Lecture – 20
Two-layer model

In this lecture, we'll continue our understanding of the stratosphere and the troposphere. We start with this simple model, which we discussed earlier in this course, where the outgoing longwave radiation has two parts. One part is the surface emission that goes through the troposphere, and the other is the emission by the atmosphere.

Greenhouse effect depends on lapse rate

Note that in a simple model with a one-layer atmosphere, the outgoing longwave radiation can be written as

$$OLR = (1 - \varepsilon)\, \sigma\, T_s^{\,4} + \varepsilon\, \sigma\, T_a^{\,4}$$

When $T_s = T_a$, $OLR = \sigma\, T_s^{\,4}$ There is no greenhouse effect!

If the atmosphere is isothermal and its temperature is equal to the surface temperature, there will be no greenhouse effect

If T_a is greater T_s the greenhouse effect will be negative !

This is what happens in the Stratosphere. As CO_2 increases troposphere warms but Stratosphere cools

Now, if the temperature of the atmosphere is equal to that of the surface, you will find that the OLR (Outgoing Longwave Radiation) does not change. Thus, the greenhouse effect—which is the difference between the surface emission and what is going out as OLR—will not change if the atmosphere is the same temperature as the surface. But we know that the troposphere is cooler than the surface, and so the greenhouse effect will be present.

The key thing to understand is that because the temperature of the troposphere decreases with height, the greenhouse effect is positive. In the stratosphere, however, the temperature increases with height and the greenhouse effect is negative. But right now, the positive greenhouse effect in the troposphere is dominating. So, we have to worry about it.

You must understand the difference between the troposphere and the stratosphere. They behave differently because the lapse rate is different. The way the temperature changes in the troposphere and stratosphere has a big impact. The emission from the top of the stratosphere is higher than at the bottom because the temperature increases with height. On the other hand, the emission from the top of the troposphere is lower than at the surface because the temperature decreases with height.
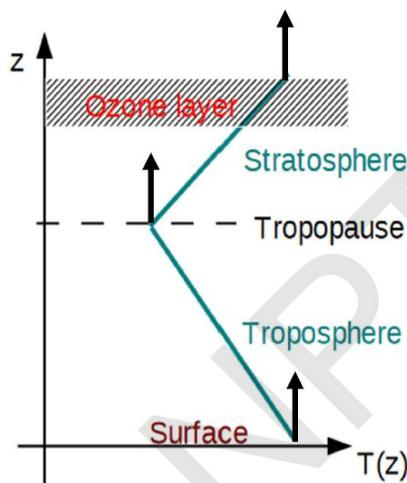
The emission from the top of the Stratosphere will be **higher** than the emission from the bottom of the Stratosphere because temperature **increases** with height in the Stratosphere

The emission from the top of the Troposphere will be **lower** than the emission from the bottom of the Troposphere because temperature **decreases** with height in the Troposphere

Example: Assume that the Ozone in the Stratosphere absorbs 10 W/m² from the incoming solar radiation and the emissivity of the Stratosphere is ε. Let The upwelling radiation at Tropopause be 240 W/m². Then the energy balance equation gives
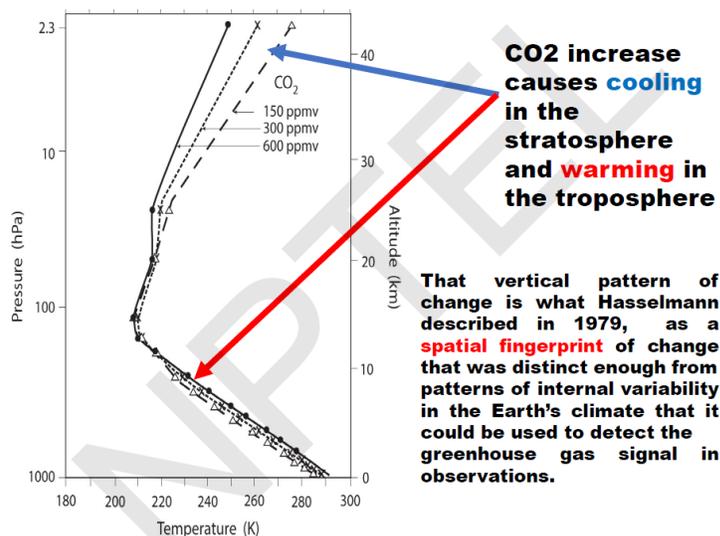
$$10 + 240\, \varepsilon = 2\, \varepsilon\, \sigma\, T^4_{strat}$$

$$\sigma\, T^4_{strat} = 120 + 5/\varepsilon$$

As CO_2 increases, ε will increase and hence the temperature of the Stratosphere will **decrease**

Let us do a simple energy balance in the stratosphere. Assume that the stratosphere gains about 10 W/m² because ozone absorbs ultraviolet radiation. Ozone also absorbs the emission from the troposphere, which is about 240 W/m², and it emits in both directions. So, we rewrite the equation such that the temperature of the stratosphere is expressed as $(120 + 5/\varepsilon)$. What this equation shows is that as epsilon (ε) increases due to an increase in CO_2 (i.e., as emissivity increases), the stratosphere will cool. This is the phenomenon that sometimes puzzles people, and you must understand that this is a consequence of the fact that in the stratosphere the temperature is increasing with height because of the absorption of solar radiation in the ultraviolet by the ozone layer. I will talk about this further because it is very important.

Now, this was shown more than 60 years ago by Nobel laureate Manabe when he ran a climate model and showed that as CO_2 increases from 150 to 600 parts per million, the temperature in the stratosphere decreased and, in the troposphere, increased. Simultaneously, CO_2 is causing a warming of the troposphere and a cooling of the stratosphere. So, this is a special effect of CO_2 increase, and the other Nobel laureate Hasselmann called it a fingerprint.

CO2 increase causes cooling in the stratosphere and warming in the troposphere

That vertical pattern of change is what Hasselmann described in 1979, as a **spatial fingerprint** of change that was distinct enough from patterns of internal variability in the Earth's climate that it could be used to detect the greenhouse gas signal in observations.

It is a spatial fingerprint because the effect in the troposphere and stratosphere is exactly opposite. This will not be the case if the incoming solar radiation increases for whatever reason; it will warm both the stratosphere and the troposphere. With CO_2, it does not happen. So, we can say with some confidence that the present cooling of the stratosphere and warming of the troposphere shows that carbon dioxide increase is playing some role. If this were due to an increase in solar radiation, then an increase would occur both in the troposphere and the stratosphere.

Because it does not happen, we know that CO_2 is playing a role in changing Earth's climate. So, this fingerprint is very critical to identify the effect of any gas on Earth's climate. Greenhouse gases affect Earth's climate in one way, and other gases affect it in another way. This difference is visible in the observations of the troposphere and stratosphere. So, this fact has to be kept in mind.

This is also seen in today's Arctic and Antarctic. The Arctic has a very unusual feature. Today in the Arctic, the temperature does not decrease with height in the troposphere. In the first few kilometers in the Arctic and Antarctic, the temperature increases. This is called an inversion (temperature inversion).
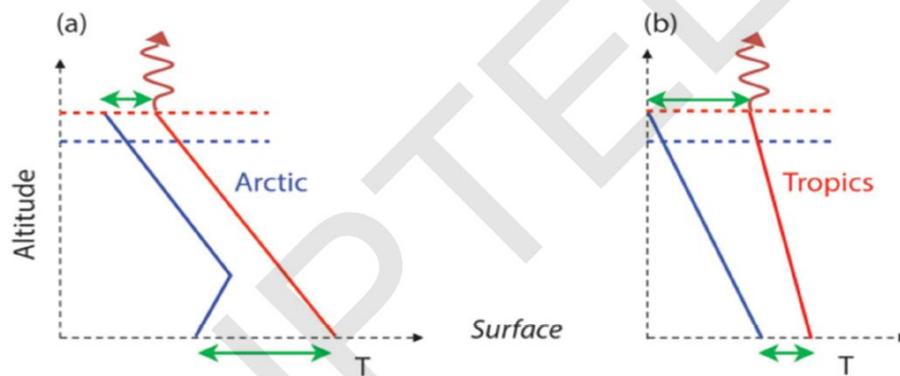

Figure 3.5: Schematics of the Arctic (a) and tropical (b) lapse rate feedbacks.
Solid blue (red) lines show the temperature profiles before (after) warming. Dashed blue (red) lines show the emission level before (after) the warming (see section 2.1.3). The green double arrows show the warmings at the surface and at the emission level.

Now, why does this happen? Because both the Antarctic and the Arctic have almost six months of no sunlight. For those six months, there is continuous cooling of the surface of the Arctic and Antarctic during the winter due to cooling by emission to space. So, even after summer arrives, that inversion still is there; it does not go away. When the CO_2 goes up in the Arctic, in the region near the surface, there is a negative greenhouse effect, which you will not see in the tropics. In the tropics, as global warming occurs, the temperature increases at the surface, but it decreases more at upper levels, while in the Arctic it is very different because of the inversion.

So, we already see that any change in the temperature gradient in the troposphere and stratosphere has an impact on the greenhouse effect. In the Arctic region, there is a negative greenhouse effect because there is an inversion, which is not present in the tropics, for example. So, please keep this in mind. Make sure to understand the greenhouse effect correctly: it need not always increase temperature. It increases temperature in the troposphere because the temperature decreases with

height, but in the stratosphere it increases. In the Arctic, in a small region near the surface where the temperature increases, again there is a negative greenhouse effect.

Let us go back and look at the radiation budget of the Earth once more to understand some of the results. The key point about the Earth's radiation budget is that of the 341 W/m^2 of the Sun's radiation that comes to the Earth's surface, about 160 W/m^2 are absorbed by the surface. About 23 W/m^2 are reflected by the land and ocean. About 79 W/m^2 are reflected by the clouds and the atmosphere.

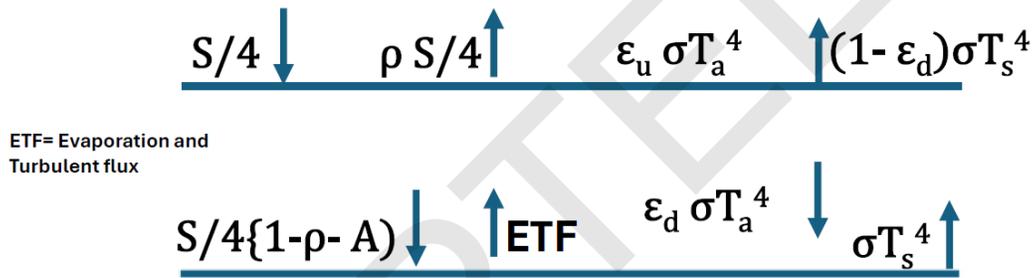Trenberth and Fasullo (2012) Surv. Geophys.
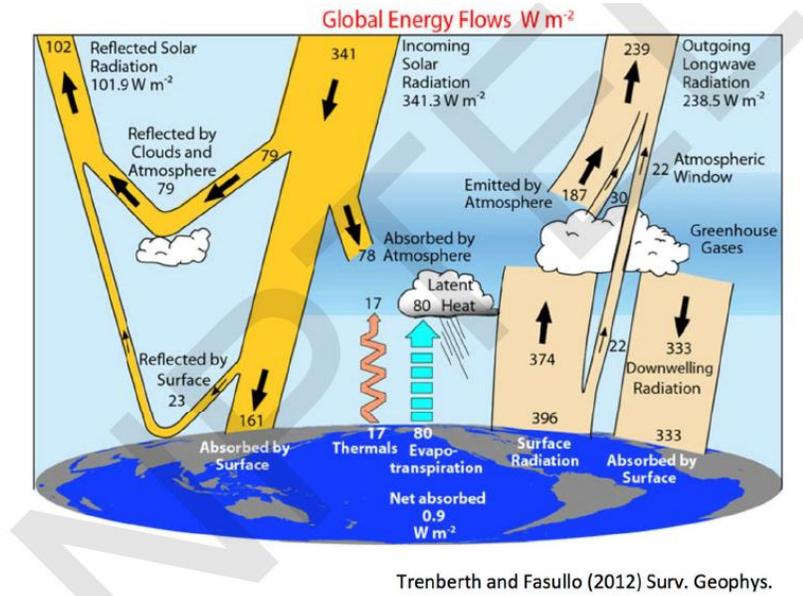
So, about one-third, or about 30 percent of the incoming radiation, is reflected back. This is an albedo of around 0.3. Now, let us go to the thermal infrared, where the Earth emits radiation of about 396 W/m^2. Notice that about 333 W/m^2 s are returned back to the Earth by the greenhouse gases.

Thus, greenhouse gases are returning most of the radiation from the Earth back to itself. And, of course, these gases emit radiation to space—that's around 239 W/m^2. But notice that out of the 396 watts per square meter emitted by the Earth's surface, only 22 W/m^2 escape in the 10-to-12-micron region, the so-called window. So, 95% of the emission from the Earth's surface is returned back to the surface.

Only a small portion reaches space. This is the role played by greenhouse gases like CO_2 and water vapor. Now, if you recall the simple energy budget model that we talked about earlier to understand the global mean temperature, in that model, we assume that the radiation emitted by the troposphere downwards and upwards were equal. It was assumed that the Earth's atmosphere, which is mainly the troposphere, emitted equally in both directions.

Liou's model with Evaporation and turbulent flux

$$S/4 \downarrow \qquad \rho\, S/4 \uparrow \qquad \varepsilon_u\, \sigma T_a^4 \qquad \uparrow (1-\varepsilon_d)\sigma T_s^4$$

ETF= Evaporation and Turbulent flux

$$S/4\{1-\rho-A) \downarrow \qquad \uparrow \text{ETF} \qquad \varepsilon_d\, \sigma T_a^4 \downarrow \qquad \sigma T_s^4 \uparrow$$

$$S/4\{1-\rho\} = \varepsilon_u\, \sigma T_a^4 + (1-\varepsilon_d)\, \sigma T_s^4$$

$$S/4\{1-\rho-A\} + \varepsilon_d\, \sigma T_a^4 = \text{ETF} + \varepsilon_u\, \sigma T_a^4 + \sigma T_s^4$$

Now, we want to modify that in the light of the radiation budget we just saw: we want to make sure that the emission downward is more than the emission upward because of the decreasing temperature with height. In the original model, we ignored this for simplicity. Now, let us include it and see the effect. Based on observation, we want to make epsilon downward (ε_d) larger than epsilon upward (ε_u).

We will assume epsilon downward is 0.95 and epsilon upward is 0.54, which reflects the fact that the troposphere is not isothermal. We also want to include the evaporation turbulent fluxes, which we ignored in the initial simple model. In Liou's model, this was ignored. So, we add that.

There are two changes: one is that the emission is not equal upward and downward, which was not a good assumption, and the second is the neglect of non-radiative heat transfer between the surface and the atmosphere, which is also important. It is around 100 watts per square meter, so we cannot ignore it. If you include these two factors, the equation changes.

You can see that at the top of the atmosphere, the radiation absorbed from the Sun is equal to what is emitted upward by the atmosphere and what is transmitted from the surface. At the surface, the radiation absorbed from the Sun after reflection and absorption by the atmosphere, plus the downward radiation from the atmosphere (which is absorbed by the Earth as a black body), and the radiation transferred from the Earth to the atmosphere by evaporation turbulent fluxes (ETF)—this is the energy balance for the atmosphere, which absorbs the radiation emitted by the surface and also the radiation emitted by the atmosphere upward.

Now, if you take the ratio of these two equations, let us look at the equations here again.

$$\sigma T_s{}^4 = B/C$$

where

$$B = [\,\{1 + (\varepsilon_d/\,\varepsilon_u)\}\{S/4\{1-\rho\} \ -A(S/4) - ETF]$$

$$C = [1 + (\varepsilon_d/\,\varepsilon_u)(1-\varepsilon_d)\,]$$

With $\varepsilon_d = 0.9$ and $\varepsilon_u = 0.54$ $\rho = 0.3$ $S = 1360$, $A = 0.2$ and $ETF = 110$
We get $T_s = 288.25$ K

Note that when $\varepsilon_d = \varepsilon_u = \varepsilon$ and $ETF = 0$ we recover the results of Liou's simple model
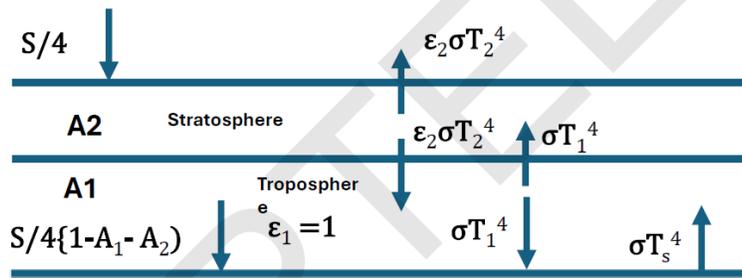
$$\sigma T_s{}^4 = S/4\{2(1-\rho) \ -A\}/\{2-\varepsilon\}$$

The ratio of these two equations is shown here, and we get the ratio of B to C as the surface temperature. I am assuming the upward emissivity is 0.9, the downward is 0.54, the albedo is 0.3, the incoming radiation is 1360, the solar absorptivity is 0.2, and the evaporation turbulent fluxes are 110. If you do all this, you will get an answer close to 288. What I have done is slightly modify Liou's model to include the fact that the radiation upward and downward by the troposphere is not equal, and I have included the non-radiative process while still being able to get 288. So, this is a model that is superior to the model obtained from Liou's textbook.

So, what I am trying to point out is that the simple model was useful but not totally correct. We can make small changes and still get almost the same answer. Now, let us further modify the model to include the stratosphere. The stratosphere is important because it contains the ozone layer. In this model as well, we treated the entire troposphere and stratosphere as a single layer, which is not correct because the temperature variation is not the same.

Now, let us look at a two-layer model (Please refer to the image shown below). We treat the troposphere at one temperature, T_1, and the stratosphere at another temperature, T_2. We do an energy balance at the stratosphere, at the troposphere, and at the top of the atmosphere. All these are shown below. The surface is here, the troposphere is here, and the stratosphere is here.

Two layer atmosphere model

$S/4$ $\varepsilon_2 \sigma T_2^4$

A2   Stratosphere $\varepsilon_2 \sigma T_2^4$ σT_1^4

A1   Troposphere

$S/4\{1-A_1-A_2)$ $\varepsilon_1 = 1$ σT_1^4 σT_1^4 σT_s^4

Energy balance at the surface $S/4\{1 - A_1 - A_2\} + \sigma T_1^4 = \sigma T_s^4$

Energy balance in troposphere $S/4\, A_1 + \sigma T_s^4 + \varepsilon_2 \sigma T_2^4 = 2 \sigma T_1^4$

Energy balance in Stratosphere $S/4\, A_2 + \varepsilon_2 \sigma T_1^4 = 2 \varepsilon_2 \sigma T_2^4$

If you add all three equations, you will get the top-of-the-atmosphere balance. Now, you will see that for simplicity, I have assumed the emissivity of the troposphere to be 1; last time it was 0.95, but I have assumed 1 to make the equations easier. However, the emissivity of the stratosphere cannot be 1 because it contains very little atmosphere, very little water vapor, some CO_2, and some ozone. So, the emissivity of the stratosphere will not be much higher than 0.1 because the mass of the stratosphere is very small. If you take all that into account and solve this equation, you get the given equation for the stratospheric temperature, which depends on the solar absorptivity of the stratosphere (due to the ozone layer) and also on the emissivity of the stratosphere. This is a general equation.

Now, we look at two limiting cases. If we neglect the solar absorption of the stratosphere, then we get a simple equation for temperature.

Solution for the temperature of the Stratosphere

$$\sigma T_2^4 = \{S/4(1-A_2) + S/4\, A_2/\varepsilon_2 \}/ \{2-\varepsilon_2\}$$

Limiting cases

1) when $A_2=0$ No absorption of solar radiation in the Stratosphere

$$\sigma T_2^4 = [S/4]/[2-\varepsilon_2]$$

As CO_2 increases, ε_2 increases. hence $(2-\varepsilon_2)$ increases and T_2 **increases.**
**The Stratosphere warms**

2) If $A_2 \neq 0$ and $\varepsilon_2 \ll 1$ (good assumption because the amount of O_3 and CO_2 is small in Stratosphere)

$$\sigma T_2^4 = \{S/4(1-A_2) + S/4\, A_2/\varepsilon_2 \}/ 2$$

As CO_2 increases, ε_2 increases T_2 decreases, **The Stratosphere cools!**

Now, if you consider increasing CO_2—that is, an increase in emissivity—you will find the temperature decreases. The key point is that if we neglect the absorption of solar radiation by the stratosphere, then the greenhouse effect in the stratosphere will be positive, as in the troposphere. So, the greenhouse effect is negative only when you include the absorption of solar radiation, as shown here in part 2. In part 2, I say A_2 is not zero, but epsilon$_2$ is very small because it is close to 0.1. Then you look at this equation. Then you see that if you now increase emissivity, this term will reduce. So, T_2 will decrease. If you make A_2 zero and that term drops out, you get a very simple equation. If A_2 is not zero, this term becomes very important.

Let us address this equation. I would argue that understanding the stratospheric warming and cooling due to increasing greenhouse gases is best done through energy balance. I want you to look at this carefully to make sure you understand what this does. So far, we have only talked about one layer or two layers, but the atmosphere is very thick and has many layers. You can extend the analysis we did with Liou's model to n layers.
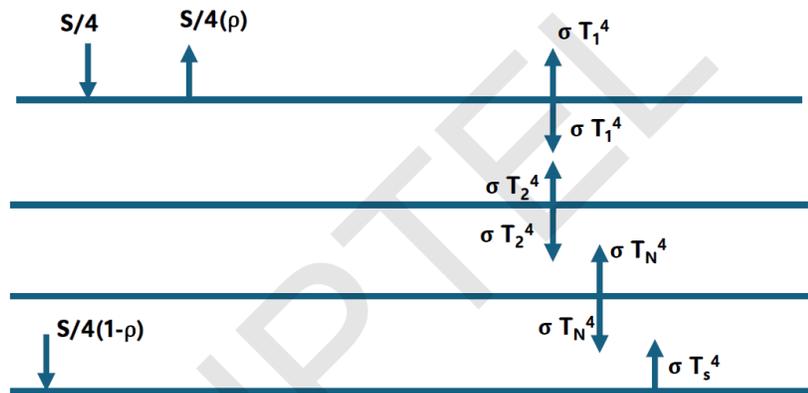
N layer model of the earth's atmosphere. All the layers are transparent to solar radiation but opaque to thermal radiation emitted by the surface

We can extend the above analysis to N atmospheric layers. To keep the analysis simple, we will assume that the N layers are **transparent** to solar radiation but totally **opaque** to the radiation emitted by the earth. Since atmosphere is opaque in the infrared, the emissivity of all layers is 1

The radiation balance at TOA
$$S/4(1-\rho) = \sigma T_1^4 \quad \text{.........(1)}$$

$\sigma T_2^4 = 2 \sigma T_1^4$ for layer 1 and therefore $\quad \sigma T_2^4 = 2\{ S/4(1-\rho) \}$

$\sigma T_3^4 + \sigma T_1^4 = 2 \sigma T_2^4$ and hence $\quad \sigma T_3^4 = 3\{ S/4(1-\rho) \}$

and so on to layer N $\quad \sigma T_N^4 = N\{ S/4(1-\rho) \}$

$S/4(1-\rho) + \sigma T_N^4 = \sigma T_S^4$ for the earth's surface

Hence $\quad \sigma T_s^4 = (N+1)\{ S/4(1-\rho) \}$

Each atmospheric layer absorbs infrared radiation emitted by layers below and above and emits radiation in both directions

These n layers will have balances at various levels. To make the analysis simple, I will assume that these n layers are transparent to solar radiation. It is not correct, but for simplicity we will do that. We will assume that the n layers absorb the Earth's radiation as a black body. So, there are two approximations: one is the neglect of the solar radiation by these layers, and the other is the total absorption of the Earth's radiation. Then this analysis becomes simpler, and I will continue this discussion in the next lecture to show how if you have a large number of layers that completely absorb the Earth's radiation, then the greenhouse effect increases rapidly. That is the point I want to make before we go on to further discuss the greenhouse effect. Thank you.