ENVIRONMENTAL GEOSCIENCES

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Lecture-35

Law of Groundwater Movement - Darcy's Law and Applications (Part-1)

Welcome to the SWAYAM NPTEL course on Environmental Geosciences. Now today we will start the module seven in which we will learn about the law of groundwater movement, that is Darcy Law and its applications, groundwater fluctuations, pollution of groundwater resources. Today we will cover the lecture one that is law of groundwater movement, Darcy's Law and Applications. In this lecture, the important concepts will be covered like water movement, various parameters of groundwater flow and factors affecting permeability of soils. So we have seen in the previous module also that water needs energy to make it move against the friction offered by the particles and the narrow circuitous route that the subsoil water had to follow.

One way of expressing the energy of water is to measure the height to which it can raise itself above an arbitrary given level or datum and this height is generally known as the hydraulic head. In the diagram also, you can see the hydraulic head is there in tube B and tube A both. Consider the arrangement in figure. You can see tube B contains more water than tube A. So here more water we are seeing in tube A and here less water we are seeing.

But if we open valve C, suppose if we will open this valve, then what will happen? Water will flow from A to B. Water will flow from A to B until the water level in both the tubes becomes at the same height above the datum level. So the head difference can be a result of pressure difference as well as result of a difference in elevation. These two are generally designated as pressure head or elevation head. So the pressure at any depth h, below the surface of a liquid is higher by an amount p, than the atmospheric pressure acting on the liquid surface. Therefore,

$$p = \rho g h$$

Therefore,

$$\text{Pressure head } (h) = p/\rho g$$

So, in the same figure, you can see what we have seen in the earlier figure. The pressure head is more in this one, less in B tube. But if we will open the valve C, then both the tube will have the same height of the liquid. So the pressure energy and elevation energy are both forms of potential energy. These are the forms of potential energy which general term is energy which a body possesses by virtue of its position or state.

Elevation energy is potential energy possessed by virtue of its position. Pressure energy is potential energy possessed by virtue of state analogous to the energy of a compressed spring. There is a third way in which water can possess energy by virtue of movement. This energy is called as kinetic energy and the contribution which it makes to the total hydraulic head is called the velocity head or dynamic head. Now,

Consider a horizontal long straight length of pipe whose cross section is the same throughout its length. So a pipe is here whose cross section is same throughout its length. Near each pipe a manometer for measurement of pressure is fitted. So each pipe we are fitting the manometer for measurement of pressure. Now a steady flow of water passes through the pipe.

We observe the water level in the manometers. We see that the water level in the manometer near to the outflow end of the pipe is lower than the level in the manometer at the inflow end. So this we are seeing. Now the reduction in the head is the result of decrease in pressure. Part of the energy is converted into the heat as a result of friction between the water molecules to a lesser extent and between the water and the surface of the pipe.

The fall in head between the two manometer in the figure is commonly referred as head loss. It is referred as head loss. Now, the faster the velocity. More energy is dissipated as heat and greater the head loss along the pipe. Head has units of length and when the head loss between the ends of the pipe is divided by the length of the pipe we get the hydraulic gradient. So in this way we are finding the hydraulic gradient. Groundwater flow in the subsurface is driven by differences in energy and as water flows from the high energy areas to low energy areas, so the mechanical energy of a unit volume of water is determined by the sum of gravitational, potential energy, pressure energy and kinetic energy. So we can calculate energy per unit weight as

$$\text{Energy per unit weight(Head)} = \frac{V^2}{2g} + \frac{P}{\rho g} + Z$$

where Z is the elevation of the measuring point relative to the datum. This one is z. P is fluid pressure at the measurement point. Then ρ is fluid density, and g is the gravitational acceleration whereas V is the fluid viscosity. So here we are getting you can see the total head time is denoted by this level. where velocity head is denoted by this much difference and this one is the pressure head, which is generally determined by P divided by ρ into g and this line is showing the hydraulic head of this structure and this is the elevation. This one is the elevation. So we are seeing that energy per unit weight can be determined, head can be determined with the help of elevation of the measuring point, that is z, then p, fluid pressure, ρ, fluid density, g, gravitational acceleration, and v, fluid velocity.

Now, because the groundwater flows very slowly, on the order of one meter per day or less, its kinetic energy is very small relative to its gravitational potential and pressure energies, and the kinetic energy term is therefore ignored. By removing the kinetic energy term and rearranging the equation to express energy in terms of mechanical energy per unit weight, the concept of hydraulic head is developed. Therefore, energy per unit weight is equal to hydraulic head that is z plus p divided by ρ into g. Ground water therefore flows from regions of high hydraulic head areas to areas of low hydraulic head.

Because groundwater flows through a porous media, the rate of flow depends on soil property, such as degree to which pore spaces are interconnected. Groundwater flows from areas of hydraulic head, that is a measure of pressure and gravitational energy, toward areas of lower hydraulic head. The rate of change, slope of the hydraulic head is known as hydraulic gradient. If groundwater is flowing and contains dissolved contaminants, it can transport the contaminants by advection from areas with high hydraulic head toward lower hydraulic head zones or down gradient. So here you can see the higher pressure and elevation, In this one, whereas here we are seeing the, we are seeing the lower pressure and elevation. Hydraulic head is the difference of S divided by difference of L, capital L.

Now parameters of groundwater flow. Velocity of groundwater flow is the first parameter it is the rate of flow of water that is equal to hydraulic conductivity and hydraulic gradient which is the difference of water levels with respect to distance of the two wells. This is expressed generally in meter per day.

$$V = k.i$$

where,

k = hydraulic conductivity

i = hydraulic gradient $\left(\frac{h_2 - h_1}{l}\right)$

h_1 = water level at piezometer-1

h_2 = water level at piezometer-2

l = distance between two piezometers

Second parameter is the travelling time of water. It is the ratio of travelling distance of water or length to the velocity of the groundwater flow which is expressed in here.

That is, you can see the formula

$$T_w = \frac{L}{v} \times n$$

Where,

L = travelling distance of water or length(km)

v = velocity of groundwater flow(m/day)

n = porosity of aquifer material

Next parameter is related to the actual velocity of tracer through aquifer. It is the travel distance of tracer with respect to time between the two wells which is expressed in meter per day or meter per hour. The expression is

$$V_a = \frac{r}{t}$$

Where,

r = travel distance of tracer between two wells(m)

t = required time for tracer to travel between two wells(min or hour)

Now, based on this, one problem is here.

If travel distance of tracer between the two wells is fifteen meter and required time for tracer to travel between the two wells is three hours, then find the actual velocity of the tracer. So, with respect to this formula,

$$V_a = \frac{r}{t}$$

where r is the travel distance of tracer between the two wells, that is given 15 meter, and t is the time required for tracer to travel between the two wells, or in meter or hour, is given 3 hours. Then we have to find out V_a, that is the actual velocity of the tracer, is

$$V_a = \frac{15}{3}$$
$$V_a = 5 \text{ m/hour}$$

Second problem we can see here,

The flow of groundwater is in longitudinal direction in an alluvial valley of unconfined aquifer. Hydraulic conductivity of the aquifer material is 30 meter per day. Two piezometers are located at a distance of 500 meter apart from the central line of the valley. The water level in the piezometer-1, which is located at an upstream side, is 1.0 meter and is 1.5 meter in the piezometer-2, which is located at downstream side from the ground surface.

Now to find out what is the velocity of the groundwater flow.

Now from the question we can see that the distance between the two piezometer is 500 meter. Hydraulic conductivity of the aquifer material is given 30 meter per day. Now difference of water levels between two piezometer is 0.5 meter that is (1.5-1). It is coming to 0.5 meter. Now on this formula

$$V = k \times i$$
$$V = k \times \left(\frac{h_2 - h_1}{l}\right) \quad \text{where, } k = \text{hydraulic conductivity, } i = \text{hydraulic gradient } \left(\frac{h_2 - h_1}{l}\right)$$
$$V = 30 \times \frac{1.5 - 1.0}{500}$$
$$V = 0.03 \text{ m/day}$$

So this is the actual velocity of ground water flow between these two piezometers.

Now look the problem three.

The flow of groundwater is in longitudinal direction in an alluvial valley of unconfined aquifer. Hydraulic conductivity of the aquifer material is 30 meter per day. Two piezometers are located at a distance of 500 meter apart from the center line of the valley. The water level in the piezometer one which is located at the upstream side is 1 meter and is 1.5 meter in the piezometer two that is located at downstream side. Then, if the porosity of the aquifer material is 30 %, compute the travelling time of water from the head of the valley to the point of 15 kilometer downstream.

This is the question. Now solution, we can see here this is the expression formula from where we can find out the travelling time that is where L is the travelling distance of water or length in kilometer, small v is the velocity of groundwater flow in meter per day, 0.03 meter per day, as calculated earlier in the previous numerical, now n is equal to porosity of aquifer material. Now, with the formula

$$T_w = \frac{L}{v} \times n$$ Where,

L= travelling distance of water or length(km)

v= velocity of groundwater flow(m/day) = 0.03 m/day, as calculated earlier

n= porosity of aquifer material

$$T_w = \frac{15000}{0.03} \times \frac{30}{100} \times \frac{1}{365}$$, since unit of T_w is calculated in Year

T_w=410.96 or 411 years

Now factors affecting permeability of soils. So following are the factors affecting the permeability of soils.

The first factor is the size of soil particle.

Second is the specific surface area of soil particle.

Third is the shape of the soil particle.

Fourth, the void ratio.

Fifth, the soil structure.

Sixth, the degree of saturation.

Seventh, the water properties.

Eighth the temperature,

Ninth the adsorbed water and

Ten the organic water.

So these are the factors which are affecting the permeability of the soil.

Permeability means ability of movement of water in the soils.

Now first factor is the size of soil particles. So permeability varies according to the size of soil particle. If the soil is coarse grained, permeability will be more and if it is fine-grained, permeability will be less.

The relation between coefficient of permeability k and particle size of the soil can be shown from the equation:

$$k \ \alpha \ D^2$$

So this is the relation between permeability and the particle size. Second important factor is the specific surface area of particles. Specific surface area of soil particles also affects the permeability. The specific surface area of soil sample is the total surface area contained in a unit mass of soil.

Soils with high specific surface areas have high water holding capacities. More adsorption of contaminants and greater soil potentials. So, therefore, the higher the specific surface area, lower will be the permeability. Formula is.

$$k \ \alpha \ \frac{1}{Specific\ Surface\ Area}$$

Third is the shape of the soil particle. Rounded particles will have more permeability than the angular shaped particles. It is due to the specific surface area of angular particles is more compared to the rounded particles. Next is the void ratio. In general, permeability increases with void ratio, but it is not applicable to all types of soils. For example, clay has high void ratio than any other types of soil but permeability for clays is very low.

This is due to the flow path through voids in case of clays is extremely small such that water cannot permit through this path easily.

$$k \ \alpha \ \frac{e^3}{1+e}$$

Next is the soil structure. Structure of any two similar soil masses at same void ratio need not be same. It varies according to the level of compaction applied.

If a soil contains flocculated structure, the particles are in random orientation and permeability is more in this case. Here you can see the flocculated structure. Particles are random in orientation. But if the soil contains dispersed structure, the particles are in face to face orientation and hence permeability is very low. The permeability of stratified soil deposits also varies according to the flow direction.

If the flow is parallel, permeability is more. If it is perpendicular, permeability is less. So this is the example of the dispersed condition. Now degree of saturation. Partially saturated soil contain air voids, which are formed due to entrapped air or gas released from the percolating fluid or water.

This air will block the flow path thereby reduces the permeability. Fully saturated soil is more permeable than the partially saturated soil. Next is the water properties. Various properties of water or fluid such as unit weight and viscosity also affects the permeability. However, unit weight of water will not affect much since it does not change much with temperature.

But when temperature is increased, viscosity decreases rapidly. From equation, you can see the permeability increases when viscosity decreases. It will increase when this will decrease. Next is the temperature. Temperature also affects the permeability in soils.

Permeability is inversely proportional to the viscosity of the fluid. It is known that viscosity varies inversely to the temperature. Hence permeability is directly related to temperature. Here permeability is directly related to temperature. Greater the temperature, higher will be the permeability.

That is the reason seepage is more in summer seasons than in winter. Next is the adsorbed water. Absorbed water is the water layer formed around the soil particle especially in the case of fine-grained soils. This reduces the size of the void space by about ten percent. Hence, the permeability reduces.

Next is the organic matter. Presence of organic matter decreases the permeability. This is due to the blockage of voids by the organic matter. So, based on the factors affecting the permeability of the soil, let us see some problems. So, the first problem is, a soil sample has a coefficient of permeability k, 2.5×10^{-4} centimeter per second. When the average particle size is D, is equal to 0.08 centimeter. Assume

$$k \propto D^2$$

then calculate the permeability when the average particle size is

$$D = 0.06 \text{ cm}$$

of soil sample. So this is the problem. Here we have to find out the permeability k when the average particle size is given.

Solution: For the first soil sample:

$$k_1 = 2.5\times10^{-4}\frac{cm}{s} \text{ and } D_1 = 0.08\,cm$$

$$k \propto D^2, \text{so } k = C \cdot D^2$$

Using k 1 and D_1:

$$\frac{k_1}{k_2} = \frac{D_1}{D_2}$$

$$\frac{2.5\times10-^4\,cm/s}{k_2} = \frac{0.08\,cm}{0.06\,cm}$$

Thus, by solving the equation for permeability becomes:

$$k = 1.875\ x10^{-4}\,cm/s$$

So, in this way, we can solve the simple numerical when the coefficient of permeability and the average particle size is given in the problem.

Now let us see the second problem,

here a fine sand sample has specific surface area of s_1 = 100 cm-square per gram and its permeability is measured as $k_1 = 2.0 \times 10^{-4}$ centimeter per second. Now a clay sample has a specific surface area of s_2 1200 centimeter square per gram, then calculate the permeability of the clay sample. So, two different specific surface area has been given and the fine sand sample k permeability value is also given here.

We have to find out the permeability value of the clay sample.

Solution: For the first soil sample:

$$k_1 = 2.0 \times 10^{-4}\,cm/s \text{ and } S_1 = 100\,cm^2/g$$

$$k \propto \frac{1}{S}, \text{so } k = \frac{C}{S}$$

Using k 1 and S_1:

$$\frac{k_2}{k_1} = \frac{S_1}{S_2}$$

$$\frac{k_2}{2.0\times10^{-4}cm/s} = \frac{100\,cm^2/g}{1200\,cm^2/g}$$

Thus, by solving the equation for permeability becomes:

$$k = 1.67\ x10^{-5}cm/s$$

Now, this is the third type of problem on the basis of the factors affecting the permeability of the soil. A fine soil sample having volume of void 4 centimeter cube and the volume of soil solids is equal to 10 centimeter cube. Then its permeability is measured as K_1 is equal to 2.0×10^{-4} centimeter per second. Now calculate the permeability K_2 of the fine soil sample if the volume of void is increased to value of eight centimeter cube. Means from 4 centimeter cube, the void volume is increasing to 8 centimeter cube. So, we have to find out the permeability value. Now, for the first soil sample, K_1 is given, that is 2.0×10^{-4} centimeter per second. Now, void ratio relationship we have seen in the earlier part. So volume of void divided by volume of soil solids.

Volume of void is given four centimeter cube and volume of soil solid is also given 10 centimeter cube. It is coming to 0.4. The question is that volume of void of the next sample, that is volume of void by volume of solids we will do, it is coming to 8 centimeter cube, value given eight centimeter cube and this will remain the same for the soil solid that is the ten centimeter cube. So it is coming to 0.8. By using the relation which we have discussed earlier,

By using this relation $k \propto \frac{e^3}{1+e}$, and using k_1, e_1 and e_2:

$$\frac{k_2}{k_1} = \frac{e_2^3 x (1+e_1)}{e_1^3 x (1+e_2)}$$

$$\frac{k_2}{2.0\times10^{-4} cm/s} = \frac{0.8^3 \text{ x } (1+0.4)}{0.4^3 x (1+0.8)}$$

$$k_2 = \frac{0.8^3 \text{ x } (1.4)}{0.4^3 x (1.8)} \text{ x } 2.0\times10^{-4} cm/s$$

Thus, by solving the equation for permeability becomes:

$k_2 = 1.6 \times 10^{-4}$ cm/s

In this way, we can measure the permeability when the void ratio is given.

Now, the next problem is,

Problem 4: At T_1 =27° C, the permeability of soil is $k_1 = 2.5 \times 10^{-4}$ cm/s, and the viscosity of water is $\mu_1 = 0.89$ mPa.s. At T_2 =60° C, the viscosity of water is $\mu_2 = 0.47$ mPa.s. Find the permeability of the soil (k_2) at T_2 =60° C.

So the first temperature value is 27°C, second is the 60°C. First given value is given here, viscosity of both the condition is given here. We have to find out the permeability that is the k_2 value.

Solution: Using the relationship:

$$k_2 / k_1 = \mu_1 / \mu_2$$

Rearranging for k 2 :

$$k_2 = k_1 \text{ x } (\mu_1 / \mu_2)$$

Substitute the given values:

$$k_2 = (2.5\times10^{-4}) \times (0.89/0.47)$$

Simplify:

$$k_2 = (2.5\times10^{-4}) \times 1.8936$$

$$k_2 = 4.734\times10^{-4} \text{ cm/s.}$$

Now let us summarize the chapter. Here we have seen the law of groundwater movement. That is, the water movement, first we have seen, water needs energy to make it to move against the friction offered by the particles and the narrow circuitous route that the subsoil water had to follow. Ground flow in the subsurface is driven by energy in energy as water flows from high energy areas to low energy. Groundwater flows from areas of higher hydraulic heights as a measure of pressure and gravitational energy toward areas of lower hydraulic head. Secondly, we have discussed the parameters of groundwater flow in which we have discussed the velocity of groundwater flow and travel in time of water and then the actual velocity of pressure through the aquifer.

And lastly, we have discussed the factors affecting the permeability of soils. We have discussed about the size of soil particle, specific surface area of soil particle, shape of soil particle, void ratio, soil structure, degree of saturation, water properties, temperature, adsorbed water, and organic matter. So, this is all about the summary of this lecture one. Thank you very much to all.