ENVIRONMENTAL GEOSCIENCES

Prof. Prasoon Kumar Singh

Department of Environmental Science and Engineering

Indian Institute of Technology (Indian School of Mines), Dhanbad

Lecture-37

Law of Groundwater Movement - Darcy's Law and Applications (Part - 3)

Welcome to the SWAYAM NPTEL course on Environmental Geosciences. We are continuing the module 7. In the module 7, we have discussed the law of groundwater movement, Darcy's law and applications. Today, we will cover the lecture 3, that is law of groundwater movement with Darcy's law and its applications. In this lecture, the important concepts will be covered like permeability of stratified soil, application of Darcy's law, discharge through an aquifer and important terms related to pumping.

Now, permeability of stratified soil. Actually in field, soil is present in the form of different stratum which has different permeabilities. For calculation of discharge, it is necessary to determine an average value of permeability. Generally, three common types of computed averages of permeability are, first is the arithmetic average, second the harmonic average and third the geometric average. Now, the selection of the averaging techniques should be based on primarily on the geometry of the flow system.

First, arithmetic average, it is also known as horizontal flow that is flow parallel to the bedding plane. When the flow is parallel to the strata then the flow is taking place through all the layers at same time and hence hydraulic gradient remains same in each layer. So these are the different different layers so when the parallel flow will take place. This will flow in all the layers at the same time. Now, the arithmetic average can be calculated by this equation, which is the summation of permeability, and your thickness of individual soil strata, hydraulic gradient of individual soil strata. With this equation we can see this one here the parallel flow remains in all the three different strata so this is the example of arithmetic average where we are getting the horizontal flow. Harmonic average it is also known as vertical flow here the flow remains normal to the bedding plane so when the flow is normal to the strata then the flow is taking place through all the layers at the same discharge in each layer.

And with this formula, we can find out the harmonic average. Here it is related with the average permeability can be found out by permeability of individual soil and thickness of individual soil strata. Now, third one is the geometric average for random flow. So for random flow, generally geometric average formula is being considered. And this is the formula which can be find out by knowing the permeability of individual soil strata and thickness of the individual soil strata. So, these are the three different methods through which we can find out the arithmetic average, geometric average and harmonic average. Now, based on these three averages, the numerical we can solve. First numerical is a soil deposit consists of three layers, 0.1 meter, 0.2 meter and 0.15 meter thickness. Now calculate the average value of permeability within the following consideration.

Means the permeability of different layers is given. K1, K2 and K3 are given here. So by using this formula what we have learnt earlier. We will just simplify it in this way. And then we will put the value what we are getting through the numericals.

That these are the values. By solving this one we are getting 0.32 centimeter per second. Now, the second problem we will see is soil deposit consists of three layers of 2 meter, 4 meter, 3 meter thickness with permeability value of 10^{-4}, 10^{-5} and 10^{-6} meter per second respectively. Then find out the horizontal and vertical permeability of the soil deposit. So for horizontal deposits, the average permeability will be, we can find out by this formula,

$$\text{Horizontal deposits average permeability (K)} = \frac{L_1 \times k_1 + L_2 \times k_2 + L_3 \times k_3}{L_1 + L_2 + L_3}$$

$$= \frac{2 \times 10^{-4} + 4 \times 10^{-5} + 3 \times 10^{-6}}{2 + 4 + 3}$$

$$= 2.7 \times 10^{-5} \text{ m/sec}$$

if we will put the value from the numerical which has been given it is simplifying it is coming 2.7 x 10^{-5} meter per second. Whereas vertical deposits average probability will be with the formula

$$\text{Vertical deposits average permeability (K)} = \frac{L_1 + L_2 + L_3}{L_1/k_1 + L_2/k_2 + L_3/k_3}$$

$$= \frac{2 + 4 + 3}{2/10^{-4} + 4/10^{-5} + 3/10^{-6}}$$

$$= 3.7 \times 10^{-4} \text{ m/sec}$$

and if we will put the value and simplify it. It will come near to the 3.7 x 10^{-4} meter per second.

Now third numerical we will see. A three-layered porous medium is subjected to horizontal flow. The thickness and permeability of each layer are 3 meter, 2 meter, 5 meter and permeability are 10 meter per day, 5 meter per day and K_3 meter per day. Now, find the value of K_3, the permeability of the third layer. So, how we will find out?

We will use the formula for equivalent permeability of horizontal flow. Here, because horizontal permeability is being asked, so horizontal flow formula we will take. We have learnt in the previous numerical. We have seen that

$$\text{Equivalent permeability (k)} = \frac{h_1 \times k_1 + h_2 \times k_2 + h_3 \times k_3}{h_1 + h_2 + h_3}$$

$$7.5 = \frac{3 \times 10 + 2 \times 5 + 5 \times k_3}{3 + 2 + 5}$$

$$k_3 = 7 \text{ m/day}$$

We have to find out the k_3 because k_1, k_2 has been given already. h_1, h_2, h_3 has been given. So if we will put the value we will find out the k_3 is 7 meter per day. So in this way we can solve the problem with the help of the formula even. Now fourth one is, A three-layered soil profile consists of the following layers arranged vertically. A constant head difference is applied across the entire soil profile causing vertical flow.

So flow is vertical. The thickness and permeability of each layer are 2 meter, 3 meter and 5 meter thick and permeability are 5 meter per day, 2 meter per day and 1 meter per day. Now we have to calculate the equivalent vertical permeability k_v of the soil profile first. Second if the total head difference across the profile is 10 meter what is the seepage velocity if the porosity of the soil is 30 percent. So these two we have to calculate by the given value so first we will use the formula for equivalent probability in vertical flow. And this is the formula we have learnt in the previous slides. Now just we have to put the value and just to add here.

And here 2/5, 3/2 and 5/1 we have to simplify it, solve it. And it is coming here to this 1. We will solve it. It will come to 1.5 meter per day. So vertical permeability is coming to 1.45 meter per day. Now seepage velocity we have to find out.

So seepage velocity formula we have read this formula also in the previous slides. So

$$v_s == k_v.i/n,$$

where i is the hydraulic gradient. So i is the head difference by total thickness. So it will be near to one and v_s will be 1.45, k_b is 1.45. We have found out in the previous steps.

So,

$$i = \frac{\text{Head Difference}}{\text{Total Thickness}} = \frac{10}{10} = 1$$

$$v_s = \frac{1.45 \times 1}{0.3} = \frac{1.45}{0.3} = 4.83 \text{ m/day}$$

So, it is now into 4.83 meter per day. So, this is the seepage velocity calculation. Now, the application of Darcy law. Darcy law is a fundamental principle in hydrogeology that describes the flow of fluids through porous media. It is used in many areas of science including hydrogeology, fluid dynamics, geophysics, etc.

One application of Darcy law is to flow water through an aquifer. Aquifer we are knowing that any rocky formation which is holding the water inside the surface. So Darcy law with the conservation of mass equation is equivalent to the groundwater flow equation being one of the basic relationships of hydrogeology. Darcy law is also applied to describe oil, gas and water flows through petroleum reservoirs. It helps in designing wells and pumping systems and site suitability analysis for construction projects.

So these are some of the applications of the Darcy's law. Now discharge through an aquifer. Aquifer we are knowing it is a permeable formation which allows a significant quantity of water to move through it under field conditions. Aquifers may be unconfined or confined aquifer. Unconfined aquifer is one in which the groundwater table is the upper surface of the zone of saturation and it lies within the test stratum.

It is also called free, phreatic or non-artesian aquifer. Whereas confined aquifer is one in which groundwater remains entrapped under pressure greater than atmospheric pressure by overlying relatively impermeable strata. So it is also called artesian aquifer. So unconfined aquifer is called as non-artesian aquifer whereas confined aquifer is called as artesian aquifer. Now coefficient of transmissibility is defined as the rate of flow of water through a vertical strip of aquifer of unit width and extending the full height of saturation under unit hydraulic gradient.

So this is the coefficient of transmissibility. This coefficient is obtained by multiplying the field coefficient of permeability by the thickness of the aquifer. When a well is penetrated into a homogeneous aquifer, the water table in the well initially remains horizontal. When water is pumped out from the well, the aquifer gets depleted of water and the water table is lowered resulting in a circular depression in the phreatic surface and this circular depression is generally referred as drawdown curve or cone of depression. The analysis means when it will remain, when the, this will be the water table, water table, if the aquifer is homogeneous.

But as soon as the pumping will take place, means the water will be withdrawn from here. So this level of the water, this level of the water will goes down. In this way it goes down. So cone of depression will be created. This is referred to as drawdown curve or cone of depression.

The analysis of flow towards such a well was given by Dupit in eighteen hundred sixty three and modified by Thiem in eighteen hundred seventy. In pumping out test, drawdowns corresponding to a steady discharge are observed at a number of observation wells. Pumping must continue at an uniform rate for an adequate time to establish a steady state condition in which the drawdown changes negligibly with time. The following assumptions are relevant to the discussion. The first is that the aquifer should be homogeneous with uniform permeability and is of infinite aerial extent.

Second, the flow is laminal and Darcy law is valid for laminar flow only. The flow is horizontal and uniform at all points in the vertical section. The well penetrates the entire thickness of the aquifer. Natural groundwater regime affecting the aquifer remains constant with time and the velocity of flow is proportional to the tangent of the hydraulic gradient that is the Dupit's assumption. Now important terms related to pumping we will see.

Water in an unconfined aquifer being pumped out at a constant rate from the well. So this is the unconfined aquifer you can see here and water is being pumped out from the well. Earlier this was the table, water table. You can see. But this was the level of the water, water table.

But as soon as we have started pumping, the level is gradually going down. You can see here it is going down. Okay, so water in an unconfined aquifer being pumped out at a constant rate from the well. Prior to the pumping, the water level in the well indicates the static water table. So this is the static water table.

First condition is the static water table. A lowering of water level takes place on pumping. If you will start the pumping, the lowering of water table will take place. If the aquifer is homogeneous and isotropic and the water table horizontally initially Due to the radial flow into the well through the aquifer, the water table assumes a conical shape and this conical shape is nothing but it is the cone of depression.

It is called as the cone of depression. You can see the cone of depression in the first month. It will remain up to here. Then second month it is here, third month it is here and fourth month it is here. In this way we can see that the cone of depression is generating while we will start the pumping.

The drop in the water table elevation at any point from its previous static level is called drawdown. So this drop in the water table elevation is called as drawdown. The aerial extent of the cone of depression is called the area or zone of influence and its radial extent is called as radius of influence. So this is the zone of influence. The aerial extent of cone of depression is called as zone of depression.

And its radial extent is called as radius of influence. At constant rate of pumping, the drawdown curve develops gradually with time due to the withdrawal of water from storage. This phase is called an unsteady flow as the water table elevation at a given location near the well changes with time. On prolonged pumping, an equilibrium state is reached between the rate of pumping and the rate of inflow of water from the outer edges of the zone of influence. The drawdown surface attains a constant position with respect to time when the well is known to operate under steady flow conditions.

Unconfined aquifer case you can see here a well penetrating an unconfined aquifer to its full depth. Here this is the diagram you can see. Let R_0 be the radius of the central well. So R_0 is the radius of the central well. R_1 and R_2 are the radial distances from the central well to the two of the observation wells.

So here these two are the observation well. This is one observation well and this is second observation well. Now Z_1 and Z_2 be the corresponding heights of the drawdown curve, drawdown curve, you can see the height drawdown curve. Above the impervious boundary j_0 be the height of water level after pumping in the central well above the impervious boundary. So this is the central well, this one is the central well now, D_0, D_1, D_2 be the depths of water level after pumping. So, D_1, D_2 we are getting the depths of water level after pumping from the initial level of water table or the drawdowns at the central well and the two observation well respectively.

h be the initial height of the water table above the impervious layer, that is, you can see, h is equal to z_0 plus d_0, r be the radius of influence or the radial distance from the central well of the point where the drawdown curve meets the original water table. So, this is the original water table, and this is the Drawdown curve. This one is the central well, whereas this one is the observation wells. Now, let r and z be the radial distance and the height above the impervious boundary at any point on Drawdown curve. So, the, by Darcy's law, the discharge q is given by

$$q = k.A.dz/dr$$

Since the hydraulic gradient i is given by dz/dr by Dupuit's assumption, here k is the coefficient of permeability and area is equal to 2pi.r.z, then this q will become this much.

And further simplification of this equation, we can get this value. Integrating the limits r_1 and r_2 for r and z_1 and z_2 for z, we are getting, just by integration you can see, we are getting this much equation. Now, in the case of confined aquifer, a well penetrating a confined aquifer to its full depth, here, This is the case of confined aquifer. In the confined aquifer, we are having the impermeable stratum at the down as well as at the top.

So here we can see the h_w value. Then r_1, r_2 is also mentioned here. h_1, h_2 is given. And this is the observation wells. These two are the observation wells. This one and this one is the observation well, whereas this one is the central well.

And this curve is the drawdown curve. This curve is the drawdown curve. The notation in this case is precisely the same as that in the case of unconfined aquifer. H indicates the thickness of confined aquifer bounded by impervious strata. By Darcy Law, discharge q is, we have seen, this is the formula of the discharge q. But the cylindrical surface area of flow is given by

$$A = 2pi.r.H.$$

So here, we will just put the value and we will get the discharge formula in this way. And ultimately, in this way, we can write the formula.

In integrating again both sides we will get this much of the permeability value. Now we will solve the problem. First problem is for a well founded in confined aquifer. So this we have to keep in memory that which type of aquifer. It is confined aquifer.

Drawdown is doubled, the discharge will become, how much it will become? Question is, if the drawdown will double, what will be the discharge? So, as per Dupuit's formula, we are knowing the equation, this equation. Then, here the difference (z_2 - z_1) represents the drawdown in well and it is directly proportional to discharge. Hence, q is directly proportional to (z_2 - z_1).

Now, if the drawdown is doubled, the discharge q will also be doubled. So, this is the answer of this problem. Second problem is, an unconfined aquifer has upper boundary at a depth of thirty meter and a lower boundary at a depth of ten meter. The discharge rate through the aquifer is 500 meter cube per day. The radius of outer boundary r_2 is hundred meter and the radius of inner boundary r_1 is 10 meter.

Then calculate the hydraulic conductivity of the aquifer. So again we will put the Dupuit formula. We have seen this one for unconfined aquifer. So data is also given in the new problem. That is q value given to 500 meter per day discharge rate.

Then upper boundary depth and lower boundary depth is also given. z_2 is 30 meter. z_1 is 10 meter. Outer radius is also given 100 meter and inner radius is given 10 meter. Now we will put the value.

By putting the value in this formula, we are getting this much first. And then by solving this one, this will be the next value. And again by solving this one, this is coming to 0.458 meter per day. So, this is the way through which we can find out the k value. Now this is another problem.

Here we see it is a case of confined aquifer. 50 meter thick and extends over an area of 2 square kilometer. A fully penetrating well is drilled into the aquifer and is pumped at a steady rate of 800 cubic meter per day. The following observations are made from two observation wells, which is located four meter and twelve hundred meter from the pumping well. Drawdown in the first observation well has been noticed to 3.5 meter, whereas drawdown in the second observation well, it was noticed that it is 1.2 meter.

Now what we have to determine? We have to determine the permeability of the aquifer and the transmissivity of the aquifer. So, again we will take the help of the Dupuit formula where we will see this one is the formula we have discussed already. We have to find out the permeability. So, this value q value is given eight hundred kilometer per day that is the charge value.

s_2 that is the drawdown in the second observation well one point two meter. s_1 3.5 meter in the first observation well drawdown. r_2 is the distance to second observation well, 1200 meter. r_1, 400 meter distance to first observation well. Aquifer thickness is also given, v value is given, 30 meter.

Area of aquifer is also given, that is 2 square kilometers. Now, by putting these values in this formula, Dupuit formula, we can solve to this much and then and again by solving this it will come k value will come to 1.218 meter per day second we have to find out the transmissivity of the confined aquifer so this formula we are knowing aquifer thickness and your permeability value. So just we will put the value 1.18 with coming here and 50 it is given there. So it is coming to 60.9 meter square per day. So in this way we can find out the permeability as well as transmissivity of any confined aquifer.

Now summarizing the lecture what we have discussed. First thing we have discussed about the permeability of stratified soil. Actually in field soil is present in the form of different stratum which has different permeability also. Three common types of computed average permeability we have seen in the slides.

First is the arithmetic average, second the harmonic average and third the geometric average.

Secondly, we have discussed about the application of Darcy law in which we have discussed that Darcy law is a fundamental principle in hydrogeology that describes the flow of fluids through porous media or aquifer. Darcy law is also applied to describe oil, gas and water flows through petroleum reservoirs as well.

Thirdly, we have discussed about the discharge to an aquifer in which we have seen that when water is pumped out from any well, the aquifer gets depleted of water and the water table is lowered, resulting in a circular depression and in the phreatic surface, that is in the water table, we are getting a circular depression. The analysis of flow towards such a well was given by Dupuit, that is in 1863, and modified by Thiem in 1870.

We have also discussed some of the important terms which is related to pumping, that is static water table, then the cone of depressions, then the area, zone of influence and the radius of influence.

Thank you very much to all.