Climate Change Science
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Lecture – 34
Milankovitch Theory(continued)

In the last class, we looked at the data of the ice cores and the incoming solar radiation. And we said that if the mean temperature of the polar region is linearly dependent upon the amount of radiation falling in the polar region, then we should expect that the dominant time periods of oscillation of the ice sheet should be dependent on the amplitude of variation of the incoming radiation at 100,000, 41,000, and 20,000 years.

> **If the mean temperature of the polar region is linearly dependent upon the amount of solar radiation falling in the polar regions, then we should expect that the dominant time periods of oscillation of ice sheets should be dependent on the amplitude of variation of incoming solar radiation in the 100,000, 41,000 and 20,000 period range. The largest amplitude of solar radiation is in the period 20,000 years (precession) but the dominant amplitude of fluctuations of the volume of ice sheets in the polar regions is around the 100,000 years period. Hence, we must conclude that some non-linear phenomena must affect the oscillations of volume of the ice sheets.**

The computations made by Milankovitch showed that the largest amplitude occurred around 20,000 years, the precession cycle. But the observed dominant amplitude of the ice sheet based on delta O-18 is at 100,000 years during the last million years. So, we concluded that some non-linear phenomena must affect the oscillation of the ice volume because a linear theory is not working. It does not show the expected result.

> **While there is a Milankovitch cycle in the range of 100,000 years, related to Earth's orbital eccentricity, the impact of variation in insolation in 100,000 years is much smaller than those of precession and obliquity. There is no simple explanation for the 100,000 year period during the past million years. In the period 5 million to 1 million years ago the dominant periodicity was 41,000 years. The unexplained transition between the two periodicity regimes is known as the Mid-Pleistocene Transition.**

So, there is no simple explanation for why 100,000 years is the dominant period of the ice ages in the last million years. So, there is a transition from what happened between 5 million to 1 million years ago, and from 1 million to today. This is known as the mid-Pleistocene transition. Now, in a recent paper, the authors say that the hysteresis loop of the North American ice sheet is such that after the inception of the ice sheet, its mass balance remains mostly positive through several precession cycles whose amplitude decreases towards the eccentricity minimum.

> The hysteresis loop of the North American ice sheet is such that after inception of the ice sheet, its mass balance remains mostly positive through several precession cycles, whose amplitudes decrease towards an eccentricity minimum.
> The larger the ice sheet grows and extends towards lower latitudes, the smaller is the insolation required to make the mass balance negative. Therefore, once a large ice sheet is established, a moderate increase in insolation is sufficient to trigger a negative mass balance, leading to an almost complete retreat of the ice sheet within several thousand years.
>
> **Insolation-driven 100,000-year glacial cycles and hysteresis of ice-sheet volume by Abe-Ouchi et al., Nature,500,2013**

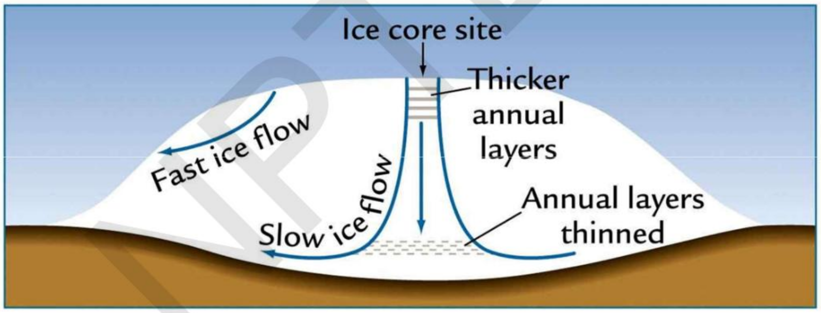
As the eccentricity goes down, you know that the effect of precession is somewhat damped out. So, the impact of precession is only significant when there is a high eccentricity. So, their argument was that the 20,000-year cycle is not working well because its radiation change is not sufficient to make changes in the ice sheets. So, this idea is the idea of skipping—that is, because the maximum amount of incoming radiation is in the 20,000-year cycle, if the ice age does not disappear every 20,000 years, this means that a couple of cycles are skipped because of some other reason. Now, that reason is not clearly stated in this paper.

The reason could be that the carbon dioxide concentration was not sufficiently high to provide the greenhouse effect to increase the temperature. That could be one reason. But the other reason could be also that—remember that ice sheets are very large, massive entities, which have their own response time. We will see later that even the large ice sheets that were there in the Northern Hemisphere—in Canada, in Greenland, and in Eurasia—actually had such a large weight that they depressed the land by many hundreds of meters. So, the argument was that as the ice sheet grows, the land starts sinking.

The land starts sinking; the ice sheet will melt because it will be at a lower altitude. So, that will slow down the increase in the ice sheet thickness. And the reverse will happen during the melting phase. When the ice melts, the land will start rebounding because there is less ice sheet, and this will prevent the ice sheet from melting further because the altitude of the land has increased. So, this is a factor which has not been accounted for by many people and which we will discuss a little later.

Several internal feedback mechanisms have been suggested as crucial in 100-kyr glacial cycles

1.delayed bedrock rebound (promising)
2.The calving of ice-sheet margins(too fast?)
3.CO2 variations(too fast ?)
4.ocean feedback (too fast?)
5.dust feedback(too fast?)



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So, this is the role of what they call bedrock rebound. As the ice sheet grows, it depresses the bedrock, thereby increasing the temperature at the top of the ice sheet, thereby preventing the ice sheet from growing. And the other proposed non-linear phenomenon is the calving of the ice sheet margin. Remember that when the ice sheets grow, they come close to the ocean from the land, and they start breaking off—calving. And that has its own time scale. It operates in its own way, and it operates in a rather abrupt way.

The ice breaking off from ice sheets has been observed in the last 50 years, and they are not linear phenomena—they are highly non-linear. Some people also propose carbon dioxide variation and ocean feedback, but my concern is also dust feedback, because there was a lot of dust during the ice age. The problem with the last three—points 3, 4, and 5—is that these are too fast. They occur over a 5 to 10 year time scale. So, you cannot use this to explain the difference between 100,000, 40,000, and 20,000 years. Those are very long-time scales. But here we see are very fast time scales. So, I would argue that the only promising proposal is about the ice sheet depressing the land. That is a slow process because solid land is not going to respond immediately. There is a time lag of many thousands of years.

So, that can explain some of the unusual things we see in the ice core data. Typically, a weight of 3 km deep ice can depress the local bedrock by 1 kilometer. And there are two responses. There is an immediate response and there is a response with memory. For example, right now, as the ice has been melting for the last 10,000 years, many parts of Eurasia are rising because ice is melting and the land is rising.

In parts of Sweden, parts of Russia, you look at harbors—they are having problems because land is going up very slowly. So, this rising of the land is occurring now, although the ice sheet disappeared in those regions 10,000 years ago. That shows the lag—the late response of the land—and so that will affect any model that you use. So, bedrock has an elastic response and a delayed response. The elastic response is immediate, which is small, and the delayed response is large but takes a long time. That here is given as 3,000 years.

•Weight of a 3 km deep can depress the local bedrock by ~ 1km !!!!

•Bedrock responds to the forcing by ice with two distinct time constants.

•Bedrock sags immediately in an elastic response

•the slow flow of rock in the softer layer of the upper mantle (100-250 km below surface) produces a viscous response that occurs with a time constant ~3000 years

•some parts of Canada and Scandinavia are still rebounding from the last glaciation !!!.

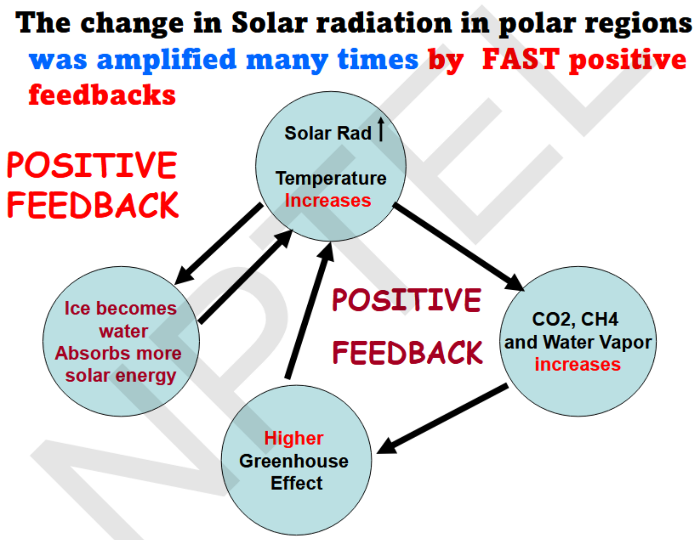
Some parts of Canada and Scandinavia are still rebounding from the last deglaciation. There are also some people concerned about the accuracy of the time in the ice core data. Now, that you may find surprising, but you must remember that it is difficult to estimate accurately at what time the snow was deposited on the Antarctic region, and as it compacted and formed an ice layer, what is the exact date when that ice was formed. That is very critical to understand these cycles.

The deep-sea sediment oxygen isotopic composition record is dominated by a 100,000-year cyclicity that is universally interpreted as the main ice-age rhythm. Here, the ice volume component of this signal was extracted by using the record of in atmospheric oxygen trapped in Antarctic ice at Vostok, precisely orbitally tuned. The benthic marine record is heavily contaminated by the effect of deep-water temperature variability, but by using the Vostok record, the d18 O signals of ice volume, deep-water temperature, and additional processes affecting air d18O were seperated. At the 100,000-year period, atmospheric carbon dioxide, Vostok air temperature, and deep-water temperature are in phase with orbital eccentricity, whereas ice volume lags these three variables. Hence, the 100,000-year cycle does not arise from ice sheet dynamics; instead, it is probably the response of the global carbon cycle that generates the eccentricity signal by causing changes in atmospheric carbon dioxide concentration.

Shackleton, N. J. , The 100,000-Year Ice-Age Cycle Identified and Found to Lag Temperature, Carbon Dioxide, and Orbital Eccentricity. Science, 289, 1897–1902. doi:10.1126/science.289.5486.1897

One proposal by Shackleton, a well-known geologist, is that the 100,000-year cycle does not arise from ice sheet dynamics—as I indicated earlier, response of the land—but probably the response of the global carbon cycle that generates the eccentricity signal by causing changes in the CO_2 concentration. My only problem with this paper is that the CO_2 changes will occur at a much faster response time than the ice sheets. So, to understand the difference between the 100,000-year cycle and the 41,000-year cycle, to invoke the carbon cycle is a bit tough because you have to talk about some other phenomenon which makes the carbon cycle respond after many, many thousand years. So, that part has not been explained well by Shackleton. Now, remember that although

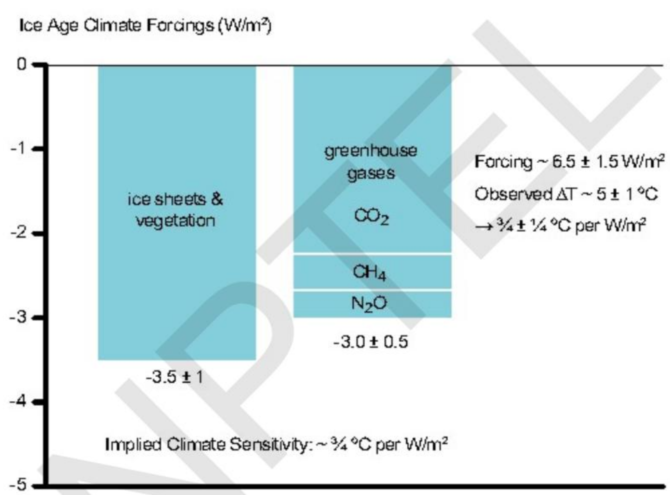
Milankovitch mainly focused on solar radiation, that solar radiation increase in the polar region will trigger other changes.



One is it will increase carbon dioxide, methane, and water vapor because of the temperature rise. Those gases will increase the greenhouse effect and cause further warming. This chain reaction, called the positive feedback, is very important for the ice melting. When ice becomes water, we all know that the region will absorb 10 times more solar radiation than before. Both these positive feedbacks will play a role in rapidly melting the ice sheet.

But again, these feedbacks occur at a much faster time scale than the 100,000-year cycle. Although these are important to quantitatively calculate the temperature rise, they are not going to help us understand the paradox that we saw in the last million years.

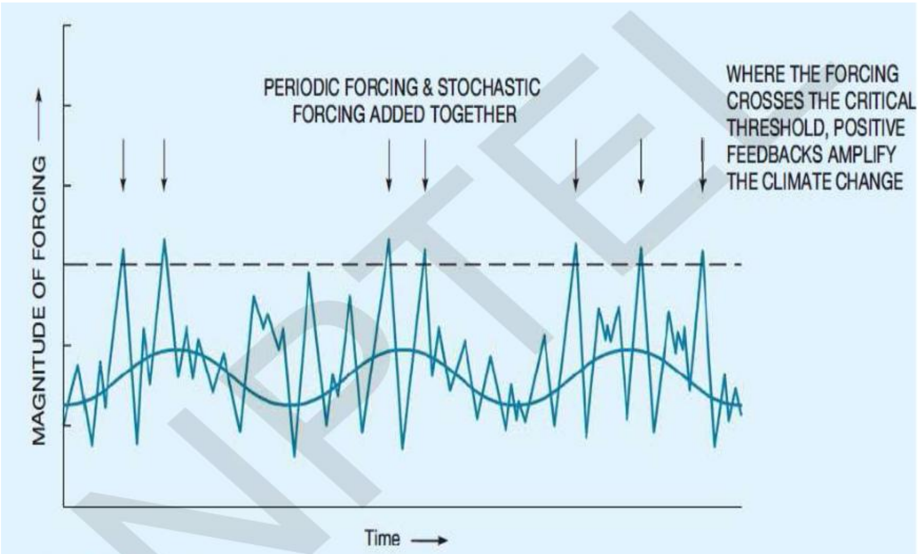
So, let us look at what role carbon dioxide and ice sheet albedo played in the feedback system.



Climate forcings during ice age 20 ky BP, relative to the present (pre-industrial) interglacial period.
(Source: Hansen et al. 2008)

According to the models by Professor Jim Hansen from the Goddard Institute of Space Studies, he has indicated that during the last ice age, on account of larger ice sheets and less vegetation, there was a negative -3.5 W/m^2 climate forcing—that is, 3.5 W/m^2 was the contribution of the ice in increasing the albedo and cooling. This is the contribution to the cooling due to the ice age, and greenhouse gases like carbon dioxide, methane, and nitrous oxide contributed another 3 W/m^2. So, a total of additional 6.5 W/m^2 on top of whatever changes the incoming solar radiation had made.

So, this shows that both the changes in greenhouse gases as well as changes in the albedo of the polar regions will play a role in the way the ice age collapses or the ice age accumulates. Both will be affected by these feedbacks.



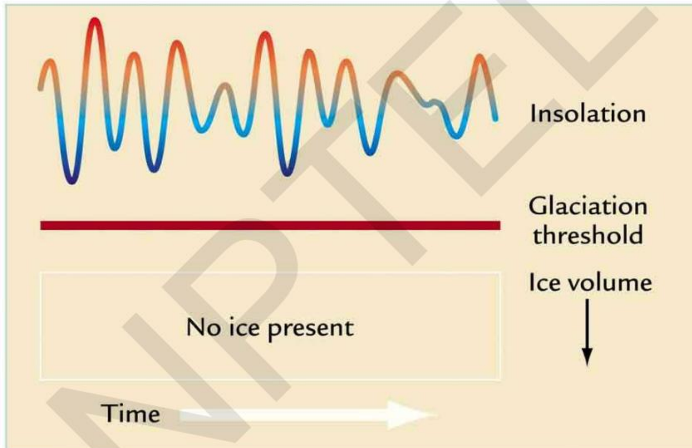
BOX FIGURE 14-4 Schematic representation of stochastic resonance.

Now, there are some people who are arguing that Earth is a complex system and some of the forcing can be random—so-called stochastic forcing. So, we can also think of this stochastic forcing as due to a volcanic eruption, which occurs at random. We cannot predict any specific periodicity for this volcanic eruption.

They feel that on top of the smooth Milankovitch cycle variation of solar radiation, there was an additional stochastic forcing due to either volcanic eruption or some other cause. Now, this is an interesting hypothesis, but there is not enough data to support it.

Now, the other important idea about the non-linearity of the feedback during the ice ages is the idea of thresholds. Remember that each time the solar radiation in the polar region decreases, you need not have an increase in ice volume because there is a value of solar radiation—unless it goes below that, ice will not form. Ice starts forming only when the temperature goes below zero.

Threshold theory

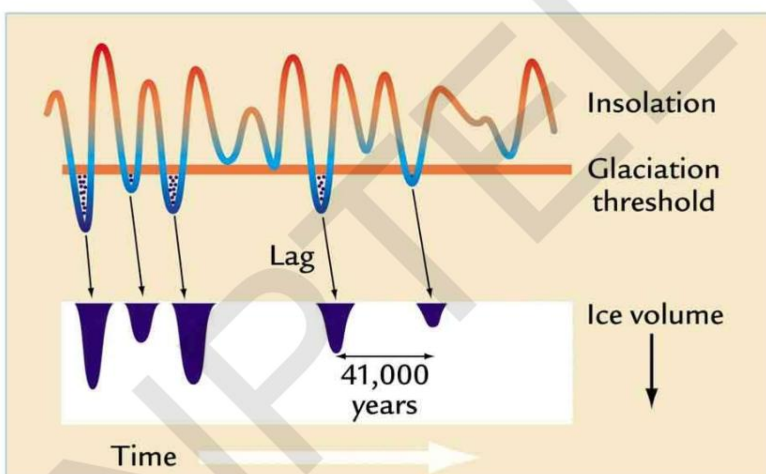


Preglaciation phase

Suppose there is a polar region which is almost ice-free, and the incoming radiation changes, let us say in the 100,000-year cycle, but the minimum radiation is such that it cannot initiate an ice age, then there will be no ice present. So, radiation is changing, but it is unable to trigger an ice age because it is not below a certain threshold value.

So, this is the threshold theory—that you cannot just depend on changes in solar radiation; you also have to look at some absolute value of this radiation. Unless it goes below that value, ice will not form. So, this can be shown further with a cartoon in a situation where the radiation falls below the threshold value. Whenever it falls below the threshold value, ice starts forming.

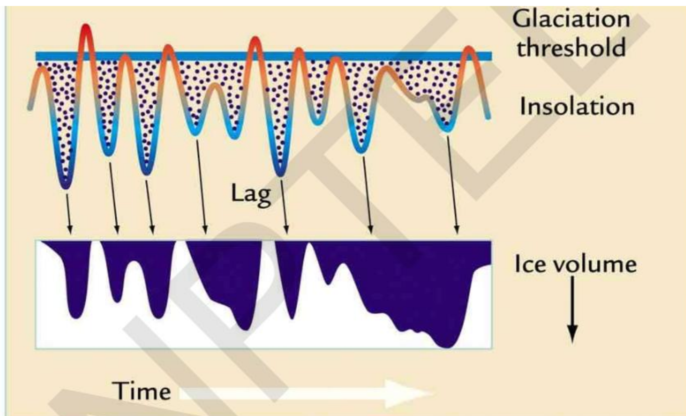
So, the argument for this threshold hypothesis is that although the incoming solar radiation varies with a time scale where the dominant periodicity is a 20,000-year cycle, every time in the 20,000-year cycle, the radiation may not go to a sufficiently low value to trigger an ice age.



Small glaciation phase

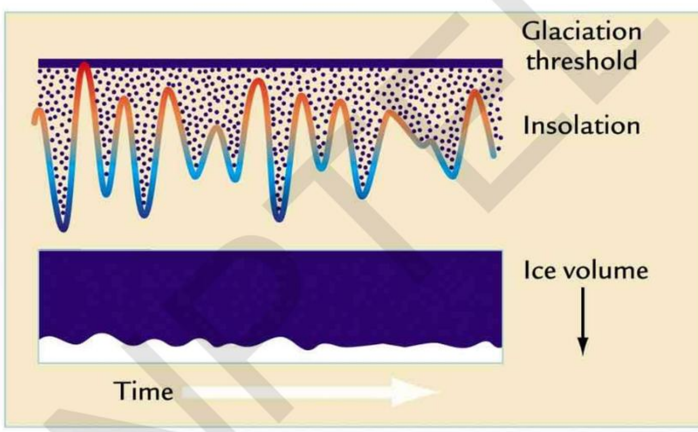
Let us take this (figure shown above) as an example: it starts from a high value, it comes to a low value, but it was still above the threshold—so no ice is formed. It decreased again and again, but did not form ice. Then the third time, it went below and formed an ice age.

So, these are the two periods when Shackleton will say that the ice age is not triggered; it skipped, because the solar radiation did not fall below a certain glaciation threshold. The threshold hypothesis looks quite realistic because, depending on the other conditions in terms of carbon dioxide and water vapor, the radiation input has to go below a certain value for the ice to start forming. It makes sense.



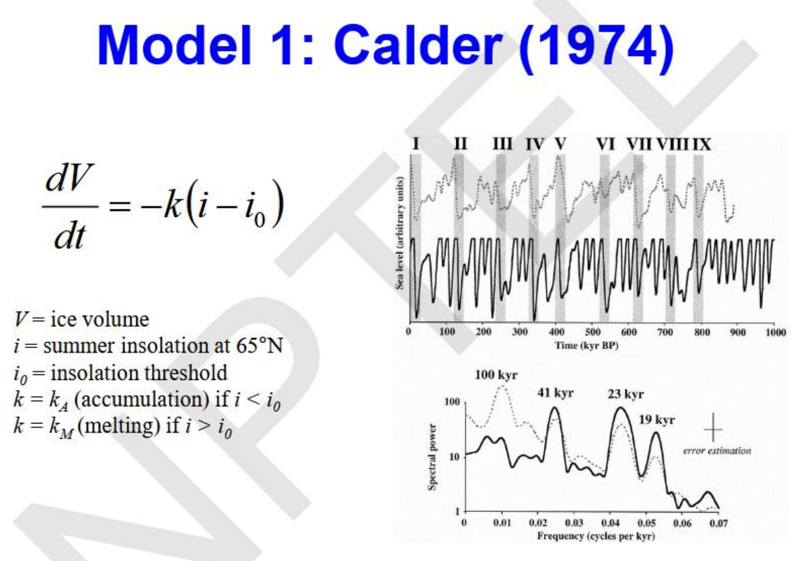
Large glaciation phase

Now, we go to a more dramatic case—so-called the large glaciation phase—when the solar energy falls to such a low value that we will see ice forming almost every time. So, the idea is this: the periodicity of radiation is always there, but whether it will trigger an ice age depends on whether the lowest value of radiation goes below the threshold value for that particular period, depending on carbon dioxide, depending on methane, and depending upon the ocean circulation patterns.



Permanent glaciation phase

And here is one phase where the threshold is so high that all the time the incoming radiation promotes an ice age, in which case ice will be there for a long time. This can also happen, and we will discuss this when we discuss Snowball Earth, which happened 600 million years ago, when under some unusual conditions, the ice age grew all the way to the equator. When that happened, the Earth was completely ice-covered for millions of years. The incoming radiation change did not affect the ice cover because it did not go below some threshold for those conditions. So, this threshold depends on other parameters in that period. It can be CO_2, it can be methane, it can be the configuration of the land. Land configuration also has an impact. So, whether the land is in the tropics or in the polar region will affect it.



Now, this was taken up by people like Calder, who had a model. He said the rate of change of volume of the ice in the polar region is

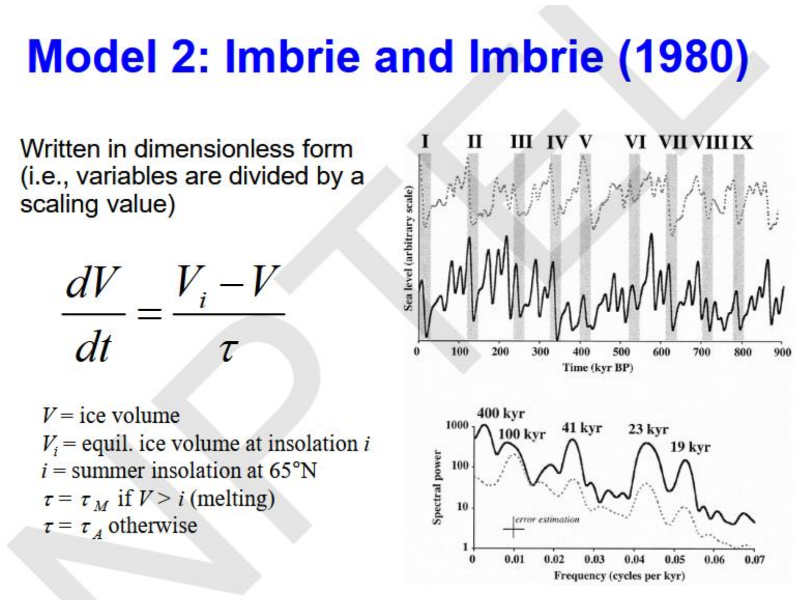
$$\frac{dV}{dt} = -k(i - i_0)$$

So, in this model, unless i goes above i_0, volume will not increase. And notice that the value of k—the rate at which volume will increase—depends on accumulation and melting. It is not the same. That is because ice accumulation and ice melting have different characteristics. So, the k value has to be adjusted for the accumulation phase and adjusted for the melting phase.

When he does that, he gets a fluctuation of ice volume, okay, and it is a spectrum. That spectrum is able to pick up the 41,000 and 20,000-year cycles, but not the 100,000-year cycle. So, if you want to explain what happened in the period between 1 million years and 5 million years, you can invoke Calder's hypothesis. He will show you the peaks that occur in those periods. But you will not get the 100,000-year peak, which is what we observed in the last million years.

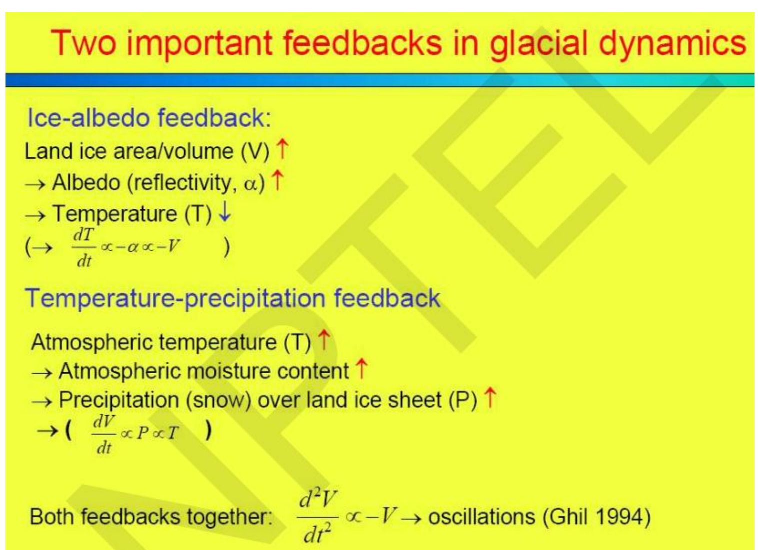
So, Calder's hypothesis works well for the period between 1 million and 3 million years ago, but not between 0 and 1 million years. So, that is the problem.

Now, Imbrie and Imbrie proposed somewhat later a slightly different model in which the rate of change of ice volume depends on whether the volume is above or below an equilibrium volume, Vi. Vi is the equilibrium volume corresponding to some incoming radiation. So, if V is above Vi—the ice volume is very high—then ice volume starts declining.



However, if V is below Vi, it will grow. So, that threshold value for ice volume is Vi, and that determines whether ice occurs or not. And you can see that with that model, Imbrie and Imbrie were able to get the 400,000-year, 100,000-year, and all these cycles, as per the data of the last million years. So, I would argue that the proposal by Imbrie and Imbrie—that there is a threshold ice volume above which the ice volume will decrease, and if the volume of the ice is below the threshold, it will grow—with that model, they were able to reproduce the observed cycle dominating at 100,000, 40,000, and 20,000 years. So, many people are happy with this model of Imbrie and Imbrie, which was proposed more than 40 years ago.

So, you can follow the logic of change in temperature in proportion to ice volume, or you can talk about the ice volume depending on rainfall and temperature.



So, there are various models proposed with threshold values which will determine the spectrum of the ice volume. Now, this did not satisfy Saltzman, who is a well-known name in paleoclimatology.

Glacial Cycles

Salzman-Maasch Model

The Salzman-Maasch model shows how the carbon cycle and the ocean currents can interact to produce unforced oscillations with periods of about 100,000 years. The same model with slightly different parameters can exhibit stationary behavior. By forcing the model with Milankovitch cycles and by slowly varying the parameters over the last two million years, they can produce a bifurcation from small oscillations tracking the Milankovitch cycles to large oscillations with a dominant 100,000 year period.

Seems like a nice idea, but it is not widely accepted as the explanation.

He has a book, which is a very nice book. Saltzman and Maasch developed a model which is able to demonstrate the dominant 100,000-year cycle. And it is a non-linear model. It looks like a nice idea, but it is not yet widely accepted. Now, let us see why. Let us see what the model is and why it is not accepted.

SALTZMAN'S MODEL

$$\frac{dX}{dt} = -\alpha_1 Y - \alpha_2 Z - \alpha_3 Y^2$$

$$\frac{dY}{dt} = -\beta_0 X + \beta_1 Y + \beta_2 Z - (X^2 + 0.004Y^2)Y + F_Y$$

$$\frac{dZ}{dt} = X - \gamma_2 Z$$

where in this particular case X, Y and Z are the ice mass, deep ocean temperature and atmospheric carbon dioxide.

where X is ice mass,

Y is ocean temperature

Z is CO_2

So, the model has three non-linear differential equations. The first equation is for ice volume. The second equation is for deep ocean temperature. And the third equation is for carbon dioxide. So, Saltzman's argument essentially is that the ice volume does not just depend on the incoming radiation—which is here in his model, F(I)—it also depends upon the present temperature and the present concentration of CO_2. So, he has coupled three non-linear differentials, and he gets—as you see—a lot of good results. But he has not convinced everyone about the nature of the non-

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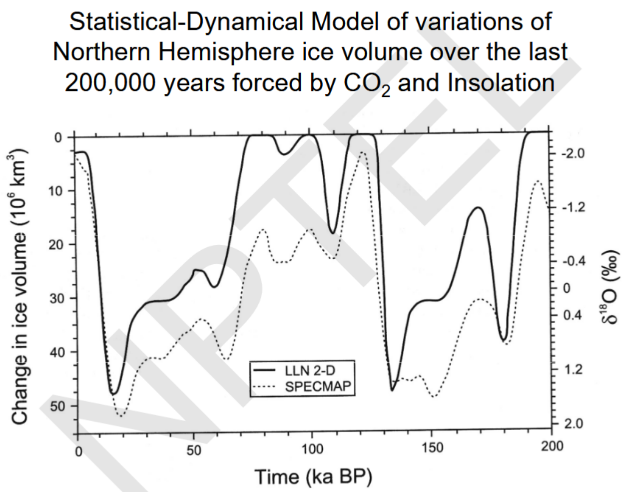
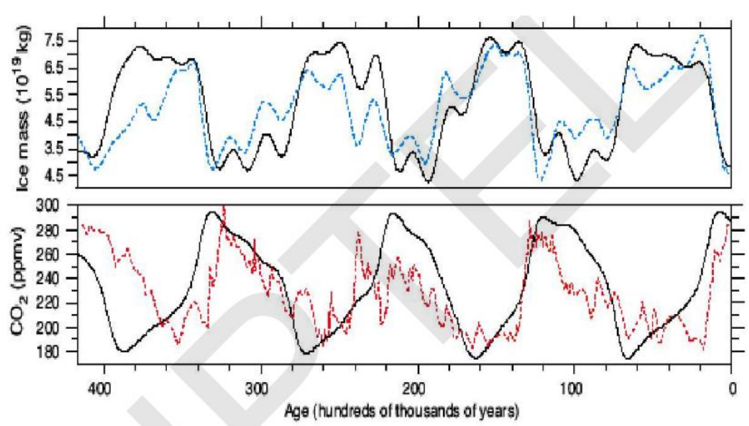
linearity. So, that is the only problem. The non-linearity that he has evoked is what you call ad hoc or empirical, and the proof from either climate models or actual observation is not yet convincing.



But you can see that this paper, published by Saltzman and Maasch, is quite impressive because the black line here is the model of Saltzman and Maasch, and the dotted line is the observed data from SPECMAP. And although there is no perfect agreement for the change in ice volume, I would argue that the broad peaks are matching quite well. So, I would say there is some merit to this model because the oscillations of the delta O-18 and the oscillation of the change in ice volume have broad agreement in the peaks. But broad agreements. So, there is some logic in Saltzman's thinking, but the concern many people have is that the non-linearities introduced here—y^2, x^2, x^2y, only three non-linearities—are not obtained from first-principle derivation from any equation. So, it has been invoked empirically because Saltzman realized that there has to be some non-linearity. So, he used a simple non-linear function, but we need to see whether these can be supported from climate models.

Here is the example of ice mass, and in this case, carbon dioxide.



Solution of the dynamical system climate model of Saltzman and Maasch (1988) for the past 400 thousand years subject to the earth orbital radiative forcing. The model prediction for ice (top panel) and carbon dioxide (bottom panel) are shown. For comparison, the dashed blue curve in the top panel is the SPECMAP ∂18O estimate of ice variations and the dashed red curve in the bottom panel is the Vostok core estimate of CO_2 variation.

The blue curve is the estimate of delta O-18, and the green is carbon dioxide from the ice core data. So, I would say that the Saltzman–Maasch model broadly reproduces the peaks in the ice volume and delta O-18—the broad peaks, that is, the 100,000-year peaks—but not the detail. There is hope here that if we further improve the model, we can get a much closer match between the model and observation.

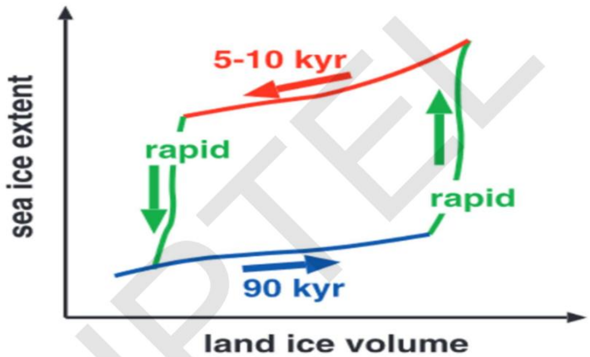
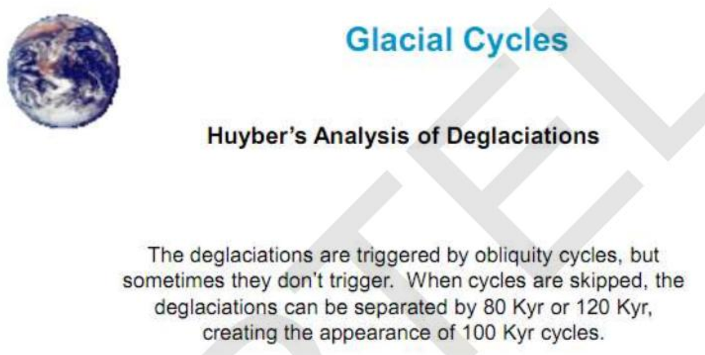


Figure 1. A schematic figure of the hysteresis between land ice and sea ice during glacial cycles, predicted using a simple box model by *Gildor and Tziperman* [2000] and examined here using a model that is continuous in the meridional direction.



Now we go on to understand how the ice ages are tipped due to volatility in the climate models.

$$c\frac{d\tilde{T}}{dt} = Q(1-\tilde{\alpha}(\tilde{T})) - A - B\tilde{T}$$

Lyapunov Potential V
Vmin stable while Vmax is unstable

$$\frac{d\tilde{T}}{dt} = -\frac{dV}{d\tilde{T}}.$$

Clearly,

$$V = -\frac{1}{c}\int [Q(1-\tilde{\alpha}(\tilde{T})) - A - B\tilde{T}]\, d\tilde{T}$$

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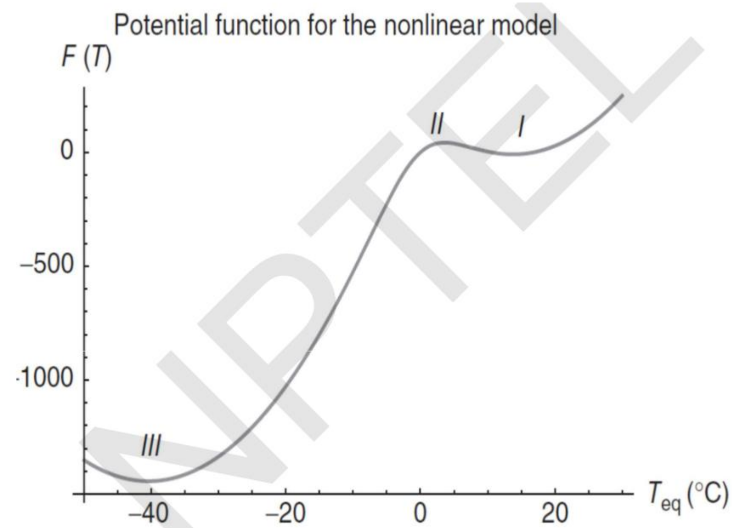
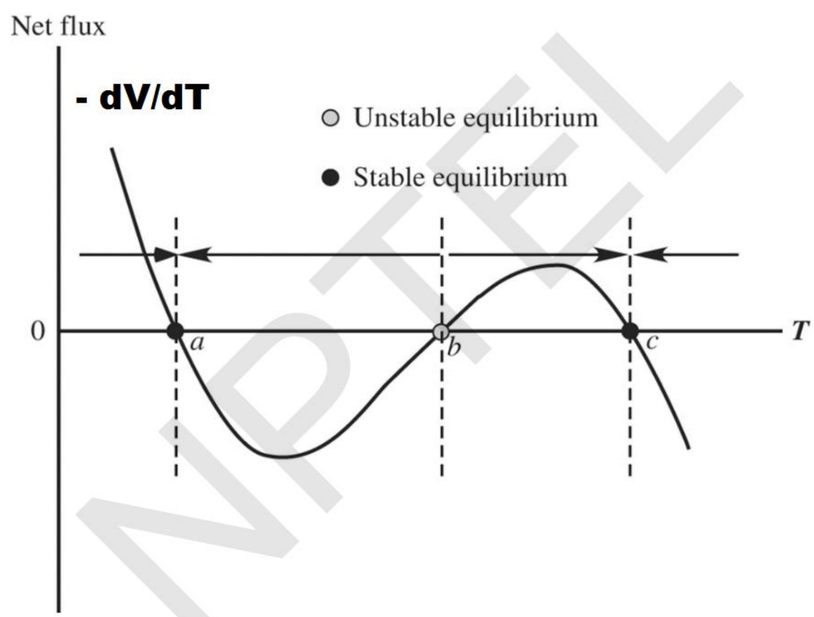
There are equations that invoke inequality (shown in the picture above). That has to be addressed. So, I go back to my original equation of rate of change in temperature. Q is incoming radiation, Q(1- albedo), which is the absorbed radiation. So, this equation we have used many times before. So, we can look at it. People in physics like to reformulate the equation in terms of a potential called the Lyapunov potential. You define the left-hand side, dT/dt, as a change in volume of ice with temperature. This velocity potential is defined this way through a rate of change in temperature with time.



The velocity potential is the integral of this quantity, and this can be seen as follows. We saw this earlier also when we discussed the simple model, that between A and B, the model is very unstable. There is no solution.



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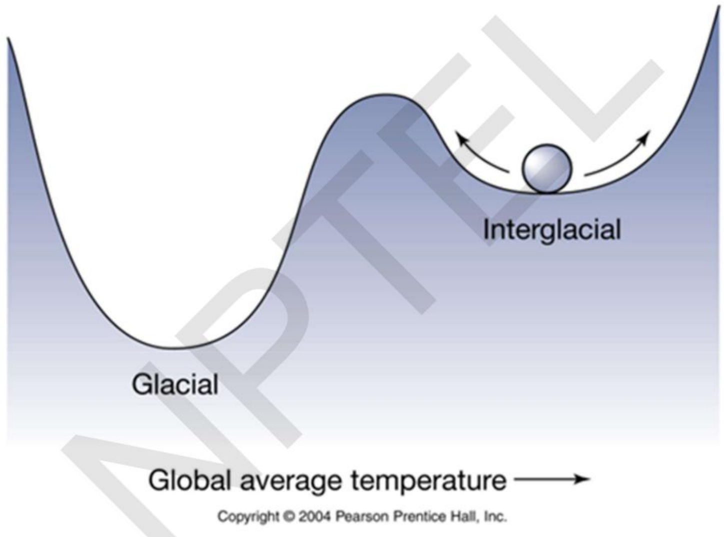
Well, between B and C, and below A, the model is able to argue that these two states are more stable than this state. So, the argument is that the Earth came out of the last glacial age due to changes in the Sun's geometry and some trigger posed by either volcanic forcing or factors which control the bedrock.



So, the idea is that the Earth will go from glacial to non-glacial provided there is enough solar radiation and there is a trigger to push it to that other side. This trigger is very important because we will see later in the course that one of the major debates in the Paris Agreement was about the 2-degree limit. When the scientists proposed that we should not let the global mean temperature go beyond 2 degrees, the immediate reaction was: why 2 degrees? The answer was that that looks like the threshold beyond which abrupt changes can occur. So, this is an example of abrupt change, which we will discuss in some detail later.

So, we need to understand what factors trigger the Earth to go from a glacial state to an interglacial state through some non-linear phenomenon. We will take this up in the next lecture.

Thank you.