values to these parameters, we are simplifying the inherent complexity of the Earth’s climate system. Nonetheless, this model serves as a foundational tool to build intuition about the Earth's global energy balance. As the course progresses, the various complex interactions and feedbacks that this model hides will be explored in greater depth.

This simple model is also valuable for explaining how each of the four key parameters influences Earth's climate. It serves as an effective communication tool, especially in the context of public discussions about climate change. Around 30 years ago, when the issue of global warming first gained widespread attention, there was significant debate, with many suggesting that variations in solar radiation were primarily responsible for the observed warming rather than human activities. While it is true that solar radiation plays a crucial role in Earth's climate, at that time, the lack of long-term, accurate data on solar input made it difficult to validate or refute this claim. However, satellite observations over the last 60 years have provided high-precision measurements of the incoming solar radiation, 'S'. These data show that the annual mean value of 'S' has varied by only about 0.1% over this period. This minimal variation makes it clear that changes in solar radiation cannot explain the 1 to 1.5°C warming observed over the past 150 years. Therefore, the argument that solar variability is the dominant factor in recent global warming is not supported by evidence.

To evaluate how different parameters in the simple climate model influence Earth's surface temperature, one can use calculus to compute sensitivities. Since the model provides an analytical formula for the global mean surface temperature, it can be differentiated with respect to each of the four parameters. Starting with incoming solar radiation ($S$), since the surface temperature depends on the fourth root of $S$, a 1% increase in $S$ leads to a 0.25% increase in temperature. It is demonstrated as below.

$$\sigma {T_g}^4 = \frac{S}{4}\frac{[2(1-\rho)-A]}{[2-\varepsilon]}$$

On differentiating the above equation with respect to $S$,

$$4\sigma {T_g}^3 \frac{\partial T_g}{\partial S} = \frac{1}{4}\frac{(2(1-\rho)-A)}{(2-\varepsilon)}$$

On rearranging the terms in the first equation we obtain,

$$\frac{\sigma {T_g}^4}{S} = \frac{1}{4}\frac{(2(1-\rho)-A)}{(2-\varepsilon)}$$

Substituting this relation in the second equation and rearranging the terms will yield,

$$4\sigma {T_g}^3 \frac{\partial T_g}{\partial S} = \frac{\sigma {T_g}^4}{S}$$

$$\boldsymbol{\frac{\Delta T_g}{T_g} = \frac{1}{4}\frac{\Delta S}{S}}$$

For example, with a mean surface temperature of ~288 K, this corresponds to an increase of only 0.72 K. However, satellite observations over the last 60 years show that S has varied by no more than 0.1%, indicating that recent warming trends cannot be attributed to changes in solar radiation.

Next, the solar absorptivity ($A$) of the atmosphere affects surface temperature because increased absorption of solar radiation by the atmosphere means less energy reaches the surface.

$$\frac{\Delta T_g}{T_g} = -\left(\frac{S}{4}\right) \times \frac{A}{4\sigma {T_g}^4(2-\varepsilon)} \times \frac{\Delta A}{A}$$

For a mean surface temperature of 288 K, solar constant of 1364 $Wm^{-2}$, and an atmospheric emissivity of 0.95, a 1% increase in $A$ from an initial value of 0.2 leads to a cooling of about 0.12 K.

Similarly, we derive the sensitivity corresponding to planetary albedo as below,

$$\frac{\Delta T_g}{T_g} = -\left(\frac{S}{2}\right) \times \frac{\rho}{4\sigma {T_g}^4(2-\varepsilon)} \times \frac{\Delta \rho}{\rho}$$

A 1% increase in the Earth's albedo ($\rho$), due to more cloud cover or increased snow and ice, leads to an even larger cooling of about 0.36 K.

On the other hand, the emissivity (ε) of the atmosphere, which controls how effectively it emits longwave radiation, has a warming effect.

$$\frac{\Delta T_g}{T_g} = \left(\frac{1}{4}\right) \times \frac{\varepsilon}{(2-\varepsilon)} \times \frac{\Delta \varepsilon}{\varepsilon}$$

A 1% increase in emissivity, typically due to an increase in greenhouse gases like $CO_2$ or methane, leads to a warming of about 0.65 K.

In conclusion, this analysis shows that a 1% change in any of the four parameters causes an approximate 0.5 K change in surface temperature. Of these, increases in $S$ and $\varepsilon$ cause warming, while increases in $A$ and $\rho$ cause cooling. These sensitivities are crucial for understanding how human-driven changes in greenhouse gas concentrations, aerosol loading, and land surface properties influence future climate change.

Currently, the primary human-induced driver of climate change is the alteration of the atmospheric emissivity ($\varepsilon$), primarily through increasing concentrations of greenhouse gases. A change in emissivity causes a change in the Earth’s temperature, but this temperature change in turn affects other parameters of the climate system, namely solar absorptivity ($A$) and albedo ($\rho$). For example, as the surface warms, ice melts, exposing darker water surfaces, which absorb more solar radiation than ice. This leads to an increase in absorptivity and a decrease in reflectivity, both of which further warm the planet.

This illustrates a crucial limitation of partial derivative-based sensitivity analysis: it assumes all other parameters are held constant. However, in the Earth system, this assumption fails because a change in one parameter typically leads to changes in others. These interdependencies are collectively referred to as feedbacks. Feedbacks describe how an initial change (e.g., in temperature) leads to further changes in the system that either amplify or dampen the original change.

The concept of feedback is central to understanding climate dynamics. It originates from control theory, where positive feedback refers to a process that reinforces the initial change (e.g., warming leads to more warming), while negative feedback refers to a process that counteracts the change (e.g., warming leads to cooling mechanisms). In the context of climate science, positive feedback is destabilizing and undesirable, while negative feedback is stabilizing and beneficial. It is important to remember that although the term “positive” often has a favourable connotation in everyday language, in climate dynamics, positive feedback tends to exacerbate climate change, whereas negative feedback can help mitigate it.

According to the simple climate model, a 1% increase in incoming solar radiation (S) would result in an approximate 0.72 K rise in global mean temperature. However, satellite observations over the last 60 years, spanning several 11-year solar cycles, indicate that changes in S have remained below 0.1%. Therefore, the observed 1.5°C warming over the last 170 years cannot be attributed to variations in solar input. This is a critical conclusion, as earlier hypotheses by some astronomers attributed Earth's warming and cooling trends primarily to solar variability. While such solar influence may have played a major role millions of years ago, it is not the dominant factor in the recent century of global climate change.

Additionally, many introductory climate models include a parameter '$f$' to represent the greenhouse factor, often assumed to be constant. However, the Liou model used here demonstrates that ' $f$ ' is not constant; it is dependent on multiple parameters, specifically emissivity ($\varepsilon$), solar absorptivity ($A$), and albedo ($\rho$).

$$f = \frac{(2-\varepsilon)}{\left(2 - \frac{A}{(1-\rho)}\right)}$$

Since these parameters are linked to the physical and chemical state of the atmosphere, $'f'$ varies with atmospheric composition, cloud cover, surface properties, and other factors. This realization drawn from a simple yet insightful model is important for deeper discussions on climate sensitivity and feedbacks, and cautions against oversimplifying complex atmospheric processes.

The simple model discussed so far assumes an isothermal atmosphere, meaning the atmosphere is treated as having a uniform temperature. However, this is not physically accurate. Observations indicate that the temperature of the Earth's atmosphere decreases with height, at an average lapse rate of about 6.5 K per kilometer. Despite this simplification, the model still yields reasonable results for Earth due to a fortuitous cancellation of errors from neglected processes. However, this model cannot be applied to planets like Venus, which has a much denser atmosphere. On Venus, the surface temperature reaches around 700 K, while at an altitude of about 50 km, the temperature drops to approximately 200 K, a change too large to justify an isothermal approximation.

Because of its thick atmosphere and significant vertical temperature gradient, Venus requires a more complex model. The Liou model, being a single-layer representation, is suitable only for Earth or Earth-like planets with moderate atmospheric thickness. It may also work for Mars, which has a very thin atmosphere. But for Venus or other planets with dense atmospheres, a multi-layer (n-layer) radiative model is necessary. In this approach, the atmosphere is divided into several layers, often 100 or more in modern climate models, with each layer treated as isothermal and having its own energy balance. This method is computationally intensive and cannot be handled analytically or manually. Instead, such models require numerical simulations.

In conclusion, while the simple model provides foundational insight, it has clear limitations. For realistic climate simulations, especially on other planets or for detailed Earth system studies, layered radiative-convective models are essential.