Climate Change Science
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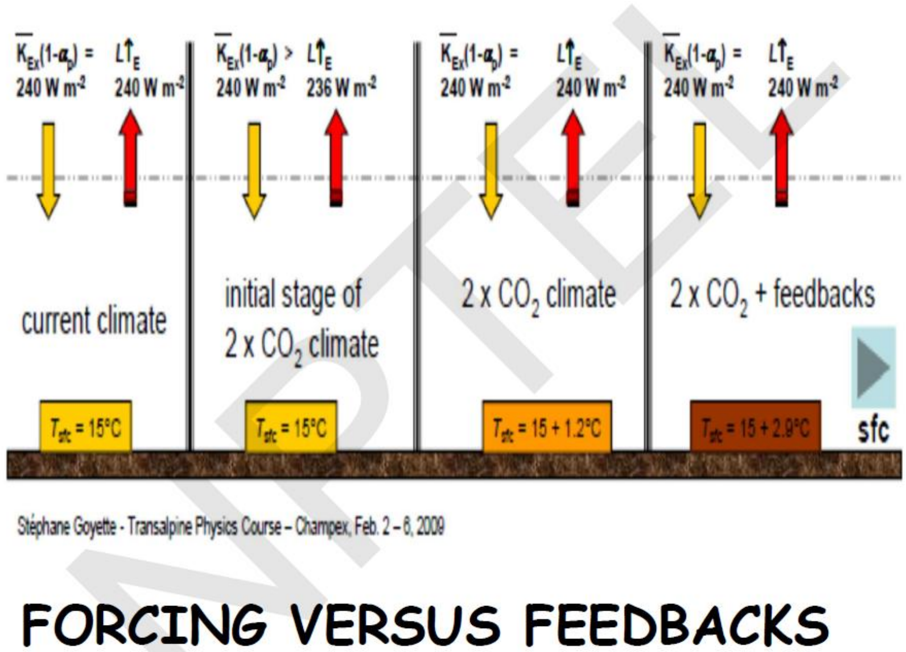
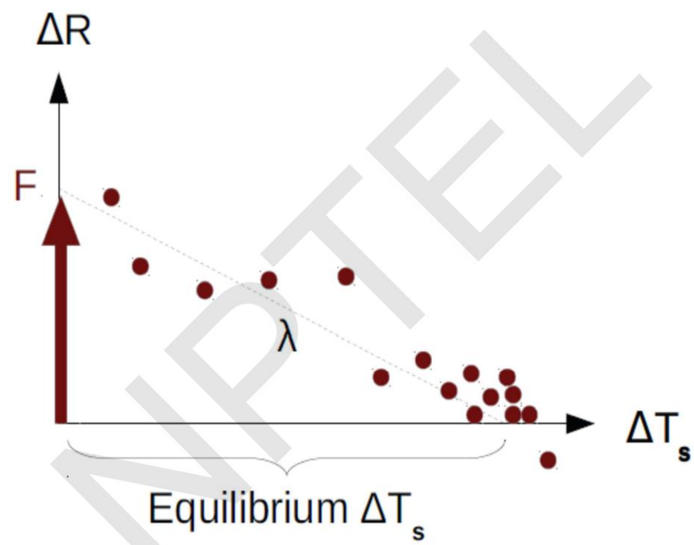
Lecture – 10
Feedback Analysis

In the last class, we discussed the role of feedback and forcing. And I showed this slide in which you can see how starting from the current climate there is an imbalance at the top of the atmosphere when you double CO_2.



That imbalance is immediately adjusted by Earth's temperature going up by 1.2 °C due to CO_2 increase alone, without any feedback. After many years, water vapour and other quantities change and the system finally comes to equilibrium again at 240 W m^{-2}. So, we can see that the forcing causes the initial change in temperature, that leads to further changes, and after many years the system reaches a new equilibrium in which both forcing and feedback contribute to temperature change.

The best way to illustrate this is to show how the change in net radiation, which was caused by the forcing, slowly reduces in time as the temperature of the Earth increases. After a couple of hundred years, the system will reach a new equilibrium where the net radiation change is zero.



It will not go down smoothly. There will be up and downs because remember energy is being exchanged between ocean and atmosphere continuously. There are imbalances occurring there. So, there will be year to year fluctuations that we saw in the last lecture. I will highlight that here.

From: Hall A, Manabe S. The Role of Water Vapor Feedback in Unperturbed Climate Variability and Global Warming. *J. Climate. 1999;12(8):2327-2346.*

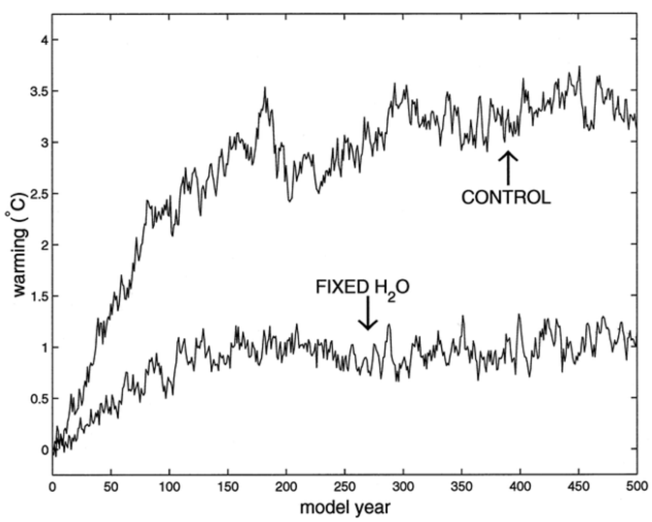


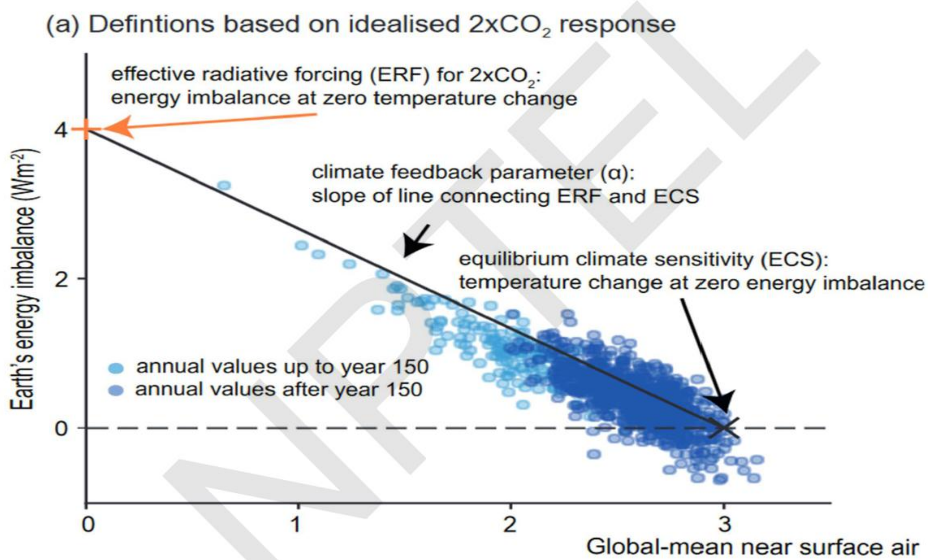
Fig. The 500-yr annual-mean time series of the global-mean surface temperature change in the integrations where CO2 is doubled to 720 ppm relative to the unperturbed variability experiments, where CO2 is fixed at 360 ppm.

You can see here, in this figure above, the year-to-year changes in global mean temperature is not climate changes, but climate variability, because in the Earth system, energy is exchanged continuously between ocean, land and atmosphere.

Due to these adjustments, in some years the temperature is higher and in some years it is lower. This is a natural fluctuation of the order of half a degree or so. When we talk about climate change, we talk about an average of 10 years or 100 years, in which case these will get completely smooth.

So, you see that temperature went on increasing in response to the increase in CO_2 and water vapour. Finally, after hundreds of years, it started fluctuating a little bit, which is natural variability. In the new equilibrium, this is zero.

Now, this is shown more clearly in a recent IPCC report, where we start with a 4 W m^{-2} imbalance due to a sudden increase in CO_2.



And after about a year, temperature changes by about 0.7 °C, then it goes on increasing. And as you go in time, we ultimately reach a state of equilibrium where there is some fluctuation of the order of 0.1–0.2 °C. But the net radiation at the top of the atmosphere has gone to 0. The perturbation has been reduced to 0.

Now, let us define some parameters which are routinely used in this field. The temperature change is written as equal to a parameter called lambda, the **climate sensitivity parameter**, times forcing which is in W m^{-2}.

$\Delta T = \lambda \, \Delta F$ **where λ is in $K/(W/m^2)$**

λ is climate sensitivity parameter

Some people define

$\Delta F/\Delta T$ as λ (in $W/(m^2 K)$ and call it a feedback parameter

Therefore, the units of lambda is Kelvin change per 1 W m^{-2}. So, it tells you for the Earth system, for 1 W m^{-2} change, how much is the change in global mean temperature. Now, this is a very important parameter because we know that when we double CO_2, we change the forcing by 4 W m^{-2}. So, we would like to know for 4 W m^{-2}, how much is the temperature change in the global mean.

Now, I want to warn you that unfortunately, some people define lambda as not ΔT by ΔF but the other way round, ΔF by ΔT. In which case, we will call it, W m^{-2} K^{-1}, feedback parameter. So, there is a sensitivity parameter and there is a **climate feedback parameter**. One is the inverse of the other. So, through units, you should be able to know which one they are talking about.

Now, to understand this approach, let us see how the system works. Initially, you give a perturbation like you double CO_2 and the temperature goes up by 1.1 °C. After some time, the feedbacks kick in and water vapour goes up, cloud changes and albedo changes. So, temperature changes further. This is called **feedback**.

$$\Delta T = \Delta T_o + \Delta T_{feedback}$$

$$\Delta T = \text{Equlibrium Response}$$

$$\Delta T_o = \text{Response without feedback}$$

$$f = \Delta T_{feedback} / \Delta T$$

$$\Delta T = \Delta T_o + f \Delta T$$

$$\Delta T = \{\Delta T_o\} / \{1 - f\}$$

There is initial perturbation and there is feedback. ΔT_0 is response without feedback. $\Delta T_{feedback}$ is the feedback. If you add these two, you get the total temperature change. So, the non-dimensional parameter f is defined as feedback by final ΔT.

Notice, it is not feedback by the initial ΔT, but feedback by final ΔT. So, using this definition, I can rewrite this as $\Delta T = \Delta T_0 + f \cdot \Delta T$. And we can take it to the left hand side and then ΔT becomes $\Delta T_0 / (1 - f)$. Now, this tells you something very interesting about the final temperature change. The final temperature change will be very very large in response to initial change if f is close to 1.

$$\Delta T_s = \Delta T_0 \frac{1}{1 - f}$$

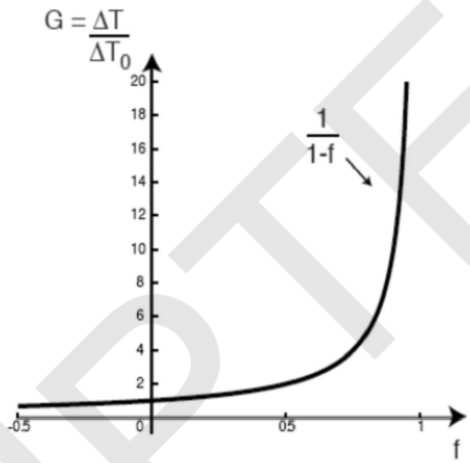
- *f > 0* ***positive feedback:*** internal response of climate system exacerbates externally forced warming
- *f* < 0 ***negative feedback:*** internal response of climate system mitigates externally forced warming
- If there is more than one feed back

$$\Delta \mathbf{T} = \Delta \mathbf{To} / \{1 - \sum \mathbf{f_i}\}$$

If f is close to 1, then $\Delta T_{feedback}$ is much larger than ΔT_0. So, this will increase a lot. So, f value which is a non-dimensional number, if it is very small, we are lucky. The feedback is not large. The initial perturbation will not lead to a large change, but if the feedback is large (0.5 or higher) it will amplify the initial temperature by a large margin. So, this is the cause of concern today as we will see in this lecture and the next lecture that a feedback parameter anywhere around 0.5 implies doubling of the initial perturbation of the Earth's climate, which is a cause for concern. Now, if f is positive, we call it positive feedback because internal changes amplify the temperature response. If f is negative, the denominator will go up and so ΔTs will not increase that much.

Now, we can talk about many feedbacks that are one feedback like albedo and water vapour and clouds. So, we can extend this derivation to $\Delta T = \Delta T_0 / (1 - \Sigma f_i)$. So, this is f as sum of all feedbacks. You can look at each feedback separately and see how much it contributes and take the sum. Because we need to understand the various feedbacks like clouds, water vapour, albedo and know the contribution of each and whether they are positive or negative and how they add.

Feedbacks: gain curve



Range of possibilities:

- $-\infty < f < 0$: $G < 1$ ⇒ response damped ⇒ NEGATIVE fdbk.
- $0 < f < 1$: $G > 1$ ⇒ response amplified ⇒ POSITIVE fdbk.
- $f > 1$: G undef. ⇒ Planet explodes…

Now let me plot that result I showed you in the last slide as the total temperature change from the initial perturbation called the gain. This is in the language of electrical engineering: gain of an amplifier. So, the gain shows you how much amplification occurs and that is 1 / (1 – f), and you can see that it is a very non-linear function. If f is small, it is linear. As f approaches 1, it becomes very large.

If G is less than 1, that means f is negative. The response is damped. It is called negative feedback. If G is greater than 1, response is amplified, called positive feedback. And if f > 1 is, of course, it is unstable. It is called an explosive feedback or runaway. The system is completely unstable.

Now, the concern we have today is our knowledge about the mean value of f is not very accurate. Presently, it is believed that the mean value of f is somewhere between 0.3 to 0.65. It is around 0.65 with an uncertainty of 0.3. That means it is anywhere between 0.35 to 0.95. So, because of this, we can predict that there is a 29 percent chance that increase in temperature may be between 2 °C to 4.5 °C; that is a 14 percent chance it is between 4.5 °C and 8 °C; and that is a 13 percent chance it is greater than 8 °C. So, this is very bad news because Earth will be not liveable at this temperature change.

Climate sensitivity: can we do better?

ΔT / $\bar{f}, \sigma_f$	2 to 4.5 °C	4.5 to 8 °C	> 8 °C
0.65, 0.3	29%	14%	13%
0.65, 0.2	43%	18%	12%
0.65, 0.1	55%	20%	8%
0.65, 0.05	95%	5%	0%

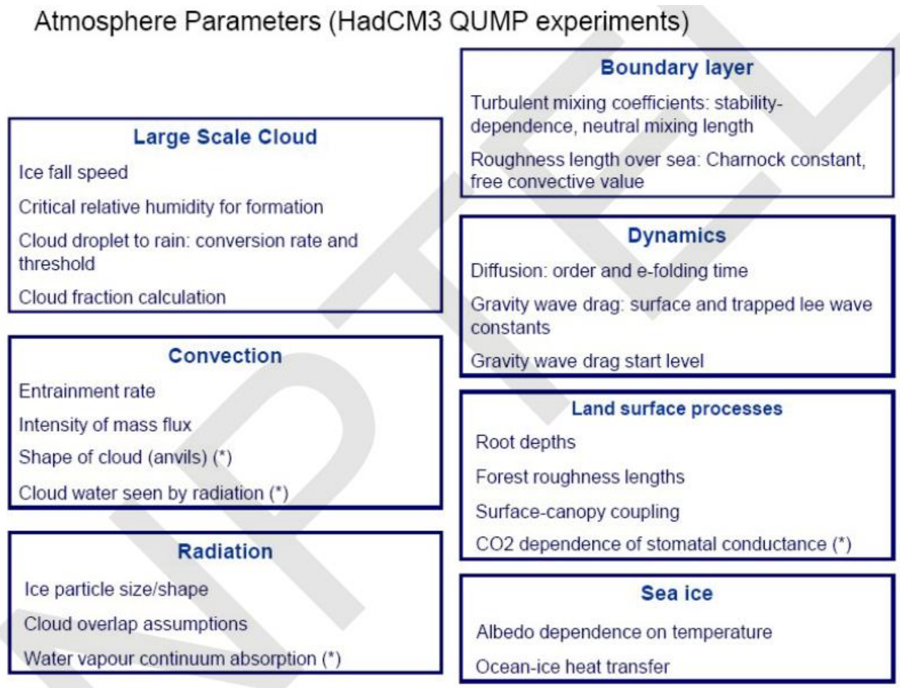
science is here

need to get here!

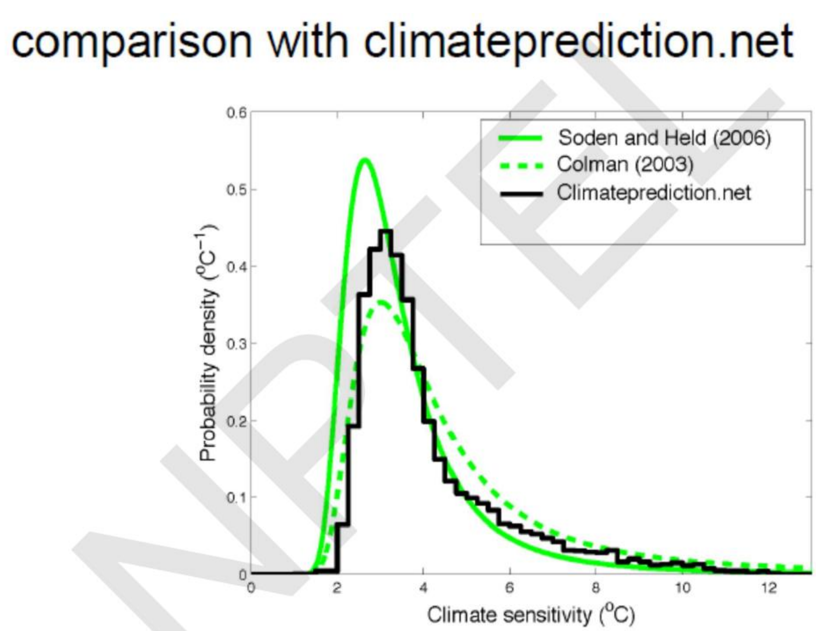
On the other hand, for the same mean value, if the uncertainty is smaller, then we are in the range which IPCC normally quotes, which is lower and safer. For example, if σ_f is around 0.1, then there is a 55 percent chance that we are in this range: 2 to 4.5°C. And if σ_f is as low as 0.05, we can say with 95 percent confidence that we will be in the same range: 2 to 4.5°C.

Our climate models are not good enough to bring down the value of σ_f to a sufficiently low value so that we can say with great confidence that we are in this range of 2 °C to 4.5 °C. This is an ongoing area of research; a lot of work is going on, but we are not yet able to bring down the uncertainty in f to around 0.05, so that we can say with 95 percent confidence that the temperature change will not be beyond this range of 2 °C to 4.5 °C. Now why is there such a large range of f in the climate models? Because today's climate models, although they are very sophisticated and advanced, they have a lot of parameters which are not accurately known. For example, the fall speed of ice is not known, so we specify a value. The critical relative humidity when clouds form, we do not know accurately. At what rate does a cloud droplet become rain, we do not know accurately. It depends a lot in different parts of the world and depends upon the kind of cloud and rain.

And the calculated cloud fraction has some inaccuracies. Now the other problem is as air parcels rise in the atmosphere, some amount of air is entrained, it gets into this parcel and it affects the rate at which clouds form. And this is not known accurately. And in the radiation calculation, we do not know accurately the ice particle size and shape. We do not know about the cloud overlap that occurs in the real atmosphere.

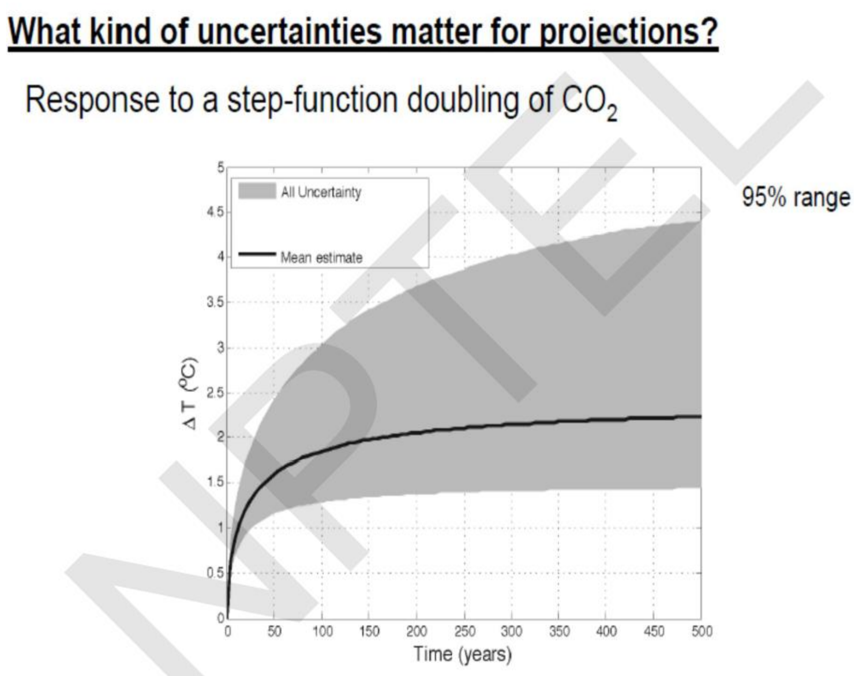


There is some uncertainty about water vapour absorption bands. I will talk about this later. There is uncertainty about turbulent mixing in the atmosphere. There is uncertainty about waves in the atmosphere due to mountains called gravity wave drag. And then there are uncertainties about roughness in the forest and roughness in the canopy and so on. And how much CO_2 is coming out of plants and albedo of ice and heat transfer. So, a lot of parameters are unknown which are given some numbers arbitrarily.

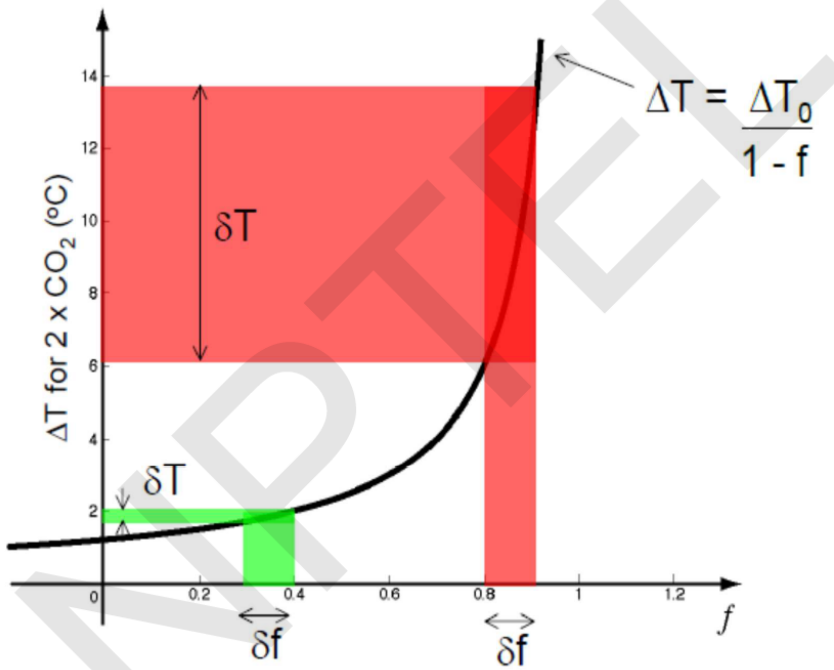


So, one of the scientists decided to see how the climate model response changes when you change these unknown parameters by a small amount. So, this shows how the **climate sensitivity**—that is, the impact of doubling CO_2—changes in response to changes in some of these parameters. The black line (in the diagram above) is the prediction from a project called climateprediction.net, where a scientist asked hundreds of people to run a climate model with slightly different parameters (small changes in f, the feedback parameter). And he got thousands of results and plotted them as a distribution curve. So, what this shows is that there is a 10 percent chance that temperature change will be as high as 6 °C, and a 1 percent chance that it may be as high as 9 °C to 10 °C, although the most probable chance is around 3 °C.

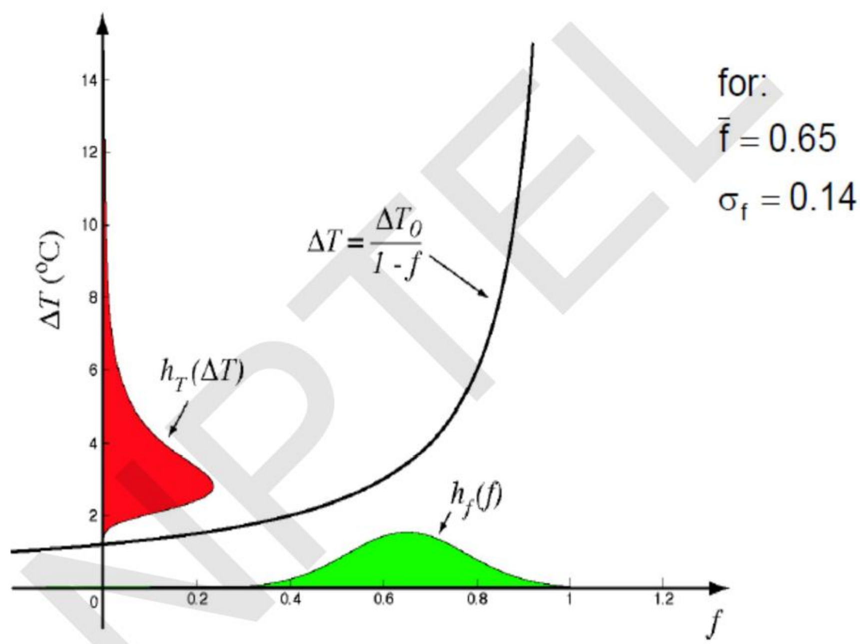
So, we should expect that doubling of CO_2 will cause about 3 °C warming, but there is a small chance that it may be 6 °C or 9 °C. This is the main issue under discussion in IPCC and the global negotiation, because it is a small chance that warming as high as 8 °C is possible. It is very bad news, because Earth will be almost not liveable for humans. But if you are a person who does not believe in extreme situations, although it is only around 50 percent, you may say, "Well, I will assume this will happen." But there is a small probability of very large changes, which we are not presently able to rule out. We cannot rule this out.



This is also shown in another way: as you run the model for hundreds of years, the probability of very high change goes on increasing. The mean may say 2 °C, but there is a small probability of 4 °C also because of the uncertainty in the feedback parameter. Now, this can be shown more clearly. You plot the $\Delta T_0 / (1 - f)$ curve and ask how a small change in f causes a change in ΔT.



If f is small, then a small change in f will not cause a large ΔT. But as you go closer to large values of f, a small change in f will cause a large change in ΔT. So, we would like to be somewhere between δf = 0.3 - 0.4. But as far as we know we are somewhere around 0.6. And there is also a fear or a possibility that we are already beyond δf = 0.8. So, this is where there is so much debate and uncertainty about how climate will change.

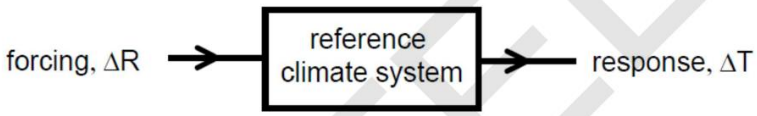


And this can be shown in terms of the probability of error in the estimate of f to the probability of error in the predicted temperature change. You can see that this is highly asymmetric.

The green curve (in the graph above) is symmetric. I am assuming that 0.65 is my mean value and it changes by plus or minus 0.14. But the response (the red curve) is one-sided: the chances of high temperature change are always present, but the chances of low temperature change are very small. This is of great concern. The asymmetry in the temperature response due to uncertainty in the value of f is what is causing so much lack of agreement among countries.

We come back to the **feedback analysis**. We say that for doubling of CO_2, without any feedback except the Planck feedback—that is, Stefan–Boltzmann's law—the warming is around 1.1 °C to 1.2 °C depending on the model.

Feedback analysis: basics



Climate sensitivity *parameter* defined by: $\Delta T_0 = \lambda_0 \Delta R$

Reference climate system:

- Blackbody (i.e., no atmosphere).
- Terrestrial flux $= \sigma T^4$ (Stefan-Boltzmann)
- $\lambda_0 = (4\sigma T^3)^{-1} = 0.26\ K\ (Wm^{-2})^{-1}$

$\Rightarrow \Delta T_0 = 1.2\ ^oC$ for a doubling of CO_2

If you add all the other terms which depend on temperature like albedo, water vapour, clouds, then it adds to the Planck term. To calculate Planck feedback, you differentiate the outgoing longwave radiation with respect to temperature, giving $4f\sigma\ T_s^3$—and for present temperature of 288 K, assuming $f \approx 0.61$, you get around $-3.3\ W\ m^{-2}\ K^{-1}$ as the feedback parameter.

Planck Term

$$\Delta R = \frac{\partial R}{\partial T}\Delta T + \sum \frac{\partial R}{\partial \alpha_i}\frac{\partial \alpha_i}{\partial T}$$

ΔR = **perturbation in net radiation at TOA**

α_i = **Variables that depend on T**
(e.g., albedo, water vapor, clouds)

Net rad $= S/4\ (1 - \rho) - f\ \sigma\ T_s^4$

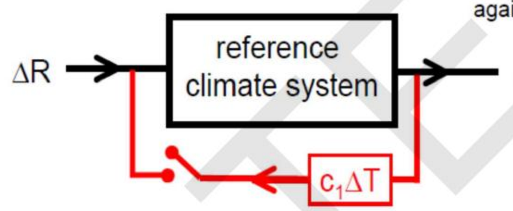
∂ net rad/∂ T $= -4\ f\ \sigma T_s^3 = -3.3\ W/m^2K$

Now, we look at an alternative way to look at feedbacks. Here, ΔR is the perturbation to the net radiation at the top of the atmosphere. The Earth's response to that is a change in global mean temperature on the right. And this change in global mean temperature causes a change in the net radiation. This is the feedback. This C_1 tells you how change in ΔT causes a further change in ΔR.

Feedback analysis: basics

- def^n: input is a function of the output

(n.b. Feedbacks are only meaningful when defined against a reference state.)



So now $$\Delta T = \lambda_0(\Delta R + c_1\Delta T)$$

Additional rad^n forcing due to system response to ΔR

Rearrange for ΔT

$$\Rightarrow \quad \Delta T = \frac{\lambda_0 \Delta R}{1 - c_1\lambda_0}$$

So, now the new ΔT becomes the old ΔT_0 plus an additional term due to change in ΔT. So, we look at this further: additional radiative forcing due to the changes in the system gives a new value of ΔT. And if you rearrange this equation, you see that ΔT is linearly proportional to ΔT_0 by $(1 - C_1)$ times λ_0. So, this quantity is, in the early derivation, what we call the **feedback factor**. This is a non-dimensional quantity.

Feedback factor: $f = c_1\lambda_0$ (f $\propto$ to fraction of output *fed back* into input)

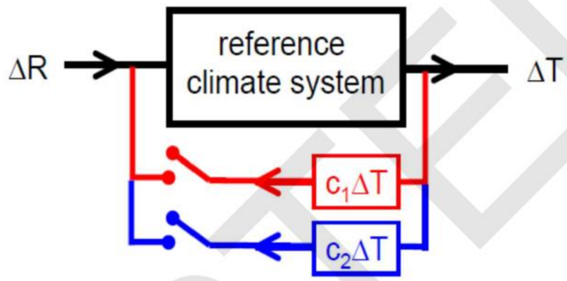
Gain $= \dfrac{\text{response with feedback}}{\text{response without feedback}} = \dfrac{\Delta T}{\Delta T_0}$ (Gain is proportion by which system has *gained*)

From before $$\Delta T = \frac{\lambda_0 \Delta R}{1 - c_1\lambda_0} = \frac{\Delta T_0}{1 - f}$$

And since $\Delta T = G\Delta T_0$: $$G = \frac{1}{(1 - f)}$$

C_1 has dimensions and λ_0 has dimensions, but $C_1 \lambda_0$ is dimensionless. And so, if you go back and define gain as before, $\Delta T / \Delta T_0$, you get the same result, $\Delta T = \Delta T_0 / (1 - f)$. So, this is an alternative derivation which some of you may find easier to understand. Now, once you understand this, you can extend it to multiple feedbacks.

Feedback analysis: more than one feedback



Now have $\Delta T = \lambda_0(\Delta R + c_1\Delta T + c_2\Delta T)$ (two nudges)

Gives: $$\Delta T = \frac{\lambda_0}{1 - c_1\lambda_0 - c_2\lambda_0}\Delta R$$

We add one more term here—one more feedback, say water vapour or lapse rate change—and repeat the same analysis. You can proceed to any number of feedback parameters. But ultimately, we need to derive these feedback factors from actual complex climate models. What is done there is, the model is run many times keeping one of these things constant.

Climate feedbacks: calculating from models

Want to consider effect of variations in:
a) water vapor; b) clouds; c) sea-ice; d) snow cover; etc..

For i^{th} climate variable: $$c_i\Delta T = \delta R\Big)_{j,\, j\neq i} = \frac{\partial R}{\partial \alpha_i}\Big)_{j,\, j\neq i} \frac{d\alpha_i}{dT}\Delta T$$

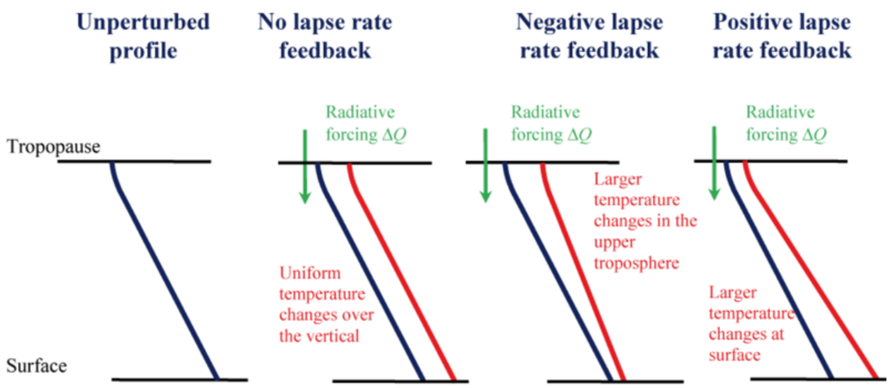
So feedback factors: $$f_i \approx \lambda_0 \frac{\Delta R}{\Delta \alpha_i}\Big)_{j,\, j\neq i} \cdot \frac{\Delta \alpha_i}{\Delta T}$$

α_i - can be a lumped property (like clouds, sea ice, etc.),
- or individual model parameter (like entrainment coefficient)
- can also calculate spatial variations in f_i if desired.

I gave one example from the work of Manabe, where he kept water vapour constant after doubling CO_2 and showed that the response was different. Similarly, you can keep clouds invariant or sea ice invariant or snow cover invariant and run the model again to see what changes. Through that you get partial derivatives: how net radiation changes when one parameter changes, and thereby derive the feedback coefficients. Now, lapse rate is important feedback in the Earth's climate system because temperature decreases with height.

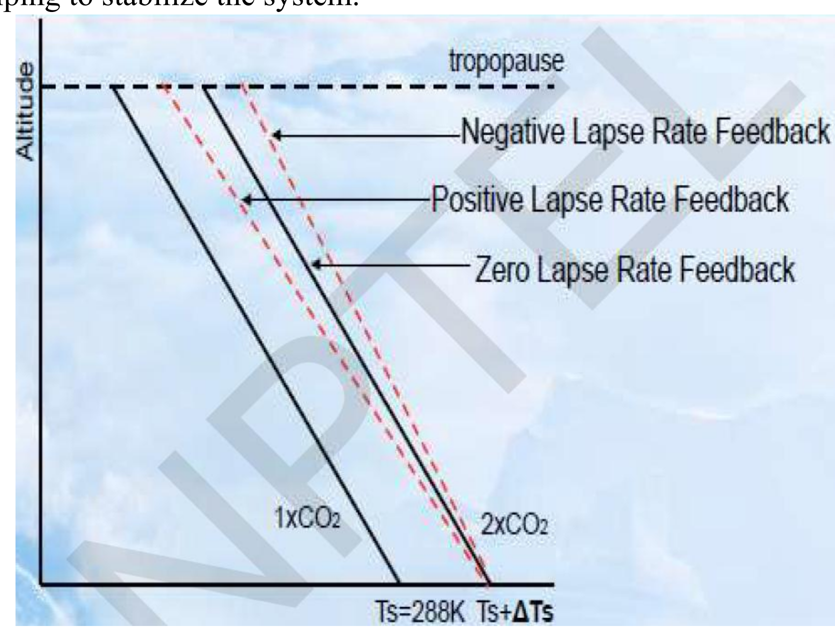
LAPSE RATE FEEDBACK

Changes of the lapse rate (T-gradient in the troposphere) also produces a feedback on radiative forcing. Overall estimated to be a negative feedback

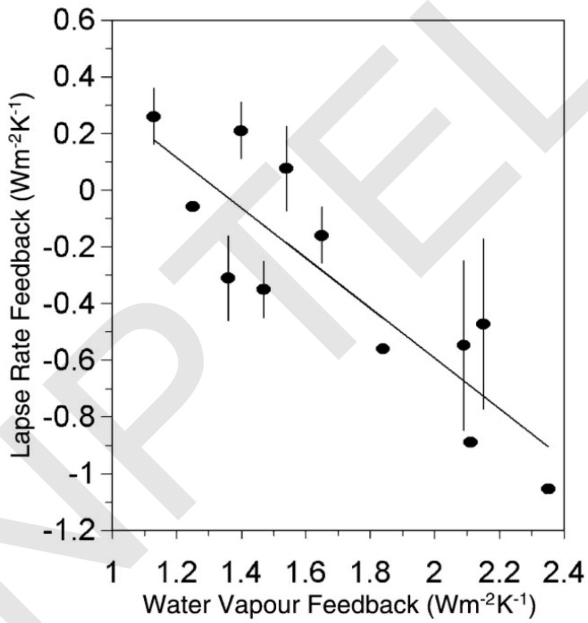


Goosse et al., 2010. Online textbook. http://www.climate.be/textbook/

The lapse rate in the tropics is about 6.5 K km^{-1} today. But as global warming proceeds, that lapse rate will change. When it does, the upper troposphere warms more than the surface. If that happens, gases in the upper troposphere emit radiation to space more effectively than before. So, that leads to negative feedback: changes in lapse rate due to warming enable the atmosphere to lose more heat to space, helping to stabilize the system.

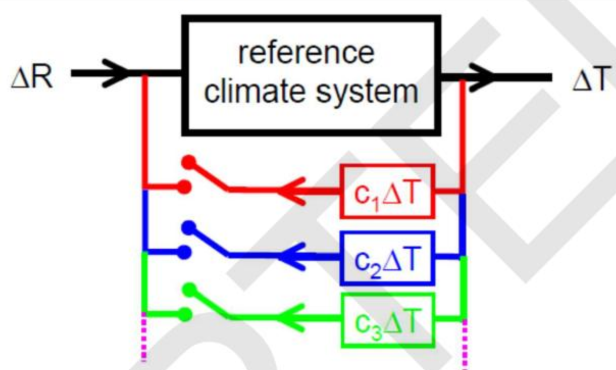


So, what this shows is that the original lapse rate changes after CO_2 doubling. Due to the change in lapse rate, the upper atmosphere can emit more radiation to space. Now, there is a connection between lapse rate change and water vapour change.



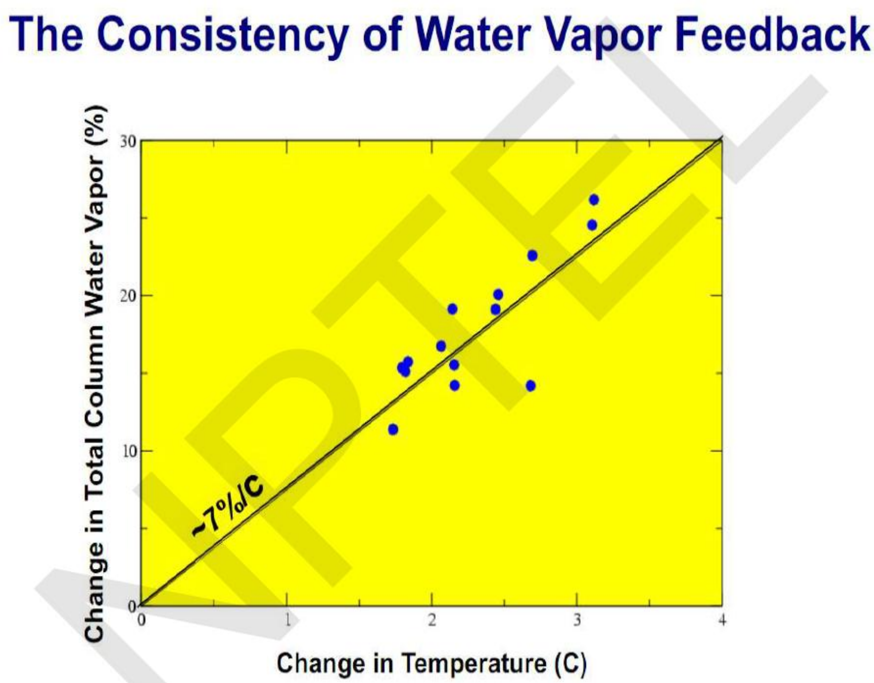
Water vapour feedback is one of the most important in Earth's climate system because warmer air holds more moisture by thermodynamics. If the upper troposphere warms, water vapour increases. Models show a correlation between lapse rate change and water vapour change, so these feedbacks are linked. Here is an example where you combine them and apply our formula.

Feedback analysis: more than one feedback

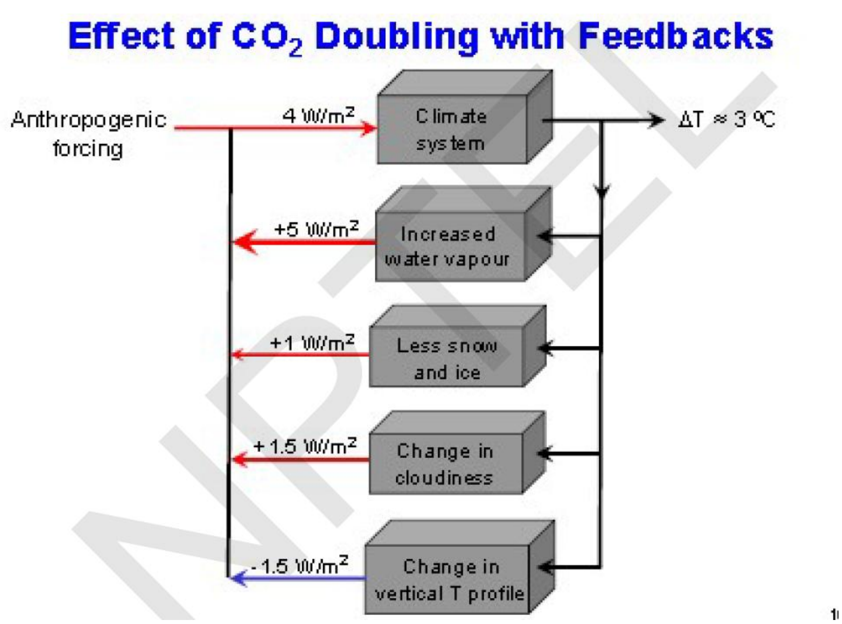


And so in general for N feedbacks:

$$G = \frac{\Delta T}{\Delta T_0} = \frac{1}{1 - \sum_{i=1}^{N} f_i}$$



This shows the relationship between global mean temperature change and total column water vapour change from the surface to the top of the troposphere; they are well correlated. Typically, a 1 °C temperature increase causes about a 7 percent rise in column water vapour. This is strong positive feedback: more water vapour traps more heat. Water vapour is positive feedback; lapse rate change is negative feedback, slightly counteracting water vapour.



Now, let us summarize the various changes. With double CO_2, you get an additional greenhouse forcing of 4 W m^{-2}. So, net radiation changes by 4 W m^{-2}, causing ≈1.1 °C warming. It increases water vapour, adding ≈5 W m^{-2} trapping. It melts snow and ice, adding ≈1 W m^{-2} absorbed heat.

It changes clouds—here +1.5 W m^{-2}. Lapse rate changes give negative feedback. The effect due to clouds and lapse rate cancel roughly. So, the original 4 W m^{-2} is increased by ~6 W m^{-2} from water vapour and clouds, giving ~10 W m^{-2} total, which causes ≈3 °C warming. This shows how feedbacks add or subtract to the initial perturbation. Now, among these, cloud feedback has the largest uncertainty, which we will discuss in detail in the next lecture. Thank you.